

# Fuzzy Control of a Three Phase Thyristorized Induction Motor

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**Abstract**— Nowadays the control of stator voltage at a constant frequency is one of the traditional and low expense methods in order to control the speed of induction motors near its nominal speed. The torque of induction motor is a nonlinear function of the firing angle, phase angle and speed. In this paper the speed control of induction motor regarding various load torque and under different conditions will be investigated based on a fuzzy controller with inverse training.

**Keywords**— Three phase induction motor, AC converter, speed control, fuzzy control.

## I. INTRODUCTION

VOLTAGE control at constant frequency is used increasingly in order to speed controlling of a three phase induction motor at low and medium powers especially when the load torque is proportional to the square of motor speed [1]. In this method the voltage of the stator is controlled between zero and its maximum value by controlling the firing angle of thyristors symmetrically. Although the speed control of induction motors by means of variable voltage and constant frequency seems to be simple but its needs to some complex analysis. Two main problems will be encountered in analyzing the behavior of induction motor by this method [2]:

- 1) The differential equation that describes the motor's behavior is completely nonlinear.
- 2) Determination of initial and bound conditions is very difficult.

In this paper by applying a variable controlled voltage through a three phase chopper upon the circuit's model of induction motor, the instant and effective values of currents and voltages are computed. It will be depicted that the torque of such a motor in the steady state is a nonlinear function of firing angle, phase angle and rotation speed. Hence achieving the produced torque for a specific firing angle and speed depends on solving nonlinear equation and calculating complex integrals that are time consuming and complicated. This problem appears when it is required to control the speed of the motor due to variation of load torque, by adjusting the firing angle of thyristor.

In the proposed method, in this paper, the induction motor plus to AC converter is considered as a nonlinear system that receives the firing angle and load torque as inputs and produces the speed as output. The proposed fuzzy controller indeed will be an inverse fuzzy model of the thyristorized induction motor with torque and speed as its inputs and the firing angle as its output.

## II. MODELING

In most of studies, the system of thyristorized induction motor is considered as shown in Fig. 1 [1].

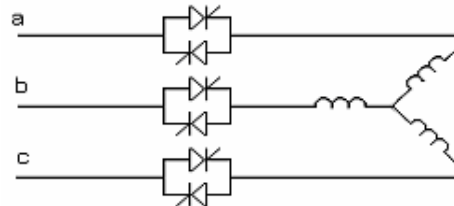


Fig. 1: The model of induction motor-AC converter.

In this system, ignoring the effect of saturation and core dissipation, it is possible to consider the induction motor as a linear circuit. In such case the equivalent circuit is considered as R-L load [2]. Fig. 2 depicts the single phase equivalent circuit of such system.

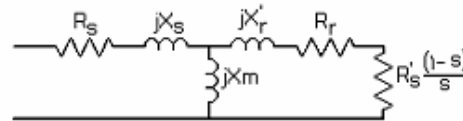


Fig. 2: The 1-phase equivalent circuit of 3-phase induction motor.

The parameters of the depicted system in Fig. 2 are:

- $R_s$ : The equivalent stator resistance  
 $R_r'$ : The equivalent rotor resistance referred to stator side  
 $X_s$ : The equivalent stator reactance  
 $X_r'$ : The equivalent rotor reactance referred to stator side  
 $X_m$ : The magnetic reactance  
 $s$ : The slip

The equivalent impedance of this circuit can be written as follows:

$$Z = \frac{\left[ DR_s + X_m \frac{R_r'}{s} (X_m + X_r') - X_m X_r' \frac{R_r'}{s} \right] + j \left[ DX_s + X_m \frac{R_r'^2}{s^2} + X_m X_r' (X_m + X_r') \right]}{D} \quad (1)$$

