

Homotopy Analysis Method for Hydromagnetic Plane and Axisymmetric Stagnation-point Flow with Velocity Slip

Jing Zhu, Liancun Zheng and Xinxin Zhang

Abstract—This work is focused on the steady boundary layer flow near the forward stagnation point of plane and axisymmetric bodies towards a stretching sheet. The no slip condition on the solid boundary is replaced by the partial slip condition. The analytical solutions for the velocity distributions are obtained for the various values of the ratio of free stream velocity and stretching velocity, slip parameter, the suction and injection velocity parameter, magnetic parameter and dimensionality index parameter in the series forms with the help of homotopy analysis method (HAM). Convergence of the series is explicitly discussed. Results show that the flow and the skin friction coefficient depend heavily on the velocity slip factor. In addition, the effects of all the parameters mentioned above were more pronounced for plane flows than for axisymmetric flows.

Keywords—slip flow, axisymmetric flow, homotopy analysis method, stagnation-point.

I. INTRODUCTION

STAGNATION flow, describing the fluid motion near the stagnation region, exists on all solid bodies moving in a fluid. There have been considerable interests in investigating plane and axisymmetric flow near a stagnation point on a surface. Hiemenz[1] was the first to discover that the stagnation point flow can be analyzed exactly by the Navier-Stokes equations and he reported two-dimensional plane flow velocity distribution. Later, Chiam[2] investigated two dimensional normal and oblique stagnation-point flow of an incompressible viscous fluid towards a stretching surface while Mahapatra and Gupta[3] studied the heat transfer of normal stagnation flow to a stretching sheet. Recently Anu-ar Ishak et al[4] investigated mixed convection flow near a stagnation point on a vertical surface.

In all the above mentioned studies no attention has been given to the effects of partial slip on the flow. The no-slip boundary condition is known as the central tenets of the Navier-Stokes theory. However, there are situations wherein such condition is not appropriate. Partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions[5]. Effects of slip conditions are very important for

some fluids which exhibit wall slip. Fluids exhibiting slip are important in technological applications such as in the polishing of artificial heart valves and internal cavities. Therefore better understanding of the phenomenon of slip is necessary. Mooney[6] initiated the study of boundary layer flow with partial slip, many researchers[7-8] had confirmed the phenomenon of wall-slip fluid. Hayat and Masood[9] examined the effect of the slip boundary condition on the flow of fluids in a channel. The non-Newtonian flows with wall slip have been studied numerically in Refs[10-12].

The stagnation slip flow on a fixed plate and on a moving one was considered by wang[13-14]. The present paper extends the results of previous authors by considering the effect of velocity slip. The method we employed here is based on the homotopy analytical method(HAM[15]) of solving non-linear equations which has already been applied to some other problems[16-18].

II. MATHEMATICAL FORMULATION

Consider the steady, two-dimensional flow of a laminar, viscous and incompressible, electrically conducting fluid near the stagnation point of a flat sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. x and y are the Cartesian coordinates with the origin at the stagnation point along and normal to the plate, respectively. A uniform magnetic field is applied in the y -direction causing a flow resistive force in the x -direction. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field will be neglected. Under these conditions and taking into account the boundary layer approximation, the system of continuity and momentum can be written as:

$$\frac{\partial}{\partial x}(x^k u) + \frac{\partial}{\partial y}(x^k v) = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x} \left(\frac{u^k}{x^k} \right) + \frac{\partial^2 u}{\partial y^2} \right) + \frac{\sigma B_0^2}{\rho} (u_e - u), \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{k}{x} \frac{\partial v}{\partial x} + \frac{\partial^2 v}{\partial y^2} \right). \quad (3)$$

Subject to boundary conditions

$$u(x, 0) = cx + \frac{2 - \sigma_v}{\sigma_v} \lambda_0 \frac{\partial u}{\partial y} \Big|_{y=0}, \quad (4)$$

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$$v(x,0) = -v_w, u(x,\infty) = u_e = ax. \quad (5)$$

Where k is the index, with $k=1$, Eqs.(1-5) is axially symmetric stagnation-point flow, while with $k=0$, it is the plane flow. x -axis is the tangential direction and x is interpreted as the radial direction for axisymmetric flow situations. u and v are the velocity components along the x -axes and y -axes, respectively. ρ is the density, ν is the kinematic viscosity, σ is the fluid electrical conductivity, B_0 is the magnetic induction. λ_0 is the mean free path and σ_v is the tangential momentum accommodation coefficient.

