

Application of Lattice Boltzmann Methods in Heat and Moisture Transfer in Frozen Soil

Wenyu Song, Bingxi Li, Zhongbin Fu, and Bo Zhang

Abstract—Although water only takes a little percentage in the total mass of soil, it indeed plays an important role to the strength of structure. Moisture transfer can be carried out by many different mechanisms which may involve heat and mass transfer, thermodynamic phase change, and the interplay of various forces such as viscous, buoyancy, and capillary forces. The continuum models are not well suited for describing those phenomena in which the connectivity of the pore space or the fracture network, or that of a fluid phase, plays a major role. However, Lattice Boltzmann methods (LBMs) are especially well suited to simulate flows around complex geometries. Lattice Boltzmann methods were initially invented for solving fluid flows. Recently, fluid with multicomponent and phase change is also included in the equations. By comparing the numerical result with experimental result, the Lattice Boltzmann methods with phase change will be optimized.

Keywords—Frozen soil, Lattice Boltzmann method, Phase change, Test rig.

I. INTRODUCTION

MOISTURE control in pavement bases, subgrades and roadbeds is acknowledged as a basic requisite to ensure a lasting road performance. Although water only takes a little percentage in the total mass of soil, it indeed plays an important role to the strength of structure. Many engineering hazards, such as slope failures, frost heaving, road boiling, pavement cracking, etc., are closely related to the relocation of water in bases, subgrades and roadbeds. Moisture transfer can be carried out by many different mechanisms which may involve heat and mass transfer, thermodynamic phase change, and the interplay of various forces such as viscous, buoyancy, and capillary forces. Therefore, it is important to understand the mechanisms of heat and moisture transfer in unsaturated soils.

At present, research on heat and moisture transfer in unsaturated soils are mainly based on two models, continuum model and discrete model. The continuum model has been widely used due to its convenience and familiarity to the researchers. Up till now, most of the research works are based

on continuum model. However, they are not well suited for describing those phenomena in which the connectivity of the pore space or the fracture network, or that of a fluid phase, plays a major role. Continuum model also breaks down if there are correlations in the system with an extent that is comparable with the linear size of the porous medium. In order to precisely describe heat and moisture transfer process in unsaturated soils, one must deal with the complex pore structure of the medium and how it affects the distribution, flow, displacement of one or more fluids, or dispersion of one fluid in another. Other than continuum model, discrete model is mostly based on a network representation of porous media and fracture networks. Once the mapping is complete, one can study a given phenomenon in porous media in great detail.

Lattice Boltzmann methods (LBMs) are a class of mesoscopic particle based approaches to simulate fluid flows. They are becoming a serious alternative to traditional methods for computational fluid dynamics [1]-[4]. LBMs are especially well suited to simulate flows around complex geometries [11], and they are straightforwardly implemented on parallel machines [12]. Historically, the lattice Boltzmann approach developed from lattice gases [5][6] although it can also be derived directly from the simplified Boltzmann BGK equation. In lattice gases [5][7][8], particles live on the nodes of a discrete lattice. The particles jump from one lattice node to the next, according to their (discrete) velocity. This is called the propagation phase. Then, the particles collide and get a new velocity. This is the collision phase. Hence the simulation proceeds in an alternation between particle propagations and collisions.

II. HEAT AND MOISTURE TRANSFER IN FROZEN SOIL

Frost heaving of soil is caused by crystallization of ice within the larger soil voids and usually a subsequent extension of this ice to form continuous ice lenses, layers, veins, or other ice masses. Frost heave results from the formation of discrete ice lenses in the soil. As depicted in Figure 1, an ice lens grows and thickens in the direction of heat transfer until the water supply is depleted or until freezing conditions at the freezing interface no longer support further crystallization. As the ice lens grows, the overlying soil and pavement will “heave” up potentially resulting in a rough, cracked pavement.

