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Dynamic Analysis of Porous Media using Finite Element Method

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***Abstract*—**The mechanical behavior of porous media is governed by the interaction between its solid skeleton and the fluid existing inside its pores. The interaction occurs through the interface of grains and fluid. The traditional analysis methods of porous media, based on the effective stress and Darcy's law, are unable to account for these interactions. For an accurate analysis, the porous media is represented in a fluid-filled porous solid on the basis of the Biot theory of wave propagation in poroelastic media. In Biot formulation, the equations of motion of the soil mixture are coupled with the global mass balance equations to describe the realistic behavior of porous media. Because of irregular geometry, the domain is generally treated as an assemblage of finite elements. In this investigation, the numerical formulation for the field equations governing the dynamic response of fluid-saturated porous media is analyzed and employed for the study of transient wave motion. A finite element model is developed and implemented into a computer code called DYNAPM for dynamic analysis of porous media. The weighted residual method with 8-node elements is used for developing of a finite element model and the analysis is carried out in the time domain considering the dynamic excitation and gravity loading. Newmark time integration scheme is developed to solve the time-discretized equations which are an unconditionally stable implicit method. Finally, some numerical examples are presented to show the accuracy and capability of developed model for a wide variety of behaviors of porous media.

***Keywords*—** Dynamic analysis, Interaction, Porous media, time domain

I. Introduction

The dynamic response of fluid-saturated porous media during earthquakes has been extensively studied in recent years. The investigation of wave motion in fluid-saturated porous media is attracting more attention because of its significance in a great number of practical engineering problems. A porous medium is an assemblage of the solid particles forming a skeleton whose voids are filled with fluid. The analysis of porous media requires a rigorous procedure that can properly characterize the interactions. The first continuum theory of porous media was developed by Biot and used to describe the behavior of porous media saturated by a fluid [1]. This theory was later generalized to determine the finite deformations of saturated porous media [2]. At a later stage the mixture theory restricted by the volume fraction concept provided a new basis for such coupled phenomena. A survey of the historical development of the porous media theory and a discussion of inconsistencies implicit in the mixture theory has recently been given by de Boer [6]. His work ends with an introduction of the model developed by de Boer [7].

Most of the problems of the two-phase behavior of a fluid-saturated porous medium can only be predicted quantitatively by elaborate numerical computation, which is possible today due to the development of powerful computers.

Only a few analytical solutions are available, for example de Boer *et al.* [8]. These analytical solutions have been used to verify the numerical results based on the same theory in a former investigation. For practical work with arbitrary boundary condition we must use the finite element method.

In this paper, the governing equations of motion of the soil mixture are coupled with the global mass balance equations and necessary assumptions are made to obtain the equivalent Biot's equations from the general balance equations. The $u_s - P - u_f$ formulation is used in the finite element spatial discretization, where u_s, P and u_f denote solid skeleton displacement, pore water pressure and fluid displacement, respectively. Obtained results show the capabilities of the proposed formulation on pore water pressure generation and strength loss occurred in loose granular soil deposits under seismic loading. The computed results show good agreement in comparison with the experimental data. It is concluded that the developed Biot formulation and computational procedure are an effective means to assess the dynamic deformation and induced hydrodynamic pressure of saturated sediment under dynamic loading. The presentation may also be used to make a critical comparison between various numerical and analytical results, as well as to provide an alternative understanding of the mechanism of wave propagation in fluid-saturated porous materials.

