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A Model Predictive Control and Time Series Forecasting Framework for Supply Chain Management

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Abstract**—**Model Predictive Control has been previously applied to supply chain problems with promising results; however hitherto proposed systems possessed no information on future demand. A forecasting methodology will surely promote the efficiency of control actions by providing insight on the future. A complete supply chain management framework that is based on Model Predictive Control (MPC) and Time Series Forecasting will be presented in this paper. The proposed framework will be tested on industrial data in order to assess the efficiency of the method and the impact of forecast accuracy on overall control performance of the supply chain. To this end, forecasting methodologies with different characteristics will be implemented on test data to generate forecasts that will serve as input to the Model Predictive Control module.

Keywords**—**Forecasting, Model predictive control, production planning.

I. Introduction

DURING the last decades, industrial goods production has shifted from the local or national level to facilities with global outreach that serve many national markets. This development has put substantial stress on the supply chains of companies, demanding a better organization of performed actions and thus calling for supply chain management methodologies that can improve customer service while reducing cost. This paper will demonstrate the coupling of Model Predictive Control with a forecasting system and investigate its effectiveness for the problem of supply chain management.

A. Model Predictive Control

Model predictive control (MPC) is nowadays recognized as a standard methodology for the control of industrial and process systems [1]. This is because of its capability to incorporate constraints for the manipulated and/or the controlled variables, to handle the nonlinearities often present in dynamical systems and to overcome modeling mismatch. The idea of model predictive control is simple: A process model is used to predict the effect of a finite number of future moves on the controlled variables. This model is incorporated in an on-line open loop optimization problem, which determines the optimal control sequence for a given performance criterion. The simplest MPC objective function is the weighted sum of the two basic control targets, namely the sum of squared differences between the predicted outputs and their set points over the future prediction horizon and the sum of squares of the control moves over the control horizon. After the solution of the minimization problem is found, only the first of the future control actions is implemented to the system. The same procedure is performed repetitively at each time step.

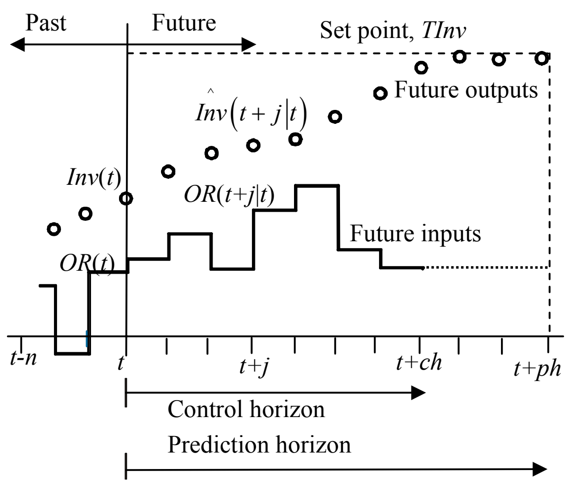

Fig. 1 MPC basic concept

MPC was first applied to inventory management by Kapsiotis and Tzafestas [2], who studied a single manufacturing site problem and included a penalty term in the objective function for deviations from a reference path for inventory in order to compensate for production lead times.

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Perea-Lopez et al [3] employed MPC to the management of a multi-level supply chain with multiple products where demand was deterministic, so the need for an inventory control mechanism was reduced. Braun et al [4] presented a linear MPC methodology for large scale supply chain problems and showed that MPC can handle uncertainties due to model mismatch and unsuccessful forecasts. Finally, Lin et al [5] presented a Minimum Variance Control system with a set point not only for the actual inventory level, but also for the WIP (Work-In-Process) level, while customer demand was expressed by an ARIMA model. Their formulation maintained inventory levels at a desired level avoiding the "bullwhip" effect and when compared, it proved superior to other frameworks.