Where:

$$D = (X_m + X_r')^2 + \left( \frac{R_r'}{s} \right)^2 \quad (2)$$

The operational mode and consequently the analysis of the

operation of the induction motor controlled by thyristorized voltage depends on the proportional values of thyristor's firing angle ( $\alpha$ ) and the phase angle ( $\varphi$ ). Where  $\varphi$  is the impedance angle (phase angle) that depends on the slip and other parameters of the motor. When the firing angle of thyristors is less than its minimum value ( $\alpha_{\min}$ ) the natural sinusoidal operation will be attained. By gradually increasing the firing angle  $\alpha$  in its permissible area at a specified slip, the system will begin to work in mode 2/3. If  $\alpha$  increases so that it exceeds a specific critical value ( $\alpha_{\text{cr}}$ ), then system will enter mode 0/3 [3]. Regarding that the main area of controlling the speed of induction motor due to variation of the load is in mode 2/3 and the resultant torque in mode 0/3 is negligible, in this work mode 2/3 is considered as control range.

$$\alpha_{\min} \leq \alpha \leq \alpha_{\text{cr}} \quad \alpha_{\min} = \varphi \quad (3)$$

$$\alpha_{\text{cr}} = \tan^{-1} \left[ \frac{\cos \varphi + \sin \left( \varphi - \frac{\pi}{6} \right) e^{p \frac{\pi}{3}}}{\sin \varphi + \cos \left( \varphi - \frac{\pi}{6} \right) e^{p \frac{\pi}{3}}} \right] \quad (4)$$

$$p = -\cot(\varphi)$$

In mode 2/3 the stator current of phase a, is obtained by:

$$i_a(t) = \begin{cases} i_1(t) & \alpha \leq \omega t < \beta + \frac{\pi}{3} \\ i_2(t) & \beta + \frac{\pi}{3} \leq \omega t < \alpha + \frac{\pi}{3} \\ i_3(t) & \alpha + \frac{\pi}{3} \leq \omega t < \beta + \frac{2\pi}{3} \\ i_4(t) & \beta + \frac{2\pi}{3} \leq \omega t < \alpha + \frac{2\pi}{3} \\ i_5(t) & \alpha + \frac{2\pi}{3} \leq \omega t < \beta + \pi \end{cases} \quad (5)$$

Where:

$$\begin{aligned} i_1(t) &= I_m \left[ \sin(\omega t - \varphi) + A_1 e^{p\omega t} \right] \\ i_2(t) &= I_m \left[ \frac{\sqrt{3}}{2} \sin \left( \omega t + \frac{\pi}{6} - \varphi \right) + A_2 e^{p\omega t} \right] \\ i_3(t) &= I_m \left[ \sin(\omega t - \varphi) + A_3 e^{p\omega t} \right] \\ i_4(t) &= I_m \left[ \frac{\sqrt{3}}{2} \sin \left( \omega t - \frac{\pi}{6} - \varphi \right) + A_4 e^{p\omega t} \right] \\ i_5(t) &= I_m \left[ \sin(\omega t - \varphi) + A_5 e^{p\omega t} \right] \end{aligned} \quad (6)$$

$$I_m = \frac{V_m}{|Z|}$$

$$A_1 = -\sin(\alpha - \varphi) e^{-p\alpha}$$

$$A_2 = A_1 + \frac{1}{2} \sin(\beta - \varphi) e^{-p \left( \beta + \frac{\pi}{3} \right)} \quad (7)$$

$$A_3 = A_2 - \frac{1}{2} \sin(\alpha - \varphi) e^{-p \left( \alpha + \frac{\pi}{3} \right)}$$

$$A_4 = A_3 - \frac{1}{2} \sin(\beta - \varphi) e^{-p \left( \beta + \frac{2\pi}{3} \right)}$$

$$A_5 = A_4 + \frac{1}{2} \sin(\alpha - \varphi) e^{-p \left( \alpha + \frac{2\pi}{3} \right)}$$

$$\sin(\varphi - \beta) = A_6 \sin(\alpha - \varphi) e^{p(\beta - \alpha)} \quad (8)$$

$$A_6 = \frac{2e^{p\pi} + e^{2p\frac{\pi}{3}} - e^{p\frac{\pi}{3}}}{2 - e^{2p\frac{\pi}{3}} + e^{p\frac{\pi}{3}}} \quad (9)$$