II. Field Equation for Saturated Soils

The governing equations for sediment domain are defined as following:

Mass conservation for the mixture:

$$\frac{n}{\beta}\frac{\partial P}{\partial t} + (1-n)(\nabla . V_s) + n(\nabla . V_f) = 0 \quad (1)$$

Balance of linear momentum in the solid phase:

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$$(1-n)\rho_s \frac{\partial V_s}{\partial t} + (1-n)\rho_s V_s.\nabla V_s - (1-n)\rho_s b_s + F_s - \nabla.\tau'_s + (1-n)\nabla P = 0 \quad (2)$$

Balance of linear momentum in the fluid phase:

$$n\rho_f \frac{\partial V_f}{\partial t} + n\rho_f V_f.\nabla V_f - n\rho_f b_f + F_f + n\nabla P = 0 \quad (3)$$

A linear constitutive relation for the interacting forces can be used for the soil. This constitutive relation can be selected as Darcy's law of flow. The balance of linear momentum for the solid and fluid phase can be written as:

$$(1-n)\rho_s \ddot{u}_s + (1-n)\rho_s(\dot{u}_s.\nabla \dot{u}_s) - (1-n)\rho_s b_s + \frac{n^2\rho_f g}{k_f}(\dot{u}_s - \dot{u}_f) - \nabla\tau'_s + (1-n)\nabla P = 0 \quad (4)$$

$$n\rho_f \ddot{u}_f + n\rho_f \dot{u}_f.\nabla \dot{u}_f - n\rho_f b_f - \frac{n^2\rho_f g}{k_f}(\dot{u}_s - \dot{u}_f) + n\nabla P = 0 \quad (5)$$

In the above equations, n is the soil porosity, ρ_S and ρ_f are the density of soil and water, F_s and F_f are the interaction forces, b_s and b_f are the global forces related to the body forces exerted on the solid and fluid phases, respectively, β is the bulk modulus of the fluid, k_f is the coefficient of permeability, P is the pore pressure magnitude, $\ddot{u}_s$ and $\ddot{u}_f$ are the soil and water accelerations, $\dot{u}_s$ and $\dot{u}_f$ are the soil and water velocities, u_s and u_f are the displacements of soil and water phases and τ'_s is the effective stress.

III. FEM Formulation of Governing Equations

The governing equations of motion for the solid and fluid phases and the mass balance of the mixture can be expressed on the matrix form, which is convenient for finite element coding. The velocities and accelerations are expressed as the first and second derivatives of displacements, respectively.
By combining the equations (1), (4) and (5), a system of discretized governing equations in the following form will be obtained:

$$[M]\{a\} + [C]\{v\} + [K]\{d\} = \{F\} \quad (6)$$

Where $[M]$ is the generalized mass matrix, $[C]$ is the generalized damping matrix, $[K]$ is the generalized stiffness matrix, $\{F\}$ is the force vector and $\{a\}$, $\{v\}$ and $\{d\}$ are the generalized acceleration, velocity and displacement vectors, respectively. Here, the term displacement vector is used for $\{d\}$ even though it contains displacements and pressures.

In the recent equation, $\{a\} = \begin{Bmatrix} \ddot{\hat{u}}_s \\ \ddot{\hat{P}} \\ \ddot{\hat{u}}_f \end{Bmatrix}$, $\{v\} = \begin{Bmatrix} \dot{\hat{u}}_s \\ \dot{\hat{P}} \\ \dot{\hat{u}}_f \end{Bmatrix}$, $\{d\} = \begin{Bmatrix} \hat{u}_s \\ \hat{P} \\ \hat{u}_f \end{Bmatrix}$ and $\{F\} = \begin{Bmatrix} F_{sb} + F_{ss} - F_{sP} \\ 0 \\ F_{fb} - F_{fP} \end{Bmatrix}$.

Generalized matrices are defined as follows:

$$[M] = \begin{bmatrix} M_{ss} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_{ff} \end{bmatrix} \quad (7)$$

$$[C] = \begin{bmatrix} R_{ss} + C_{ss} & 0 & -C_{sf} \\ S_{Ps} & Q_P & S_{Pf} \\ -C_{fs} & 0 & R_{ff} + C_{ff} \end{bmatrix} \quad (8)$$

$$[K] = \begin{bmatrix} K_{ss} & -S_{sP} & 0 \\ 0 & 0 & 0 \\ 0 & -S_{fP} & 0 \end{bmatrix} \quad (9)$$