B. Forecasting

Forecasting plays a central role in the efficient operation of a supply chain, as it provides valuable information on the expected future direction of important factors, thus enabling planners to act preemptively and more effectively. Various methodologies have been proposed for forecasting and they are typically time series algorithms that, depending on the nature of the model they are based on, can be classified as linear on nonlinear. The simplest method of all is naïve forecasting, where the forecast is assumed to be equal to the previous value, a method that is often used as a basis for comparison. Linear models are the most popular, partly due to their simplicity and ease of use. Examples of widely used linear methodologies are the autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) [6], which is a generalization of the former. The forecast in these two methodologies is produced after differencing the time series at an appropriate order (if necessary) using weighted past values of the time series and past forecast errors. A general form of the ARIMA(p,d,q) model is the following:

$$\left(1-\sum_{i=1}^{p}\varphi_i L^i\right)\left(1-L\right)^d X_t=\left(1+\sum_{i=1}^{q}\theta_i L^i\right)\varepsilon_t \quad (1)$$

where L is the lag operator, φ_i are the parameters of the autoregressive part of the model, θ_i are the parameters of the moving average part, p is the order of autoregression, d is the order of differencing, q is the order of the moving average process and ε_t are error terms. Depending on the values of the parameters in the general form depicted in Eq. (1), there are many types of ARIMA models, like the Autoregressive (AR) model, which is an ARIMA(p,0,0) model where only past values of the function are used to produce a forecast.

Another method widely used is the Holt-Winters, which is an exponential smoothing methodology. As such, it uses weighted values of past time series occurrences, where the coefficients decay exponentially with each period, thus giving more weight to recent values and less to more distant ones. Its structure can capture trends and seasonality in data, making it suitable for various types of time series data. The additive form of the Holt-Winters method is:

$$Y_{t+h}=\mu_t+b_t t+S_{t-p+h}+e_t \quad (2)$$

where Y_{t+h} is the predicted value of the h-th period ahead in time; μ_t is a mean value of the series, which is updated as in Eq. (3), where p the periodicity of the seasonality; b_t is a trend parameter of the series, updated as in Eq. (4) and S_t is the seasonal component of the series, updated as in Eq. (5).

$$\mu_t=\alpha\left(Y_t-S_{t-p}\right)+\left(1-\alpha\right)\left(\mu_{t-1}+b_{t-1}\right) \quad (3)$$

$$b_t=\gamma\left(\mu_t-\mu_{t-1}\right)+\left(1-\gamma\right)b_{t-1} \quad (4)$$

$$S_t=\delta\left(Y_t-\mu_t\right)+\left(1-\delta\right)S_{t-p} \quad (5)$$

In most methods, including the two mentioned above, the critical parameters of the equations that describe the behavior of the time series are not known and have to be established through a time-consuming procedure of trial-and error and application of statistical tests. Furthermore, the linear structure of the model is not able to represent nonlinearities possibly present in the time series. Artificial Neural Networks, ANN, and more specifically Radial Basis Function (RBF) neural networks, is a nonlinear methodology that addresses the weaknesses that were mentioned above and are present in many models. Its inherent sophisticated structure allows it to capture the complexity in the behavior of series with nonlinearity, while at the same time model parameters can be determined with algorithms that require no trial-and error procedure. The RBF neural networks consists of three layers, as shown in Fig. 2. The input layer is used to feed the input variables into the model. The hidden layer contains a number of nodes, which apply a nonlinear transformation to the input variables, using a radial basis function. The output layer serves as a linear summation unit. The neural network depicted in Fig. 2 is a typical RBF neural network with only one output node. Each hidden node is associated with a vector c with dimension equal to the number of inputs to the node, called a center. The activity v of a hidden node is the Euclidean distance between the input vector and the node center. The hidden node output is the value of the radial basis function when the activity v is its input variable. In the present work, the thin-plate-spline radial basis function is employed: $f(v)=v\log(v)$. A training algorithm for RBF networks is based on using a set of input-output data (x_i, y_i), i=1,2,…, K_1 in order to determine of the structure and the parameters of the network that lead to a minimum error between the predicted output and the actual values. This defines a Mixed Integer Nonlinear Programming (MINLP) optimization problem, which is solved using special training algorithms.