Near the sheet, the stream function for the viscous flow far from the sheet is

$$\psi(x,y) = \frac{x^{k+1}}{k+1} F(y).$$

The velocity components are then

$$u = \frac{1}{x^k} \frac{\partial \psi}{\partial y} = \frac{x}{k+1} F'(y), v = -\frac{1}{x^k} \frac{\partial \psi}{\partial x} = -F(y). \quad (6)$$

Substituting (6) into Eq.(2), the x -momentum equation then gives

$$\frac{1}{\rho} \frac{\partial P}{\partial x} = x \left(\nu \frac{F'''}{k+1} + \frac{FF''}{k+1} - \frac{(F')^2}{(k+1)^2} + \frac{\sigma B_0^2}{\rho} \left(a - \frac{F'}{k+1} \right) \right), \quad (7)$$

Partial integration of (7) yields

$$P_0 - P = \frac{1}{2} \rho K(y) + \frac{1}{2} \rho x^2 \left(\nu \frac{F'''}{k+1} + \frac{FF''}{k+1} - \frac{(F')^2}{(k+1)^2} + \frac{\sigma B_0^2}{\rho} \left(a - \frac{F'}{k+1} \right) \right). \quad (8)$$

Where P_0 is the stagnation pressure. When (8) are inserted in the y -momentum equation, we obtain

$$FF' = \frac{1}{2} K'(y) - \nu F'' + \frac{1}{2} x^2 \left(\nu \frac{F'''}{k+1} + \frac{FF''}{k+1} - \frac{(F')^2}{(k+1)^2} + \frac{\sigma B_0^2}{\rho} \left(a - \frac{F'}{k+1} \right) \right). \quad (9)$$

This equation cannot be true for arbitrary x and y unless

$$\nu \frac{F'''}{k+1} + \frac{FF''}{k+1} - \frac{(F')^2}{(k+1)^2} + \frac{\sigma B_0^2}{\rho} \left(a - \frac{F'}{k+1} \right) = H, \quad (10)$$

where H is a constant. When (10) is evaluated as $y \rightarrow \infty$ with the assumption that $F''(\infty) = F'''(\infty) = 0$ along with $F'(\infty) = (k+1)a$, we find that $H = -a^2$.

Further, introducing the following dimensionless quantities and transformations

$$f(\eta) = \frac{F(y)}{((k+1)c\nu)^{\frac{1}{2}}}, \quad \eta = y \left(\frac{(k+1)c}{\nu} \right)^{\frac{1}{2}}. \quad (11)$$

When (11) are inserted in Eq. (10), we obtain

$$f''' + ff'' - n(f')^2 + nd^2 - nM(f' - d) = 0. \quad (12)$$

The boundary conditions (4-5) may be expressed in dimensionless form as

$$f(0) = R, f'(0) = 1 + \lambda f''(0), f'(\infty) = d. \quad (13)$$

Where the local Knudsen number Kn_x , the local Reynolds number Re_x , velocity slip parameter λ , the Hartmann number M , the suction/injection velocity parameter R ,

velocity ratio parameter d and the dimensionality index n are defined respectively, as:

$$Kn_x = \frac{\lambda_0}{\sqrt{d}x}, Re_x = \frac{u_e x}{\nu}, \lambda = \frac{2 - \sigma_v}{\sigma_v} Kn_x Re_x^{\frac{1}{2}},$$

$$n = \frac{1}{1+k}, M = \frac{\sigma B_0^2}{c\rho}, R = \frac{v_w}{\sqrt{a\nu}}, d = \frac{a}{c}.$$