Solving the nonlinear equation (8) for each  $\varphi$  and  $\alpha$ , the excitation angle ( $\beta$ ) is calculated and based on it, the current of stator will be obtained as:

$$\begin{aligned} I_{RMS} &= \left[ \frac{1}{T} \int_{\alpha}^{\alpha+T} i_s^2(t) dt \right]^{\frac{1}{2}} \\ &= \left[ \frac{1}{\pi} \left\{ \int_{\alpha}^{\beta+\frac{\pi}{3}} i_1^2(t) d\omega t + \int_{\beta+\frac{\pi}{3}}^{\alpha+\frac{\pi}{3}} i_2^2(t) d\omega t + \right. \right. \\ &\quad \left. \int_{\alpha+\frac{\pi}{3}}^{\beta+\frac{2\pi}{3}} i_3^2(t) d\omega t + \int_{\beta+\frac{2\pi}{3}}^{\alpha+\frac{2\pi}{3}} i_4^2(t) d\omega t + \right. \\ &\quad \left. \left. \int_{\alpha+\frac{2\pi}{3}}^{\beta+\pi} i_5^2(t) d\omega t \right\} \right]^{\frac{1}{2}} \end{aligned} \quad (10)$$

It is seen that the value of each phase current is a nonlinear function of the phase angle ( $\varphi$ ), motor parameters, the firing angle ( $\alpha$ ) and the excitation angle ( $\beta$ ). After calculating the effective value of current in each phase of stator, the effective value of rotor current can be obtained by current dividing rule. Then the produced torque is calculated as follows:

$$T = \frac{3}{2} \frac{P}{\omega} R_r' \frac{I_r^2}{s} \quad (11)$$

T: The torque of motor

$\omega$ : The angular speed of motor

$I_r$ : Effective value of the rotor current

In order to demonstrate the relation of current and torque with firing angle a typical induction motor with following parameters is considered:

$P=4$

$V_m=380$

$f=50\text{HZ}$

$R_s=38.7\Omega$

$R_r'=25\Omega$

$X_s=25\Omega$

$X_r'=25\Omega$

$X_m=500\Omega$

By solving the previous mentioned equations and integrals for different firing angles the results shown by Figs. 3 to 5 are obtained.

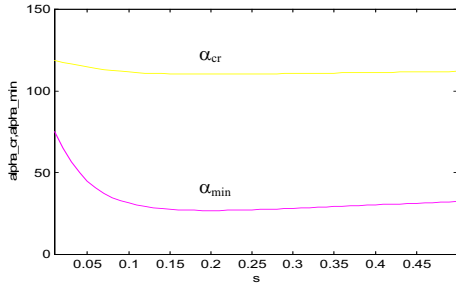


Fig. 3: Variations of  $\alpha_{cr}$  and  $\alpha_{min}$  versus the slip of motor.

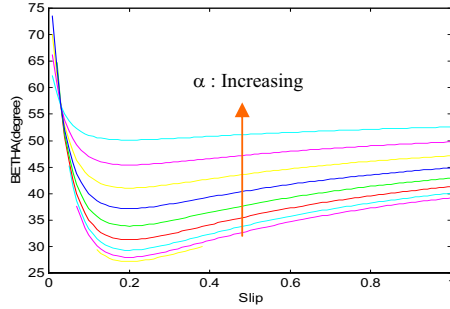


Fig. 4: Variations of  $\beta$  versus the slip of motor for different  $\alpha$ s.

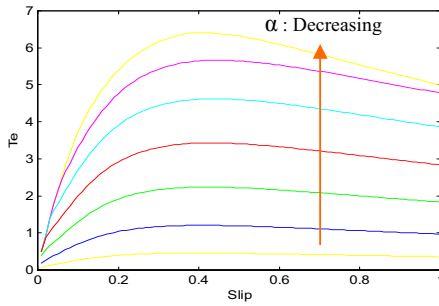


Fig. 5: Variations of torque versus the slip of motor for different  $\alpha$ s.