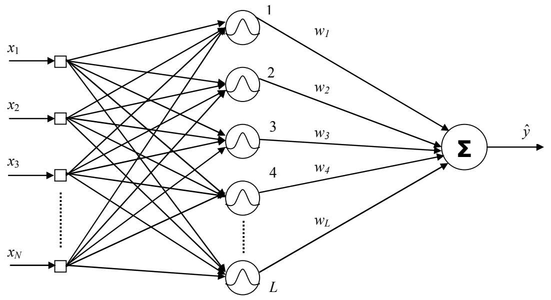

Fig. 2 An example of the RBF neural networks architecture

II. Methodology

In this section the proposed control methodology will be described in detail. Fig. 3 presents the block diagram of the control scheme, consisting of the process, the MPC law and the forecasting policy of customers demand. In the following, a brief description of the process and afterwards the MPC configuration will be presented.

A. Process Model and Material Balance

In many production-inventory systems, the production process is modelled by a pure delay unit, with a discrete transfer function equal to z^{-T}, where T is the lead time. However, such an assumption is not always realistic since the production rate may depend on orders given in different times in the past. In this work, we assume that the process dynamic behavior is described by a Finite Impulse Response (FIR) model. In this case, the system output (production rate $R(t)$) will be given by the following Eq.:

$$R(t) = \sum_{i=1}^{n} g_i \cdot Order(t-i) \tag{6}$$

where $Order(t-i), i = 1,...,n$ is the order rate at time *t-i*, *n* is the system order and $g_i, i = 1,...,n$ are the system parameters. Eq. (6) can easily lead to the transfer function between production rate and order rate *z*-transformed signals:

$$\frac{R(z)}{Order(z)} = g_1 \cdot z^{-1} + ... + g_n \cdot z^{-n} \tag{7}$$

which obviously is a generalization of pure delay.

From the block diagram of Fig. 3, inventory $Inv(z)$ is given by the following equation:

$$Inv(z) = \frac{1}{1-z^{-1}}\left(R(z) - Sales(z)\right) \tag{8}$$

where $Sales(z)$ is the *z*-transform of customers demand $Sales(t)$ and $\frac{1}{1-z^{-1}}$ is the transfer function of the integrator.

Combining Eqs. (6)-(8) we arrive at Eq. (9), which shows that inventory at time t is related to order rate with an autoregressive with exogenous input model (ARX) that also considers customer demand as an external measured disturbance.

$$Inv(t) = Inv(t-1) + \sum_{i=1}^{n} g_i \cdot Order(t-i) - Sales(t) \tag{9}$$

B. Robust Model Predictive Control Scheme

In case of inventory control (Fig. 1), manipulated variables of the proposed control scheme are the future order rates $Order(t+j|t), j = 0,...,ch-1$ and controlled variable is the predicted inventory $\hat{Inv}(t+j|t), j = 1,..., ph$. A predictor for inventory is formulated based on the material balance represented of Eq. (9). In order to test the robustness of the proposed control scheme, we assume that the predictor is based on an approximation of the process parameters $\hat{g}_i, i = 1,...,n$ and not their actual values (Eq. (6)). The inventory predictor also uses an estimation of unknown future sales $ForSales(t+j|t), j = 1,..., ph$. This estimation can be the simple projection of current sales over the prediction horizon, or can be calculated from a forecasting policy, as is the case here. So, the optimization problem solved on line is described by the set of Eqs. (10)-(17).

$$\min_{\substack{OR(t+i)\\ i=0,...,ch-1}} \sum_{j=1}^{ph}\left(w\left(\hat{Inv}(t+j|t) - TInv\right)\right)^2 + \sum_{j=0}^{ch-1}\left(r \cdot \delta Order(t+j|t)\right)^2 \tag{5}$$

$$\begin{aligned}\hat{Inv}(t+j|t) = {} & \hat{Inv}(t+j-1|t) + \sum_{i=1}^{n} \hat{g}_i \cdot Order(t+j-i) \\ & -ForSales(t+j|t) + e(t+j|t)\end{aligned} \tag{10}$$

$$\hat{Inv}(t|t) = Inv(t) \tag{11}$$

$$e(t+j|t) = \begin{cases} e(t|t), & if\ j = 1 \\ 0, & else \end{cases} \tag{12}$$

$$e(t|t) = Inv(t) - Inv(t-1) - \sum_{i=1}^{n} \hat{g}_i \cdot Order(t-i) + Sales(t) \tag{13}$$

$$\begin{aligned}&\delta Order(t+j|t) = Order(t+j|t) - Order(t+j-1|t), \\ &j = 0,...,ch-1\end{aligned} \tag{14}$$

$$u_{\min} \le Order(t+j|t) \le u_{\max}, \quad j = 0,...,ch-1 \tag{15}$$

$$\delta u_{\min} \le \delta Order(t+j|t) \le \delta u_{\max}, \quad j = 0,...,ch-1 \tag{16}$$

$$\delta Order(t+j|t) = 0, \quad j = ch,..., ph \tag{17}$$

where $\hat{Inv}(t+j|t), j = 1,..., ph$ is the *j*- step ahead prediction of inventory, *ph* and *ch* are the prediction and the control

