

# Effect of Neighbourhood Size on Negative Weights in Punctual Kriging based Image Restoration

Asmatullah Chaudhry, and Anwar M. Mirza

**Abstract**—We present a general comparison of punctual kriging based image restoration for different neighbourhood sizes. The formulation of the technique under consideration is based on punctual kriging and fuzzy concepts for image restoration in spatial domain. Three different neighbourhood windows are considered to estimate the semivariance at different lags for studying its effect in reduction of negative weights resulted in punctual kriging, consequently restoration of degraded images. Our results show that effect of neighbourhood size higher than 5x5 on reduction in negative weights is insignificant. In addition, image quality measures, such as structure similarity indices, peak signal to noise ratios and the new variogram based quality measures; show that 3x3 window size gives better performance as compared with larger window sizes.

**Keywords**—Image restoration, punctual kriging, semivariance, structure similarity index, Negative weights in punctual kriging.

## I. INTRODUCTION

IMAGE restoration is an important branch of image processing, dealing with the reconstruction of images by removing noise and blur from degraded images and making them suitable for human perception. Images are often degraded by noise due to channel transmission error, faulty acquisition device, and atmospheric electrical emissions. Due to strong amplitude of noise, human visual perception is very sensitive to it, and the removal of such noise is an important issue in image processing [1]. One of the primary tasks in developing such image restoration techniques is noise removal without destroying edge information. In the sequel, we present a brief review of spatial filtering technique, based on punctual kriging and fuzzy logic control, to remove noise while efficiently preserving the image details and edge information.

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Punctual kriging is a well-established estimation technique in the fields of mining and Geostatistics [2]. Kriging has been applied successfully in many other fields. It has proven to be a nonlinear predictor in signal processing. Costa et al [3] suggest kriging as an efficient tool for nonlinear filtering.

In the field of image processing, Pham and Wagner [4], [5] reported the first use of kriging along with fuzzy sets to enhance images corrupted by Gaussian noise. They modeled soft-thresholding by fuzzy sets. In their method, the pixel value in the processed image is a weighted sum of two values: the original (noisy) and the estimated (by kriging). Mirza et al [6] have applied fuzzy logic with punctual kriging to estimate the degraded images. In case of matrix inversion failure or negative weights, they have replaced the pixel to be estimated with the average of neighbouring pixels. Further, they have used averaging filter of size 3x3 to smooth out the resultant image and claim that their technique offers better results than Pham and Wagner [4], and adaptive Wiener filter. In our previous paper [7], we have presented spatially adaptive image restoration technique based on fuzzy punctual kriging. Based on the pixel local neighbourhood, fuzzy logic has been employed intelligently to avoid unnecessary estimation of a pixel. The intensity estimation of the selected pixels is carried out by employing punctual kriging. The problem of negative weights in punctual kriging is solved by using approximation; assigning zero to negative weights and renormalization of positive weights. Further, instead of employing smoothing filter to the resultant image, fuzzy weighted filter is used to estimate the inversion failure, and not selected pixels only.

Journel and Huijbregts (1978) [8] suggest that each lag interval ‘d’ should have at least 30 pairs for refining the semivariogram. The American Society for Testing and Materials (Standard D5922-96) [8] have suggested 20 pairs of each lag interval for better estimation of semivariance and to reduce negative weights occurring in punctual kriging. For a typical  $3 \times 3$  neighbourhood, a kriging matrix of size  $9 \times 9$  has to be inverted, which can make the overall filtering process computationally expensive. Also, due to a zero diagonal, the kriging matrix may not always be inverted. The filter weights also suffer from the problem of negative values, which may lead to overall poor performance of the filter. This paper is aimed to compare the results considering different neighbourhood sizes by utilizing the technique based on fuzzy

inference system, punctual kriging and fuzzy smoothing [7]. This paper makes the following contributions:

- Comparative analysis of the effect of neighbourhood size on reduction of negative weights and the consequent improvement in image restoration.

The remaining paper is organized as follows. Section II presents a brief review of punctual kriging. Some of the most commonly used image quality measures are briefly discussed in section III. Experimental results along with their discussion are presented in section IV. A summary of our findings and directions for future work is given in section V.

## II. PUNCTUAL KRIGING

Punctual kriging offers the best linear unbiased estimate of an unknown point on a surface [9]. The estimate is the weighted sum of the known neighbouring values around the unknown point. The weights are calculated to minimize the variance of the estimation-error. To achieve this kriging uses a *variogram model* (a concept from geostatistics). Based on the variogram model chosen, known values are assigned optimal weights to calculate the unknown value where as in the present case, image data is known at each location. Due to this characteristic of the image data, optimal weights are directly find out by solving the system of punctual kriging equations to estimate the degraded pixel.