Frost heave occurs primarily in soils containing fine particles (often termed “frost susceptible” soils), while clean sands and gravels (small amounts of fine particles) are non-frost susceptible (NFS). Thus, the degree of frost susceptibility is mainly a function of the percentage of fine particles within the soil.

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As mentioned in part I, in order to simplify the computation model, porous medium is simplified to a continuum medium. Continuum model only consider the permeability of porous medium while, the micro structure, is neglected. This often works for the heat and mass transfer problems in which there is no phase change phenomena, Darcy's law for instance. However, while describing the heat and mass transfer phenomenon in frozen soil, such continuum medium is facing great challenges. For example, while observing ice lenses in frozen soil, it is found that the ice lenses are randomly distributed in the frozen soil. Even in different years, for the frozen soil in the same location, the position and size of ice lenses are different as well. For frozen soil is a system includes multicomponent, phase change and structural changes. The micro structure, as well as the permeability, is changing simultaneously during the freezing and thawing processes.

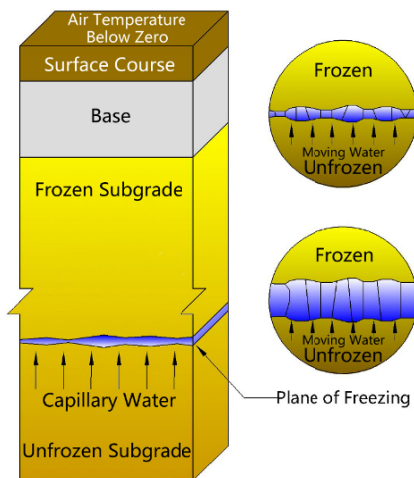


Fig. 1 Formation of ice lenses in a soil structure

III. PERMEABILITY OF POROUS MEDIUM

In order to investigate the permeability and structure of porous mediums which have the same porosity, a Matlab code is developed. In this code, a 2D porous medium is described by a 2D matrix in which solid points are labeled by 1 and void points are labeled by 0. All the elements in this matrix are initialized by 0 elements at the beginning. Then the elements are labeled by 1 elements randomly. The porosity is calculated by the amount of 0 elements divided by total amount of elements of the matrix, i.e.

$$e = \frac{n_{void}}{n_{total}} \quad (1)$$

The code ends while e reaches 0.5, then an analysis is preceded. To describe the permeability, it is assumed that the fluid comes into the porous medium from the left boundary. The criterion of deciding whether a void element is a valid void element is as follows. For a void element $A(i, j)$ which is labeled by 0, if in the following elements $A(i-1, j-1)$,

$A(i, j-1)$, $A(i+1, j-1)$, any of these three elements is labeled by 0, then $A(i, j)$ is a valid void element and keeps the label 0. Otherwise $A(i, j)$ is labeled by 2. Then the matrix is outputted to a picture, all the 0 elements are drawn by white pixels, others are drawn by black pixels.

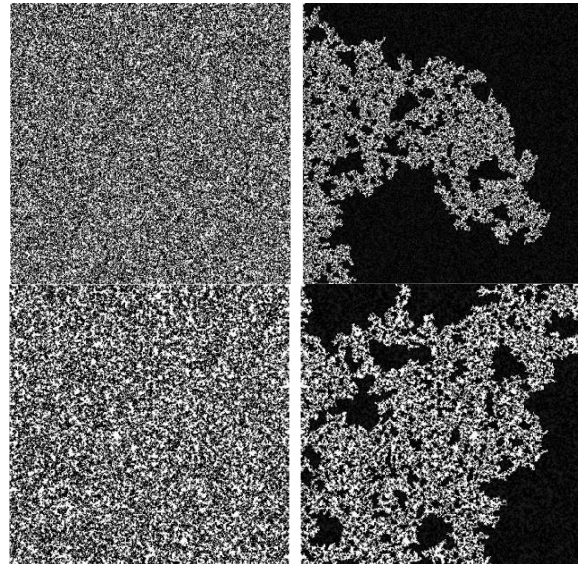


Fig. 2 Different structure and permeability of porous mediums with the same porosity

Fig 2 gives an example of different structure and permeability of porous medium. As shown in fig 2. The upper one and lower one have the same porosity e of 0.5. However, the permeability of those two porous mediums shows significant differences. The lower one is permeable while in the contrary, the upper one is obvious impermeable. This code, combined with phase change Lattice Boltzmann method will be used in simulating the freezing and thawing process of soil.