Important physical parameter for this flow is the skin friction coefficient. It is defined as follows:

$$C_f = \frac{2\sqrt{k+1}f''(0)}{\sqrt{Re_x}}.$$

III. HAM SOLUTION FOR $f(\eta)$

A. Zeroth-order deformation equations

Under the first rule of solution expression, the initial guess approximation for HAM solution is

$$f_0(\eta) = R + d\eta + \frac{(1-d)\eta e^{-\eta}}{1+2\lambda} \quad (14)$$

and the auxiliary linear operator is

$$L_f(f) = f''' + f''.$$

The operator in above equation satisfies

$$L_f[C_1 + C_2\eta + C_3e^{-\eta}] = 0,$$

in which $C_i, i=1,2,3,4,5$ are arbitrary constants.

The zeroth order deformation problem is

$$(1-q)L_f[F(\eta;q) - f_0(\eta)] = qh_f N_f[F(\eta;q)], \quad (15)$$

$$F(0,q) = R, F'(0,q) = 1 + \lambda F''(0,q), F'(\infty,q) = d. \quad (16)$$

Where $N_f[F(\eta;q)]$ is

$$N_f[F(\eta;q)] = \frac{\partial^3 F}{\partial \eta^3} + F \frac{\partial^2 F}{\partial \eta^2} - n \left(\frac{\partial F}{\partial \eta} \right)^2 + nd^2 - nM \left(\frac{\partial F}{\partial \eta} - d \right).$$

In above equations $q \in [0,1]$ is the embedding parameter, h_f is auxiliary non-zero parameter. Due to Taylor's theorem, one can write

$$F = f_0(\eta) + \sum_{m=1}^{+\infty} f_m(\eta) q^m, f_m(\eta) = \frac{1}{m!} \frac{\partial^m F(\eta;q)}{\partial \eta^m} \Big|_{q=0}. \quad (17)$$

B. High-order deformation equations

Differentiating the zeroth order deformation (15-16) k times with respect to q and then dividing by $k!$, and finally setting $q=0$. We get the following k th-order deformation equation

$$L_f[f_k(\eta) - \chi_k f_{k-1}(\eta)] = h_f R_{fk}(\vec{f}_{k-1}), \quad (18)$$

$$f_k(0) = f_k'(\infty) = 0, f_k'(0) = \lambda f_k''(0). \quad (19)$$

Where

$$R_{fk}(\vec{f}_{k-1}) = \sum_{s=0}^{k-1} \left(f_s(\eta) f_{k-1-s}''(\eta) - n f_s'(\eta) f_{k-1-s}'(\eta) \right) + f_{k-1}'''(\eta) - n M f_{k-1}'(\eta) + n(1 - \chi_k)(d^2 + dM),$$

and

$$\chi_k = \begin{cases} 0 & k=1 \\ 1 & k>1 \end{cases}.$$

C. Recursive formulae

We have the solution of problem as

$$f_m(\eta) = \sum_{k=0}^{m+1} \sum_{i=0}^{2m+2-k} a_{m,k}^i \eta^i \exp(-k\eta), \quad (20)$$

Substituting (20) into (18-19), the recurrence formulae for the coefficients $a_{m,k}^i$ of $f_m(\eta)$ are obtained for $m \geq 1$:

$$\begin{aligned} a_{m,0}^0 &= \chi_m a_{m-1,0}^0 + \sum_{k=2}^{m+1} \sum_{q=0}^{2m+2-k} (k+k^2\lambda-1) \mu_{k,0}^q \Omega_{m,k}^q \\ &\quad - \sum_{q=0}^{2m} (\lambda+1) \mu_{1,1}^q \Omega_{m,1}^q + \sum_{k=2}^{m+1} \sum_{q=2}^{2m+2-k} \mu_{k,2}^q \Omega_{m,k}^q \\ &\quad - \sum_{k=2}^{m+1} \sum_{q=1}^{2m+2-k} (1+2k\lambda) \mu_{k,1}^q \Omega_{m,k}^q - \sum_{q=1}^{2m} 2\lambda \mu_{1,2}^q \Omega_{m,1}^q, \\ a_{m,0}^i &= 0, \quad 1 \leq i \leq 2m+2, \\ a_{m,1}^0 &= \chi_m a_{m-1,1}^0 - \sum_{k=2}^{m+1} \sum_{q=0}^{2m+2-k} (k+\lambda k^2) \mu_{k,0}^q \Omega_{m,k}^q \\ &\quad + \sum_{k=2}^{m+1} \sum_{q=1}^{2m+2-k} (1+2k\lambda) \mu_{k,1}^q \Omega_{m,k}^q + \sum_{q=0}^{2m} (1+\lambda) \mu_{1,1}^q \Omega_{m,1}^q \\ &\quad - \sum_{k=2}^{m+1} \sum_{q=2}^{2m+2-k} 2\lambda \mu_{k,0}^q \Omega_{m,k}^q + \sum_{q=1}^{2m} 2\lambda \mu_{1,2}^q \Omega_{m,1}^q, \\ a_{m,1}^i &= \chi_m a_{m-1,1}^i + \sum_{q=i-1}^{2m} \mu_{1,i}^q \Omega_{m,1}^q, \quad 1 \leq i \leq 2m-1, \\ a_{m,1}^i &= \sum_{q=i-1}^{2m} \mu_{1,i}^q \Omega_{m,1}^q, \quad 2m \leq i \leq 2m+1, \\ a_{m,k}^0 &= \chi_m a_{m-1,k}^0 + \sum_{q=0}^{2m+2-k} \mu_{k,i}^q \Omega_{m,k}^q, \quad 2 \leq k \leq m, \\ a_{m,k}^i &= \chi_m a_{m-1,k}^i + \sum_{q=i}^{2m+2-k} \mu_{k,i}^q \Omega_{m,k}^q, \quad 2 \leq k \leq m, \quad 1 \leq i \leq 2m-k, \\ a_{m,k}^i &= \sum_{q=i}^{2m+2-k} \mu_{k,i}^q \Omega_{m,k}^q, \quad 2 \leq k \leq m, \quad 2m+1-k \leq i \leq 2m+2-k, \\ a_{m,m+1}^i &= \sum_{q=i}^{m+1} \mu_{m+1,i}^q \Omega_{m,m+1}^q. \end{aligned}$$

Where

$$\begin{aligned} \mu_{k,i}^u &= \frac{i!(u-i+2)}{u!}, \quad k=1, 0 \leq i \leq u+1, \\ \mu_{k,i}^u &= \frac{u!}{i!(k-1)^{u-i+1}} \left\{ 1 - \left(\frac{1}{n} \right)^{u-i+1} \left[(u-i+2) - (u-i+1) \left(\frac{1}{n} \right) \right] \right\}, \\ &\quad (k \geq 2, 0 \leq i \leq u) \\ \Omega_{m,1}^i &= h_f (e_{m-1,1}^i - n M c_{m-1,1}^i) + (\delta_{m,1}^i + n \Delta_{m,1}^i), \quad 0 \leq i \leq 2m-1, \\ \Omega_{m,1}^{2m} &= \delta_{m,1}^{2m} + n \Delta_{m,1}^{2m}, \quad \Omega_{m,m+1}^i = \delta_{m,m+1}^i + n \Delta_{m,m+1}^i, \quad 0 \leq i \leq m+1, \\ \Omega_{m,k}^i &= h_f (e_{m-1,k}^i - n M c_{m-1,k}^i) + \delta_{m,k}^i + n \Delta_{m,k}^i, \quad 0 \leq i \leq 2m-k, 2 \leq k \leq m, \\ \Omega_{m,k}^i &= \delta_{m,k}^i + n \Delta_{m,k}^i, \quad 2m+1-k \leq i \leq 2m+2-k, \quad 2 \leq k \leq m. \end{aligned}$$