Fig. 3 depicts the variations of  $\alpha_{cr}$  and  $\alpha_{min}$  by the motor slip in mode 2/3. Fig. 4 represents the variation of  $\beta$  based on slip in several firing angles. In Fig. 5 the variations of produced torque base on slip in several firing angles are demonstrated. From the last figure we find the nonlinear relation between torque, speed and firing angle. Further more, considering Fig. 5 we know that the controlling area of motor speed is in the negative region of characteristic curve of torque speed. Regarding the nonlinearity nature of the relations we use a fuzzy controller with the aim of controlling the speed of motor.

### III. FUZZY CONTROLLER [4]

The bloke diagram of proposed control system is brought in Fig. 6.

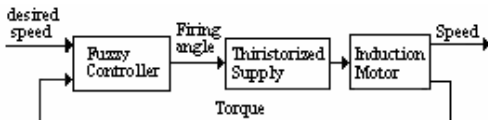


Fig. 6: The block diagram of proposed control system.

In this work a first order TSK (Takagi, Sugeno, Kang) fuzzy system is used. It has two fuzzy inputs: speed and torque and one output: firing angle. Each of the fuzzy inputs is divided to some fuzzy partitions and each fuzzy partition is specified with its own membership function. If speed input ( $S$ ) is divided to  $m$  fuzzy partitions and torque input ( $T$ ) is divided to  $n$  fuzzy partitions, assuming that  $A_i$  ( $i=1, 2, \dots, m$ ) is  $i$ th fuzzy partition of the speed with membership function  $a_i(S)$  and  $B_j$  ( $j=1, 2, \dots, n$ ) is  $j$ th fuzzy partition of torque with membership function of  $b_j(T)$ , then the fuzzy system will have at most  $m \times n$  rules where the  $l$ th rule is:

$$R_l: \text{if } s \in A_i \ \& \ T \in B_j \ \text{then } \alpha = K_{l1}s + K_{l2}T + K_{l3} \quad (12)$$

In fuzzy partitioning of inputs it is required to determine the type and specifics of any membership function. Here gaussian membership functions are considered for both fuzzy inputs.

$$a_i(S) = e^{-\frac{1}{2} \left( \frac{S-S_i}{\sigma_i} \right)^2} \quad i:1\dots m \quad (13)$$

$$b_j(T) = e^{-\frac{1}{2} \left( \frac{T-T_{ji}}{\sigma_j} \right)^2} \quad j:1\dots n$$

For  $m+n$  membership functions of inputs there are  $2(m+n)$  unknown parameters and for  $m \times n$  rules we have  $2(m \times n)$  unknown factors. Consequently  $2(m+n) + 2(m \times n)$  unknown parameters exist for the fuzzy system. After constructing fuzzy system by considering multiplication as AND operator and fuzzy implicant and Max as OR operator and fuzzy aggregator, the firing angle can be produced for each couple ( $T, S$ ). In order to produce the crisp output ( $\alpha^*$ ) the weighted average defuzzification method is used:

$$\alpha^* = \frac{\sum_{l=1}^{mn} \beta_l [K_{l1}S^* + K_{l2}T^* + K_{l3}]}{\sum_{l=1}^{mn} \beta_l} \quad (14)$$

Where  $\beta_l$  is the firing intensity of  $l$ th rule and is computed as:

$$\beta_l = a_i(S^*) \cdot b_j(T^*) \quad (15)$$

For training the fuzzy system its unknown parameters must be determined. To do this the data that are gathered experimentally or by ordinary calculations will be used. The data are arranged in such case that the speed and torque are considered as inputs and the firing angle as output. Using this arranged data and in order to train the fuzzy system, the least square method is used. In order to obtain the best result we can repeat the routine for different partitions for each of inputs. In this work it was seen that 4 partitions for speed and 3 partitions for torque take the best result. The obtained membership functions are shown in Figs. 7 and 8.