IV. LATTICE BOLTZMANN METHODS AND TEST RIG

Historically, the lattice Boltzmann approach developed from lattice gases [5][6], although it can also be derived directly from the simplified Boltzmann BGK equation [9]. In lattice gases [5][7][8], particles live on the nodes of a discrete lattice. The particles jump from one lattice node to the next, according to their (discrete) velocity. This is called the propagation phase. Then, the particles collide and get a new velocity. This is the collision phase. Hence the simulation proceeds in an alternation between particle propagations and collisions.

The major disadvantage of lattice gases for common fluid dynamics applications is the occurrence of noise. If the main interest is a smooth flow field one needs to average out over a very large lattice and over a long time. The lattice Boltzmann method solves this problem by pre-averaging the lattice gas. It considers particle distributions that live on the lattice nodes, rather than the individual particles. The general form of the lattice Boltzmann equation is

$$f_i(x + \Delta t \bar{c}_i, t + \Delta t) = f_i(x, t) + \Omega_i \quad (2)$$

The collision operator Ω_i differs for the different lattice Boltzmann methods. In the LBGK method, the particle distribution after propagation is relaxed towards the equilibrium distribution $f_i^{eq}(x, t)$ is

$$\Omega_i = \frac{1}{\tau} (f_i(x, t) - f_i^{eq}(x, t)) \quad (3)$$

The relaxation τ parameter determines the kinematic viscosity ν of the simulated fluid, as,

$$\nu = (2\tau - 1)/6 \quad (4)$$

The equilibrium distribution $f_i^{eq}(x, t)$ is a function of the local density ρ and the local velocity \bar{u} . These are the first and second order moments of the particle distribution as,

$$\rho(x, t) = \sum_i f_i(x, t) \quad (5)$$

$$\bar{u}(x, t) = \frac{\sum_i f_i(x, t) \bar{c}_i}{\rho(x, t)} \quad (6)$$

The equilibrium density $f_i^{eq}(\bar{u}, \rho)$ is calculated as

$$f_i^{eq}(\bar{u}, \rho) = t_p \rho \left(1 + \frac{\bar{c}_i \bar{u}}{c_s^2} + \frac{(\bar{c}_i \bar{u})^2}{2c_s^4} + \frac{\bar{u} \bar{u}}{2c_s^2} \right) \quad (7)$$

Lattice Boltzmann methods were initially invented for solving fluid flows. Recently, fluid with multicomponent and phase change is also included in the equations. Fig. 3 shows the results obtained from phase change Lattice Boltzmann methods; the research was carried out by Christian Huber, Andrea Parmigiani, etc.. [10]

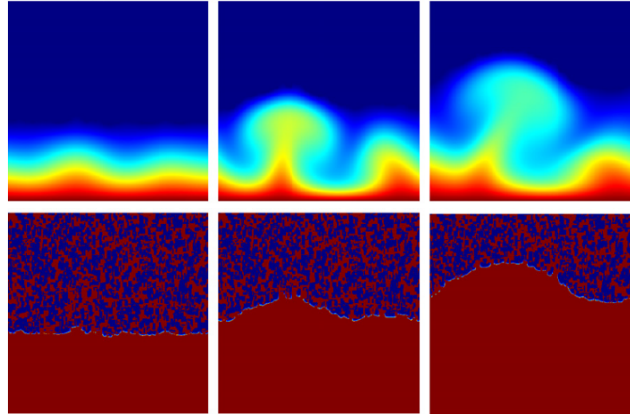


Fig. 3 Porous media convection melting carried out by Christian Huber, Andrea Parmigiani, etc

In the figure, the phase interface is a non-horizontal surface, which is, different from the results obtained by finite volume methods. The non-horizontal surface is mainly caused by capillary force in porous medium. This work gives an initial view of the potential ability of solving heat and mass transfer problems in porous medium which has highly complex geometry. However, during freezing and thawing processes in porous medium, there is still considerable amount of unfrozen water, which makes the frozen zone still permeable.