The coefficients $\delta_{m,k}^i$ and $\Delta_{m,k}^i$ for $m \geq 1$ are

$$\begin{aligned} \delta_{m,k}^i &= \sum_{s=0}^{m-1} \sum_{r=\max\{1,k+s-m\}}^{\min\{s+1,k-1\}} \sum_{t=\max\{0,i+2s+k-r-2m\}}^{\min\{2s+2-r,i\}} a_{m-1,s,k-r}^{i-t} d_{s,r}^t, \\ \Delta_{m,k}^i &= \sum_{s=0}^{m-1} \sum_{r=\max\{1,k+s-m\}}^{\min\{s+1,k-1\}} \sum_{t=\max\{0,i+2s+k-r-2m\}}^{\min\{2s+2-r,i\}} c_{m-1,s,k-r}^{i-t} c_{s,r}^t, \end{aligned}$$

TABLE I COMPARISON OF WANG^[13] OF WITH THE HAM SOLUTION FOR $R=0.0$, $M=0.0$, $D=0.0$, $N=1.0$

λ	WANG ^[13]	HAM
0.1	-1.000	-1.00026
0.3	-0.701	-0.70776
1.0	-0.430	-0.44031
2.0	-0.284	-0.28690
5.0	-0.145	-0.15111
20.0	-0.0438	-0.04678

and the coefficients $c_{m,k}^i$, $e_{m,k}^i$ and $d_{m,k}^i$ are

$$\begin{aligned} c_{m,k}^i &= (i+1) a_{m,k}^{i+1} \lambda_{m,k}^{i+1} - k a_{m,k}^i \lambda_{m,k}^i, \\ e_{m,k}^i &= (i+1) d_{m,k}^{i+1} \lambda_{m,k}^{i+1} - k d_{m,k}^i \lambda_{m,k}^i, \\ d_{m,k}^i &= (i+1)(i+2) a_{m,k}^{i+2} \lambda_{m,k}^{i+2} - 2k(i+1) a_{m,k}^{i+1} \lambda_{m,k}^{i+1} + k^2 a_{m,k}^i \lambda_{m,k}^i, \\ \lambda_{m,k}^i &= \begin{cases} 0 & i=j=0, k \geq 2 \text{ or } i>0, j=0, k \geq 1, \\ 0 & j>i+1 \text{ or } k>2(i+1)-j, \\ 1 & \text{otherwise.} \end{cases} \end{aligned}$$

Using the above recurrence formulae, we can calculate all coefficients $a_{m,k}^i$ by using only the first three

$$a_{0,0}^0 = R, \quad a_{0,0}^1 = d, \quad a_{0,1}^1 = \frac{1-d}{1+2\lambda}$$

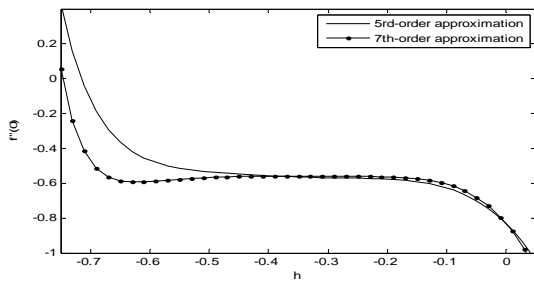
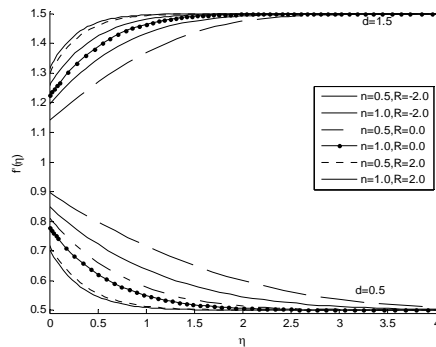
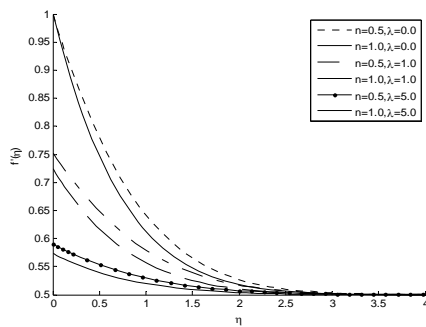
given by the initial approximation (14). Therefore, the following explicit, totally analytic solutions of the present flow is

$$f(\eta) = \lim_{N \rightarrow \infty} \left(\sum_{m=0}^N a_{m,0}^0 + \sum_{k=1}^{N+1} \sum_{m=k-1}^{2N} \sum_{i=0}^{2m+1-k} a_{m,k}^i \eta^i \exp(-k\eta) \right).$$