### A. Punctual Kriging Procedure

To formulate the punctual kriging procedure mathematically, let us define  $z$  to be the actual sample value at a point and  $\hat{z}$  be an estimate for this value. It can be represented as a linear combination of the neighbouring sample values, as given by eqn. (1).

$$\hat{z} = \sum_i w_i z_i \quad (1)$$

where  $w_i$  are the weights and the  $z_i$  are the neighbouring values of  $z$ . This is an unbiased estimator if the weights add up to 1.

The semivariance of the samples at lag 'd' is defined as:

$$\gamma(d) = \frac{1}{2} \text{Var}(z_{i+d} - z_i) \quad (2)$$

Statistical variance is measure of how different the estimated value is different from its neighbouring sample values. It can be found using the eqn. (3).

A number of such linear unbiased estimators are available, but we find the *best* one in the sense that it has the smallest estimation variance.

$$\text{Var}(e) = \text{Var}(z - \hat{z}) \quad (3)$$

Variance is a measure, which depends upon the changes in the sample value in the overall neighbourhood. Information

about the local morphological structure detail is hidden inside the variance parameter. This structural detail can explicitly be written by expanding the variance into semivariances. Using eqn. (2) and the definition of variance [10], this can be worked out as

$$\begin{aligned} \text{Var}(e) &= \text{Var}(z) - 2 \sum_j w_j \gamma(d_j) \\ &\quad + \sum_j \sum_k w_j w_k \gamma(d_{jk}) \end{aligned} \quad (4)$$

Here  $d_j$  is the distance between the location of current value and its neighbor 'j'. Also  $d_{jk}$  is the distance between neighbors 'j' and 'k'. The expression for estimation variance depends upon the basic geometry of the samples and unknown sample point, behavior of the semivariogram and the weights assigned to each sample [11]. To minimize the estimation variance, we differentiate it with respect to the weights and set it equal to zero.

$$\frac{\partial \text{Var}(e)}{\partial w_i} = 0 \quad \text{where } i = 1, 2, \dots, n \quad (5)$$

The weights obtained from eqn. (5) provide an estimator that has minimum estimation variance, but the weights may not necessarily add up to 1. This is because there is no constraint on weights in the above system of linear equations. This additional constraint on weights is given by:

$$\sum_i w_i = 1 \quad (6)$$

To obtain the Best Linear Unbiased Estimator, a Lagrange Multiplier parameter  $\lambda$  is also included and the cost function is redefined as  $\varphi(w_i, \lambda)$  in eqn. (7) and minimizes it instead of minimizing the estimation variance of error.

$$\varphi(w_i, \lambda) = \text{Var}(e) - 2\lambda \left( \sum_i w_i - 1 \right) \quad (7)$$

By differentiating the cost function  $\varphi(w_i, \lambda)$  with respect to  $w_i$  and  $\lambda$ , and after rearranging the system of equations, these can be written in matrix form as:

$$\begin{pmatrix} \gamma(d_{11}) & \gamma(d_{12}) & \cdots & \gamma(d_{1n}) & 1 \\ \gamma(d_{21}) & \gamma(d_{22}) & \cdots & \gamma(d_{2n}) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma(d_{n1}) & \gamma(d_{n2}) & \cdots & \gamma(d_{nn}) & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \\ \lambda \end{pmatrix} = \begin{pmatrix} \gamma(d_1) \\ \gamma(d_2) \\ \vdots \\ \gamma(d_n) \\ 1 \end{pmatrix} \quad (8)$$

or in matrix-vector notations

$$Aw = b \quad (9)$$

The  $A$  matrix is symmetric and has zero diagonal elements. The elements of the matrix are taken from the semivariogram (defined in eqn. (2) for the current point. Solving eqn. (9) gives us the optimal kriging weights  $\{w_1, w_2, \dots, w_n\}$  for estimating the unknown value  $\hat{z}$  using its neighbors.

### III. IMAGE QUALITY MEASURES

The most widely used qualitative measures in image processing applications are mean squared error (MSE) and signal-to-noise ratio (SNR). During computation, these qualitative measures require the original image as well as the degraded image. However, still no single measure is accepted as representing the true measure of image quality. For a detailed discussion on image quality, one should refer to [12]. Structural similarity index measure (SSIM), recently proposed by Wang et al. [13] is based on the hypotheses that human visual system is highly adopted for extracting structural information. A new image quality measure in terms of the experimental variograms of the original and degraded images is proposed in [7], [14].