In the above equations, the components of matrixes are defined as follows:

$$F_{sb} = \sum_{e=1}^{nes} \int_{\Omega_s^e} N_s^{e^T} (1-n)\rho_s b_s^e d\Omega_e^s \quad (10)$$

$$F_{ss} = \sum_{e=1}^{nes} \int_{S_s^e} N_s^{e^T} \tau_s'^e n dS_s^e \quad (11)$$

$$F_{sP} = \sum_{e=1}^{nes} \int_{S_s^e} N_s^{e^T} (1-n) P^e dS_s^e \quad (12)$$

$$F_{fb} = \sum_{e=1}^{nef} \int_{\Omega_f^e} N_f^{e^T} n\rho_f b_f^e d\Omega_f^e \quad (13)$$

$$F_{fP} = \sum_{e=1}^{nef} \int_{S_f^e} N_f^{e^T} n P^e dS_f^e \quad (14)$$

$$M_{ss} = \sum_{e=1}^{nes} \int_{R_e} N_s^{e^T} (1-n)\rho_s N_s^e \ddot{u}_s^e d\Omega_s^e \quad (15)$$

$$M_{ff}=\sum_{e=1}^{nef}\int_{\Omega_f^e} N_f^{e^T} n\rho_f N_f^e d\Omega_f^e \qquad (16)$$

$$R_{ss}=\sum_{e=1}^{nes}\int_{R_e} N_s^{e^T}(1-n)\rho_s[(N_s^e\dot{u}_s^e).\nabla N_s^e]d\Omega_s^e \qquad (17)$$

$$C_{ss}=\sum_{e=1}^{nes}\int_{\Omega_s^e} N_s^{e^T}\frac{n^2\rho_f g}{k_f}N_s^e d\Omega_s^e \qquad (18)$$

$$C_{sf}=\sum_{e=1}^{nes}\int_{\Omega_s^e} N_s^{e^T}\frac{n^2\rho_f g}{k_f}N_f^e d\Omega_s^e \qquad (19)$$

$$S_{Ps}=\sum_{e=1}^{neP}\int_{\Omega_P^e} N_P^{e^T}(1-n)(\nabla.N_s^e)d\Omega_P^e \qquad (20)$$

$$Q_P=\sum_{e=1}^{neP}\int_{\Omega_P^e} N_P^{e^T}\frac{n}{\beta}N_P^e d\Omega_P^e \qquad (21)$$

$$S_{Pf}=\sum_{e=1}^{neP}\int_{\Omega_P^e} N_P^{e^T} n(\nabla.N_f^e)d\Omega_P^e \qquad (22)$$

$$C_{fs}=\sum_{e=1}^{nef}\int_{\Omega_f^e} N_f^{e^T}\frac{n^2\rho_f g}{k_f}N_s^e d\Omega_f^e \qquad (23)$$

$$R_{ff}=\sum_{e=1}^{nef}\int_{\Omega_f^e} N_f^{e^T} n\rho_f[(N_f^e\dot{u}_f^e).\nabla N_f^e]d\Omega_f^e \qquad (24)$$

$$C_{ff}=\sum_{e=1}^{nef}\int_{\Omega_f^e} N_f^{e^T}\frac{n^2\rho_f g}{k_f}N_f^e d\Omega_f^e \qquad (25)$$

$$K_{ss}=\sum_{e=1}^{nes}\int_{\Omega_s^e} B_s^{e^T} D_s^e B_s^e d\Omega_s^e \qquad (26)$$

$$S_{sP}=\sum_{e=1}^{nes}\int_{\Omega_s^e} (\nabla.N_s^{e^T})(1-n)N_P^e d\Omega_s^e \qquad (27)$$

$$S_{fP}=\sum_{e=1}^{nef}\int_{\Omega_f^e} (\nabla.N_f^{e^T})nN_P^e d\Omega_f^e \qquad (28)$$

The numerical solutions can be obtained by putting the selected constitutive model in equation (6) and integrating the equations in the time domain as well as the space domain.