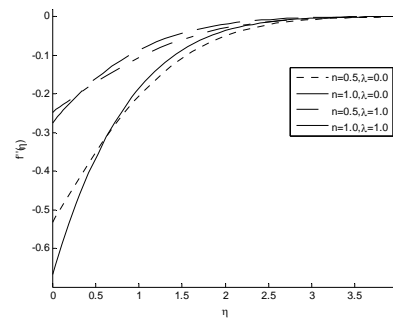
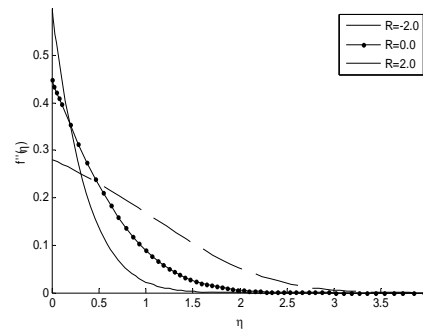
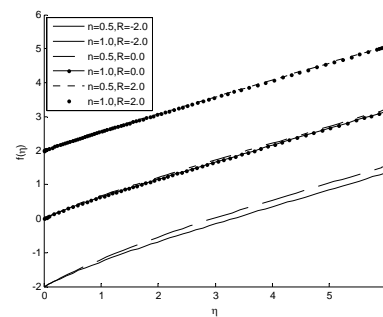
IV. RESULTS AND DISCUSSION

The convergence and rate of approximation for the HAM solution strongly depend on the values of auxiliary parameter h_f . To see the admissible values of h_f , h-curve is plotted in Fig.1. Fig.1 clearly elucidates that the range for the admissible values is $-0.5 \leq h_f \leq -0.1$. It is founded that our analytic approximations for $h_f = -0.35$ agree well with the results of wang[13], as shown in Table.1.

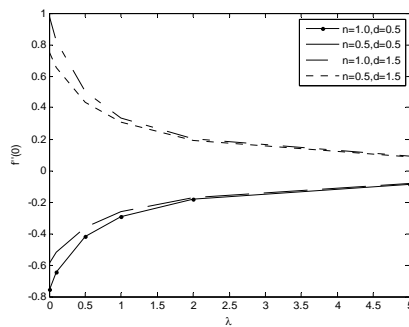
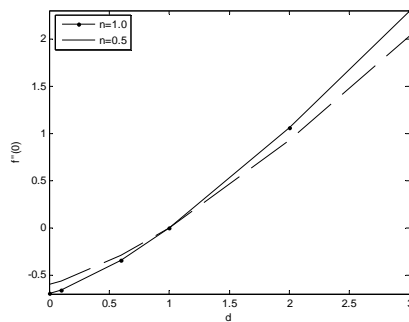
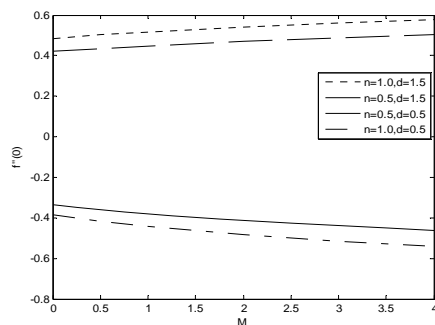
Figs 2-4 present representative profiles for the tangential velocity $f'(\eta)$ and shear stress profile $f''(\eta)$ of both plane and axisymmetric flows for various slip factor λ and velocity ratio parameter d , respectively. Fig.2 shows that the flow has a boundary layer when $d > 1$. Further the thickness of the boundary layer decreases with increase in $d > 1$. On the other hand an inverted boundary layer is formed when $d < 1$. Slip velocity has the tendency to warm up and slow down the movement of the fluid. This effect is depicted in Figs. 3-4. The effect of λ both on the tangential velocity and the shear stresses depends on d . For $d > 1$, increasing λ increases $f'(\eta)$ and decreases $f''(\eta)$, while for $d < 1$ increasing λ decreases $f'(\eta)$ and increases $f''(\eta)$. When $\lambda \rightarrow \infty$ (full slip) the solution is the potential flow $f(\eta) = d\eta + R$. These features are more pronounced for plane flows.