### IV. RESULTS AND DISCUSSION

We have compared the performance of Adaptive Fuzzy Punctual Kriging (AFPK) [7] method for different neighbourhood sizes by considering two scenarios. For details of the AFPK method, one should refer to [7]. Firstly, the performance comparison has been made for additive Gaussian white noise of different variances. Lena image is taken as a test image. Secondly, the performance of AFPK with different neighbourhood sizes is compared for different images corrupted with Gaussian white noise of the same variance. Typical results from the Fuzzy Decider are shown in Fig. 1. The Fuzzy Decider used is a Mamdani type FIS for making the decision of whether a pixel needs to be kriged or not kriged, depending upon the local properties of the neighbourhood. The white pixels are the ones that need to be kriged.

#### A. Scenario 1

In this scenario, we have considered lena image as a test image. The image is degraded with Gaussian white noise of variances ranging from 0.01 to 0.1. The results obtained using AFPK with different neighbourhood sizes have been compared.

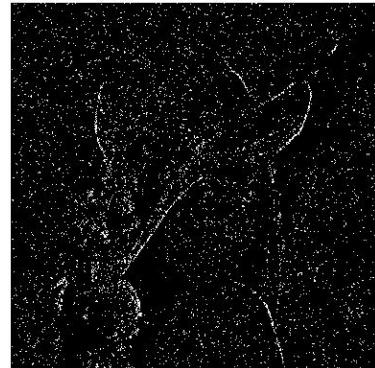


Fig 1 Decision map of Fuzzy-Decider for Lena image degraded at 0.01 variance

The effect of the additive Gaussian white noise and its removal by AFPK with different neighbourhood sizes is shown in Fig. 2. Table I gives a quantitative comparison of AFPK with different neighbourhood sizes in terms of MSE, PSNR, SSIM, VMSE, VPSNR and Negative Weights. From table I, it can be observed that by increasing the number of pairs at different lag intervals 'd' i.e.  $1, \sqrt{2}, 2, \sqrt{5}$  and  $\sqrt{8}$ , the number of negative weights reduces. However, its effect on punctual kriging based image restoration is not so fruitful as shown in table I and Fig. 2. Further, it can be observed that AFPK with  $3 \times 3$  neighbourhood size offers superior performance against the Gaussian white noise for different noise variances as compared to other neighbourhood sizes. This may be due to the fact that as neighbourhood size is increased to fulfill the condition of at least 20 pairs for each lag 'd' to estimate semivariance [8], moves toward global estimation of semivariance at different lags. Further, this estimated semivariance is used in punctual kriging to estimate noisy pixel.

The experimental variograms of the original, AFPK restored images with different neighbourhood sizes are plotted in Fig. 3. The Blood cells image is degraded with Gaussian noise of variance 0.05. Comparing the variograms produced by AFPK with different neighbourhood sizes, the AFPK  $3 \times 3$  produces a variogram that overlaps with the variogram of the original image. This is also clear from Table I, where the VMSE for  $3 \times 3$  windows is less as compared with the other two neighbourhood sizes.

#### B. Scenario 2

In this case, performance comparison has been made on 450 different images. These images have been corrupted with Gaussian white noise of variance 0.05. Performance analysis of the AFPK method with different neighbourhood sizes is carried out in terms of MSE, PSNR, SSIM, VMSE, VPSNR and Negative Weights as shown in Table II. Graphical representation of results presented in table II is shown in Fig. 4. It can be observed that the performance of AFPK with neighbourhood  $3 \times 3$  is better as compared with other neighbourhood sizes across all of the image quality measures.

This is due to the fact that by estimation of semivariance at lag 'd' using larger neighbourhood size means moving towards global estimation, and thus may not capture the local knowledge. Further, making comparison in terms of number of negative weights, Table II shows that AFPK with neighbourhood sizes 5x5 and 7x7 results negative weights for nearly same number of times. It also shows that AFPK with

higher neighbourhood size reduces in negative weights as compared with 3x3 neighbourhood. This may be due to the fact that estimation of semivariance at lag 'd' by considering large number of pairs further refine the experimental semivariogram.