IV. Numerical Examples

The finite element discretization and numerical time integration procedures developed in previous section have been implemented into a FORTRAN finite element code called DYNAMP, which is created by author. In this code, the fluid displacement, the solid displacement and the fluid pressure are used as nodal unknown variables. The 8-node quadrilateral elements are employed for both displacement and pressure. It was shown that this kind of elements is more efficient and gives accurate results than the other elements. The code allows for static and dynamic 2-D analysis of saturated porous media.

A. *Centrifuge modeling of soil liquefaction*

This model has been set in the base of Bao's experimental study in 2005. This test is a centrifuge modeling of a saturated soil with 16 meter length and 10 meter height which contains sand and water. The initial void ratio of the soil was measured as 0.776 and input acceleration was in the form of sinusoidal wave with 0.2g amplitude, 1 Hz frequency and 10 second time duration. The cross-section of developed model with the layout of instrumentation is shown in Fig. 1 [10].

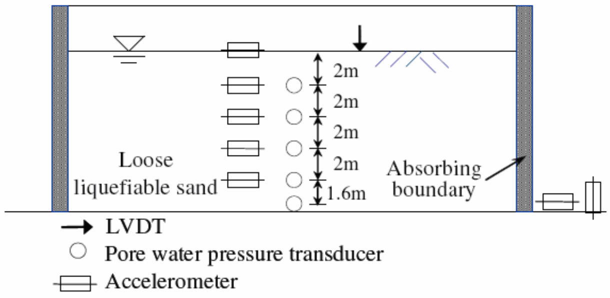

Fig. 1 The cross-section of the model and its instrumentation layout

Fig. 2 shows the results for excess pore water pressure distribution in 9.6 m depth. This figure also shows clearly that the induced progressive pore water pressure is accompanied by a cycle-by-cycle pore water pressure variation.

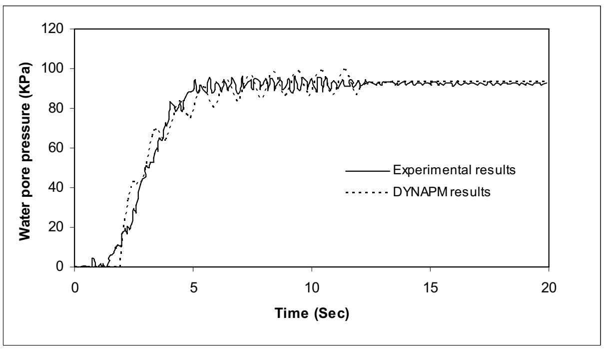

Fig. 2 The excess pore water pressure time history at depth= 9.6 m

B. Reservoir simulation

The effect of constant load on dynamic response of porous media has been investigated in this example which was performed by Zheng et al. (2005). Consider a two-dimensional porous sample shown in Fig. 3, whose dimensions are 2 meter length and 3 meter height, under a gravity loading. The horizontal motions of its left and right boundaries are restricted and the vertical displacement at its bottom is also constrained to zero [9].

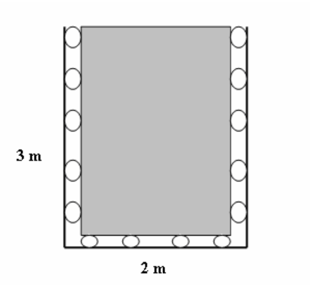

Fig. 3 A porous sample under gravity load

After analyzing the finite element model for this case, the pore pressure distribution and vertical displacement of the model is obtained. Numerical results of the pore pressure and vertical displacement are shown in Fig. 4 and 5, along with the Zheng's model results. Comparison of the finite element model results with Zheng's model results shows very good agreement. According to obtained results, it can be concluded that the pore pressure inside the porous sample shows a linear relation with the height, while the vertical displacement of the solid phase is a quadratic function of the height under gravity load.