In order to verify the result given by numerical simulation, a test rig is built. As shown in fig. 4, in the test rig, soil is filled into a steel cylinder (1) which has a height of 2.50 meters and a diameter of 1.00 meter. The cylinder has a removable base (13) connected by bolts. An organic glass tank (2) is filled with water and (2) is connected to (1) by pipe in order to control underground water table in (1). There are two heat exchangers (8) which locate in the lower surface and upper surface of soil pillar respectively. Ethylene glycol is chosen as secondary refrigerant, the temperature is controlled by a water chiller (10). The temperature range is between -20 to +50 Celsius degree. A hydraulic jack (4) which has the capability of applying 200KN force on the upper surface of soil pillar, is located on a portal frame (3). Sensors (6) and video camera (5) are installed, all the data is collected by data loggers (9) and is sent to central computer (11). The central computer also works as a controller of hydraulic jack by sending different commands to hydraulic jack controller (12).

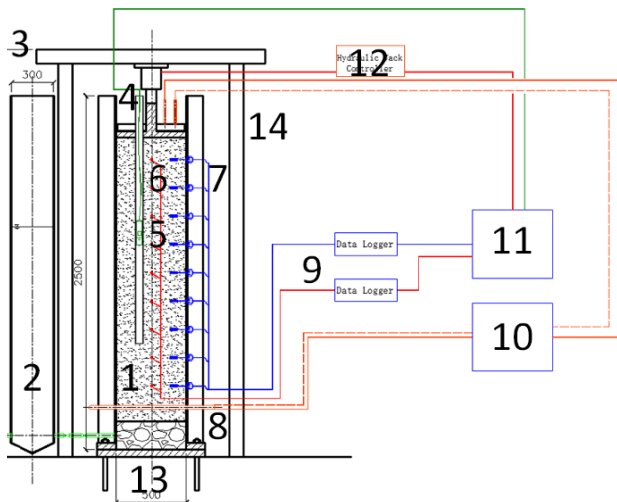


Fig. 4 Schematic diagram of frozen soil test rig

In the test rig, serials of investigation on heat and mass transfer phenomenon in soil during freezing and thawing processes will be carried out. By controlling the temperature of secondary refrigerant, the freezing and thawing rate can be precisely adjusted. The height of soil pillar is enough to reduce capillary effects so that one can give a better investigation on unsaturated soil. The micro structure of soil will be investigated by Scanning Electron Microscope (SEM) or Micro-CT. By comparing the numerical result with experimental result, the Lattice Boltzmann methods with phase change will be optimized.

V. CONCLUSION

1. Frozen soil is a system includes multicomponent, phase change and structural changes. The micro structure, as well as the permeability, is changing simultaneously during the freezing and thawing processes.

2. During the phase change stage, moisture transfer phenomenon occurs at the freezing front. Therefore, Moisture transfer phenomenon could not be ignored in further investigation.

3. Phase change Lattice Boltzmann methods can be used to solve the complex fluid flow in porous medium. However, more validation work needs to be done in the future. The micro structures of soil during freezing and thawing process are needed to be investigated.

APPENDIX

\bar{c}_i	discrete velocity
c_s	speed of sound
f_i	concentration of particles
$f_i^{eq}(x, t)$	equilibrium distribution
$f_i^{eq}(\rho, \bar{u})$	equilibrium density
Δt	time step

t_p corresponding equilibrium density

\bar{u} local velocity

Greek symbols

Ω_i collision operator

τ relaxation parameter

ν kinematic viscosity

ρ local density

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