TABLE I

COMPARISON OF DE-NOISING METHOD AFPK WITH DIFFERENT NEIGHBOURHOOD SIZES FOR LENA IMAGE DEGRADED WITH GAUSSIAN WHITE NOISE OF DIFFERENT VARIANCES/PSNR

De-noising Methods	Quality Measures	White Gaussian Noise of different variance				
		0.1	0.075	0.05	0.025	0.01
Noisy Image	MSE	4584	3681.8	2681.7	1455.7	615.55
	PSNR	11.518	12.47	13.847	16.5	20.238
	SSIM	0.0744	0.091215	0.11811	0.18138	0.30061
	VMSE	13600000	9220000	4940000	1490000	322000
	VPSNR	0.0000088	0.000013	0.000024	0.00008	0.000372
	MSE	<b>895.68</b>	<b>688.23</b>	<b>479.01</b>	<b>244.19</b>	<b>119.72</b>
AFPK 3x3	PSNR	<b>18.609</b>	<b>19.753</b>	<b>21.327</b>	<b>24.064</b>	<b>26.906</b>
	SSIM	<b>0.228</b>	<b>0.269</b>	<b>0.331</b>	<b>0.465</b>	<b>0.625</b>
	VMSE	<b>29849</b>	<b>16019</b>	<b>7144</b>	11759	<b>4963</b>
	VPSNR	<b>0.004003</b>	<b>0.00746</b>	<b>0.016726</b>	0.010162	<b>0.024076</b>
	Negative Weights	173000	155000	126000	64375	9992
	MSE	946.53	723.93	498.43	248.85	121.11
AFPK 5x5	PSNR	18.369	19.534	21.155	23.982	26.856
	SSIM	0.219	0.259	0.323	0.461	0.624
	VMSE	50124	24183	7333	<b>11425</b>	5069
	VPSNR	0.002384	0.004942	0.016295	<b>0.010460</b>	0.023573
	Negative Weights	150990	<b>136460</b>	<b>110650</b>	<b>57546</b>	<b>9319</b>
	MSE	947.32	725.18	499.89	251.52	124.70
AFPK 7x7	PSNR	18.366	19.526	21.142	23.970	26.730
	SSIM	0.219	0.259	0.323	0.460	0.623
	VMSE	50465	24341	7459	11788	5470
	VPSNR	0.002368	0.004909	0.016020	0.010138	0.021847
	Negative Weights	150840	136600	111180	58060	<b>9319</b>

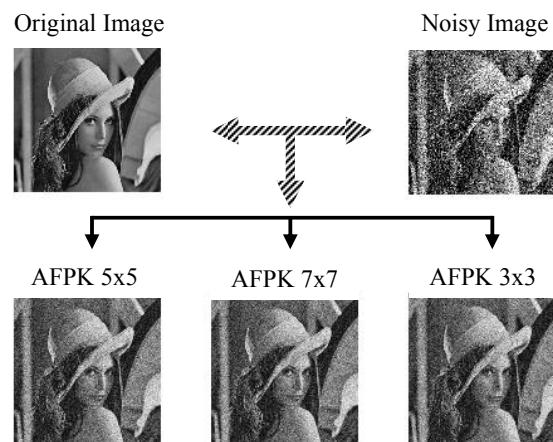


Fig. 2 The original image, noisy image of Lena and the estimated images obtained through AFPK method with different neighbourhood sizes

TABLE II

AVERAGE VALUES OF DIFFERENT QUALITATIVE MEASURES FOR 450 TEST IMAGES CORRUPTED WITH GAUSSIAN WHITE NOISE OF VARIANCE 0.05

Qualitative Measures	AFPK 3x3	AFPK 5x5	AFPK 7x7
MSE	542.76	564.80	567.43
PSNR	20.77	20.62	20.60
SSIM	0.40	0.39	0.39
VMSE	129962.71	124403.48	125563.27
VPSNR	0.00284	0.00275	0.00274
Negative Weights	19446	17161	17145

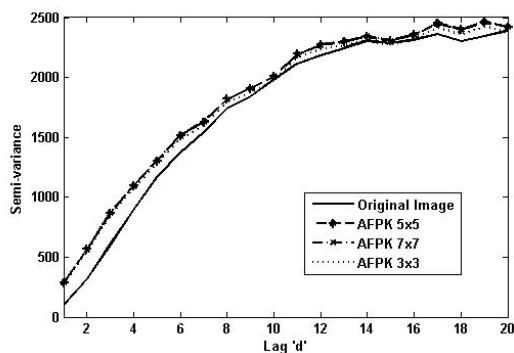


Fig. 3 Comparison of the variograms of the original, degraded and processed Blood cells image