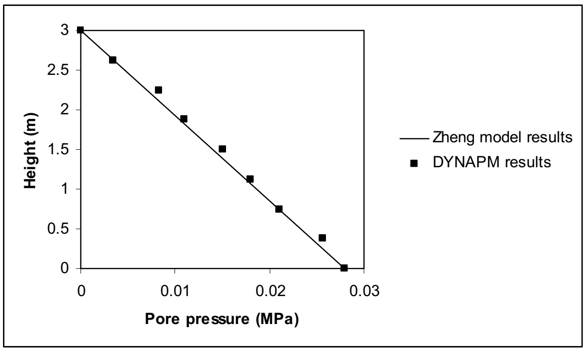

Fig. 4 The pore pressure inside the model under gravity load

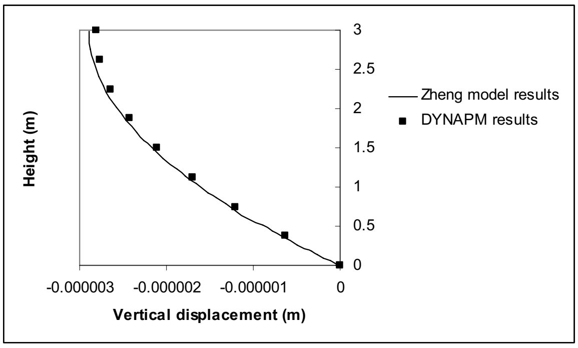

Fig. 5 The vertical displacement of the sample under gravity load

C. Elastic response of saturated soil column subjected to loading

Usually, the governing equations described should be solved by numerical methods. However, for one-dimensional small strain case, the analytical solution is available, in which the small variation of the volume fraction is approximately neglected.

In this section, an illustrative example of a soil column subjected to surface loading is investigated. A number of results obtained from the numerical solution of water-saturated soil column is presented and compared with the analytical solution. The loading function at the surface is $\sigma(0,t) = f(t)$, where $f(t)$ is chosen to be a sinusoidal function. The physical and mechanical properties of the soil are assumed as shown in Table 1.

TABLE I
MATERIAL PROPERTIES

$\rho_s = 2400 \frac{kg}{m^3}$
$\rho_f = 1000 \frac{kg}{m^3}$
$\lambda_s = 5.5833 MN/m^2$
$\mu_s = 8.3750 MN/m^2$
$n = 0.33$, $\nu = 0.2$

Where λ_s and μ_s are the lame's constants of soil skeleton and ν is the poison factor.

The problem is defined in Fig. 6. A one-dimensional infinite soil column is separated from a half space consisting of soil deposit saturated by water. On the surface, the soil column is subjected to a load $f(t)$. It is assumed that the surface is a drained boundary. Analytical solution of the problem has been presented by de Boer et al. [7]. This example is introduced to demonstrate the capability of the code in capturing the different boundary conditions and to check the efficiency of the model.

In order to model the infinite soil column by using the finite element method, a soil column with a length of 10 m is considered.

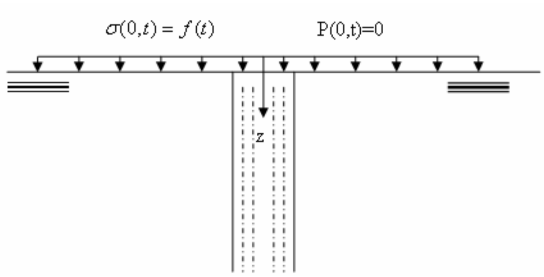

Fig. 6 Geometry of the investigated problem

In the case of sinusoidal loading, the responses of the solid and liquid displacements versus time and depth measured from the free surface are shown in Fig. 7 to11.

The numerical displacements and pore water pressure at various depths and times are reported and compared with analytical solutions in fig. 7 to10, respectively.