Fig. 1 The h_f curve for $f''(0)$ when $\lambda = 0.1, d = 0.5, M = 0.0, R = 0.0$.Fig. 2 Velocity profiles $f'(\eta)$ for different values of R and $M = 1.0$, $\lambda = 0.5$.Fig. 3 Velocity profiles $f'(\eta)$ for different values of λ and $M = 0.5$, $R = 0.0, d = 0.5$.

In Fig. 2 and Figs. 5-6, the influence of suction ($R < 0$) and injection ($R > 0$) are illustrated on the velocity profiles $f(\eta), f'(\eta)$ and shear stress profile $f''(\eta)$, respectively. The variations of $f'(\eta)$ and $f''(\eta)$ with $\lambda = 0.5$ and $M = 1.0$ are shown in Fig. 2 and Fig. 5, respectively. It is seen that the effects of parameters R on the velocity $f'(\eta)$ and shear stress $f''(\eta)$ are similar to those of the slip parameter. The variation of $f(\eta)$ with $\lambda = 0.5, d = 0.5$ and $M = 1.0$ is shown in Fig. 6. It is noted that the variation of $f(\eta)$ increases with increasing of parameter R for the each fixed value of parameter η .

Fig. 4 Effects of λ on $f''(\eta)$ at $M = 0.5, R = 0.0, d = 0.5$.Fig. 5 Effects of R on $f''(\eta)$ at $M = 1.0, \lambda = 0.5, d = 1.5, n = 0.5$.Fig. 6 Variation of $f(\eta)$ with R for $M = 1.0, \lambda = 0.5, d = 0.5$.

Finally, we compute the dimensionless shear stress at the wall for the various parameters involved in the problem. Effect of slip parameter λ on $f''(0)$ depends on d as shown in Fig. 7. It can be seen that when $d < 1$, the wall shear $f''(0)$ increase with increase in λ . But when $d > 1$, $f''(0)$ decreases with increase in λ . Fig. 8 shows that the magnitude of $f''(0)$ increases with increasing in d which is consistent with the fact that there is progressive thinning of the boundary layer with increase in d . Application of a magnetic field has the tendency to warm up and slow down the movement of the fluid. This effect is depicted by the increases in the values of $|f''(0)|$, as shown in Table 2 and Fig. 9.

Fig. 7 Effects of λ on $f''(0)$ at $M=0.5, R=0.0$.Fig. 8 Effects of d on $f''(0)$ at $M=0.5, R=0.0, \lambda=0.5$ Fig. 9 Effects of M on $f''(0)$ at $R=0.0, \lambda=0.5$

V. CONCLUSIONS

The present paper studies the plane and axisymmetric stagnation flows towards a stretching sheet with the slip boundary conditions. Exact similarity solutions are obtained for momentum equations. The resulting equation system is then solved analytically by using HAM. The obtained results were presented graphically to elucidate interesting features of the solutions. A boundary layer is formed when the stretching velocity is less than the free stream velocity and an inverted boundary layer is formed when the stretching velocity exceeds the free stream velocity. The flow due to the lateral motion of the plate depends heavily on the velocity slip factor, both on the flow field and the shear stresses through the stagnation flow.

Shear stress at the surface $f''(0)$ increases with increase in d . Also, effect of increasing values of λ is to decrease the variation of $|f''(0)|$ and the surface Shear stress $|f''(0)|$ is close

to 0 with $\lambda \rightarrow \infty$. The variation of $|f''(0)|$ increases with increasing of parameter M . The effects of all the parameters mentioned above were more pronounced for plane flows than for axisymmetric flows. It is hoped that the results obtained in this paper be of use for understanding of more complicated problems involving stagnation-point slip flows.

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