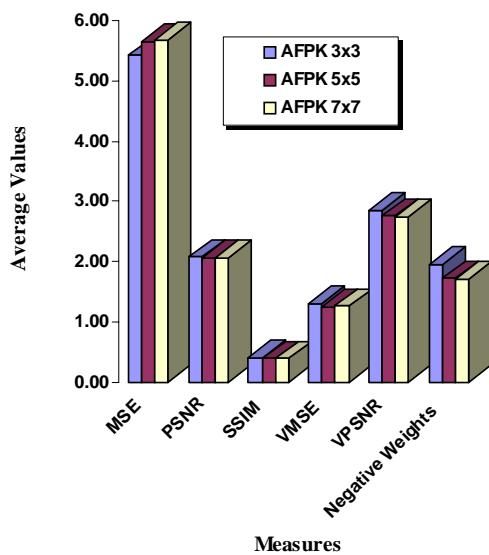


Fig 4 plot of average values of different measures for Noise variance 0.05. The values have been rescaled for elaboration purpose

## V. CONCLUSION

In this work, we have analyzed the effect of different window sizes on negative weights in punctual kriging based image restoration. To increase number of pairs at each lag to estimate semivariance by increasing the size of neighbourhood, methodology changes from local to global

estimation. Although by increasing the neighbourhood size, number of negative weights in punctual kriging reduces. However, reduction in number of negative weights does not play significant role in punctual kriging based image restoration to estimate the noisy pixel. Furthermore, approximation of initializing zero value to negative weights and renormalization of positive weights in punctual kriging based image restoration with 3x3 neighbourhood performs better than 5x5 and 7x7.

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## REFERENCES

- [1] Gonzalez R. C. and Woods R. E., "Digital Image Processing", 2<sup>nd</sup> Ed., Pearson Education Inc., 2002.
- [2] Krige D., "A statistical approach to some mine valuation and allied problems on the Witwatersrand", Master Thesis, University of Witwatersrand, South Africa, 1951.
- [3] Costa J. P., Pronzato L., and Thierry E., "Nonlinear prediction by kriging, with application to noise cancellation", *Signal Processing*, 2000, 80: 553–566.
- [4] Pham T. D. and Wagner M., "Image enhancement by kriging and fuzzy sets", *Int. J. Pattern Recognition and Artificial Intelligence*, 2000, 14(8): 1025–1038.
- [5] Pham T. D. and Wagner M., "Filtering noisy images using kriging", 5<sup>th</sup> Int. Symposium on Signal Processing & its Applications (ISSPA'99), Brisbane, Australia, August 1999.
- [6] Mirza, Anwar M. and Munir, B., "Combining fuzzy logic and kriging for image enhancement", in Proceedings of 8th International Conference on Computational Intelligence, Theory and Applications, (FUZZY DAYS 2004), Advances in Soft Computing 2, Springer-Verlag Berlin Heidelberg, 2005, pp. 435-448.
- [7] Anwar M. Mirza, Asmatullah Chaudhry, and Badre Munir, "Spatially adaptive image restoration using fuzzy punctual kriging", Journal of Computer Science & Technology, Springer, 2006 (Accepted).
- [8] Engineering and Design- Practical Aspects of Applying Geostatistics at Hazardous, Toxic, and Radioactive Waste Sites (ETL 1110-1-175), Dept. of the Army, U.S. Army Corps of Engineers, Washington, DC 20314-1000, 1997.
- [9] Naser El-Sheimy, "Digital Terrain Modeling (ENGO 573)", University of Calgary, Canada, 1999.
- [10] Walpole R. E., Myers R. H. and Myers S. L., "Probability and Statistics for Engineers and Scientists", 6<sup>th</sup> Ed., Prentice Hall International Inc., 1998.
- [11] Clark, I. and Harper, W. V., "Practical Geostatistics 2000", Ecosse North America, OH, USA, 2000.
- [12] M. Kutter and F. A. P. Petitcolas, "A fair benchmark for image watermarking systems", Electronic Imaging '99, Security and Watermarking of Multimedia Contents, Vol. 3657 Sans Jose, CA, USA, the International Society for Optical Engineering, Jan. 1999, pp.25-27.

- [13] Wang Z., Bovik A.C., Sheikh H. R. and Simoncelli E. P., "Image quality assessment: From error visibility to structural similarity", *IEEE Trans. on Image Processing*, March 2000, 13(3): 1–14.
- [14] Asmatullah Chaudhry, Asifullah Khan, Asad Ali and Anwar M. Mirza, "A hybrid image restoration approach: using fuzzy punctual kriging and genetic programming", *International Journal of Imaging Systems and Technology*, 2006 (Submitted).