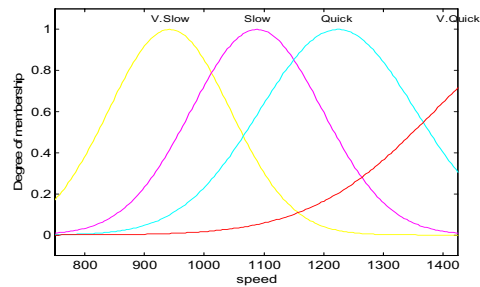


Fig. 7: Member ship functions of the speed.

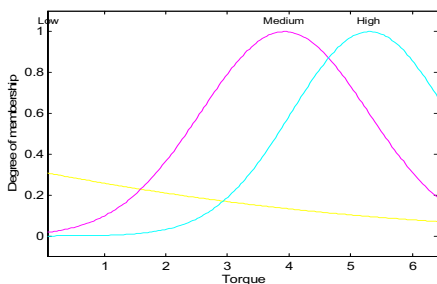


Fig.8: Membership functions of the torque.

For ensuring the validity of resultant fuzzy system, it can be tested by some specific data. The results of this process are brought in Figs. 9 and 10.

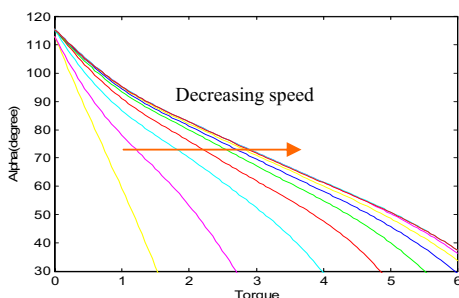


Fig. 9: The output of fuzzy controller against torque for some speeds.

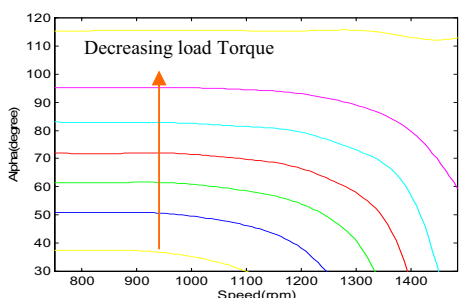


Fig. 10: The output of fuzzy controller against speed for some torques.

#### IV. FUZZY CONTROL OF THYRISTORIZED INDUCTION MOTOR

The prepared fuzzy controller can be used for controlling the induction motor. Here the fuzzy controller based on the block diagram of Fig. 11 is used for this purpose.

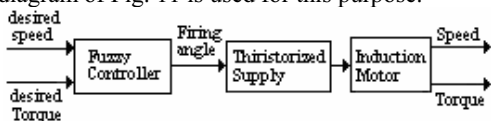


Fig. 11: The block diagram of simulated system.

The operation of controlled system is verified by different speed and torque. The results are shown in Fig. 12. As it is seen in Fig. 12, in different situations, for feasible demanded speeds the fuzzy controller will be able to produce the proper firing angle even if the load torque is varied in its respective range. In the case where a demanded speed-torque characteristic doesn't exist, the controller will represent the nearest response which adapts with the motor's maximum power capability.

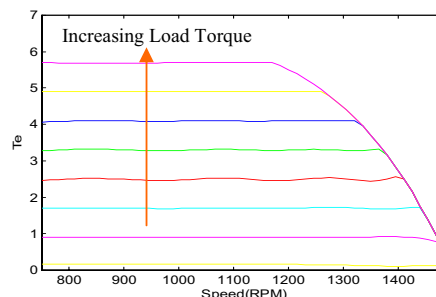


Fig. 12: Variation of electrical torque versus speed for different load torques.

#### V. CONCLUSION

In this paper the performances of the induction motor with thyristorized supply is studied. Regarding the nonlinear relation of the produced torque with the motor parameters and firing angle, a fuzzy controller is designed to control the motor speed. It was shown that the fuzzy controller can control the motor in different situations.

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