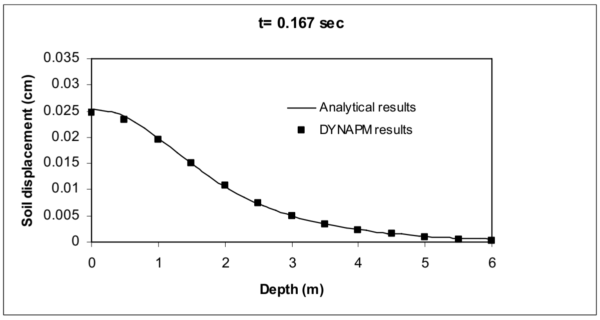

Fig. 7 Comparison of numerical and analytical response

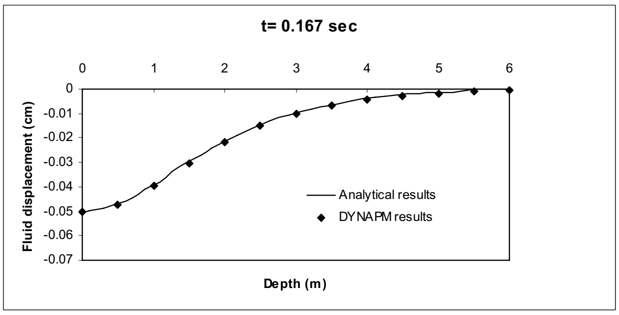

Fig. 8 Comparison of numerical and analytical response

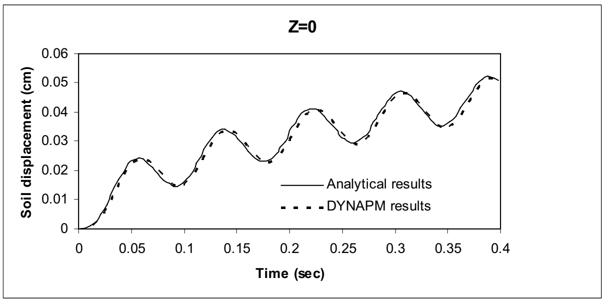

Fig. 9 Comparison of numerical and analytical response of solid displacement vs. time at depth 0.0 m

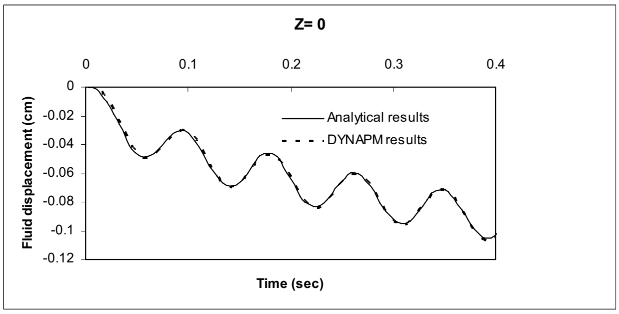

Fig. 10 Comparison of numerical and analytical response of fluid displacement vs. time at depth 0.0 m

Fig. 11 presents the time variation of displacements and pore water pressure for different depths. In all the comparisons, the numerical results agree favorably with the analytical solutions.

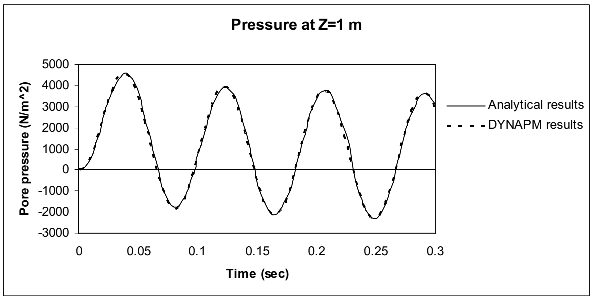

Fig. 11 Comparison of numerical and analytical response of pore pressure vs. time at depth 1.0 m

V. Conclusion

A perfect numerical model for a transient analysis of a liquid-saturated elastic porous skeleton was presented in this paper. For numerical modeling, the finite element formulation for wave propagation in poroelastic solids has been reviewed to include a standard Galerkin weighted residual formulation more general and concise than those existing in the literature. The technique is an enhanced represented as a two-phase poroelastic region with all features of wave motion in which the dynamic behavior is described by Biot's equations.

Various examples were considered to describe and validate the accuracy of numerical procedures for transient problems in such media. All comparisons in the model testing show that the obtained results from this model give excellent agreements. This work can provide the further understanding of the characteristics of wave propagation in porous materials and may be taken for a quantitative comparison to various numerical solutions.

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