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horizon respectively, $TInv$ is the target inventory value, $\delta Order\left(t+j\left|t\right.\right),\ j=0,...,ch-1$ are the future control moves (Eq. (14)), *w, r* are weight matrices and $e\left(t+j\left|t\right.\right),\ j=1,...,ph$ is the predictor error (Eq. (12)-(13)). Eq. (11) shows that the current value of the predictor is equal to the actual. Eq. (12) denotes that the predictor error should correct only the first prediction since Eq.(10) is an autoregressive model. Eq. (13) gives the predictor error from current sales and inventory value. Eqs. (15)-(16) are hard constraints that bound the manipulated variables and the control moves respectively. $u_{\min}$, $u_{\max}$, are the lower and upper bounds for order rates and $\delta u_{\min}$, $\delta u_{\max}$, are the lower and upper bounds for control moves. Eq. (17) ensures that no control moves are made after the control horizon.

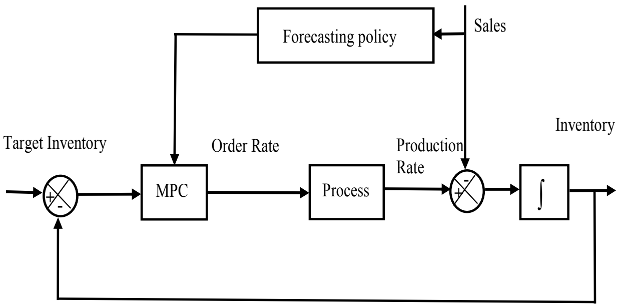

Fig. 3 Block diagram of MPC scheme for inventory control

III. Results

The proposed MPC-forecasting framework was tested in four test cases, where in each case a different forecasting method was employed in order to produce sales forecasts. The forecasting methodologies used were naïve forecasting (for comparison purposes), Linear Autoregression (Linear AR), Holt-Winters and RBF neural networks (RBF ANN).

The test data was supplied by a leading Greek dairy products manufacturer concerning the sales of a fast moving product and the results are shown in Table I. The first column contains the average error for forecasting the sales time series and the second the sum of squared deviations from the inventory set point.

TABLE I
IMPLEMENTATION RESULTS

Method	Average Forecasting Error	Deviation from inventory set point (SSQE)
RBF ANN	0,0534	170,33
Holt	0,0958	182,23
Linear AR	0,1020	179,59
Naïve model	0,1988	400,089

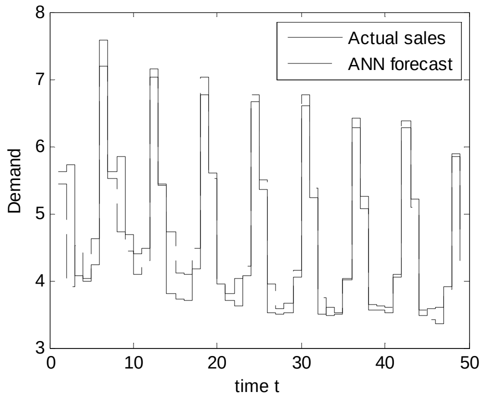

Fig. 4 Comparison between Actual sales and the ANN forecast

These results indicate, first of all, the character of the time series, which is mostly nonlinear since the nonlinear method used (RBF neural networks) provides definitively better forecasts than the two other linear methods. Secondly, it becomes clear that employment of a forecasting methodology leads to improved performance of the MPC module. In particular, the forecasting method that produced the best results, that is RBF neural networks, led to a drastic reduction of the deviation from the inventory set point, thus leading to significantly less inventory holding costs. Figs. 4-6 show the results of the ANN forecasting case of framework implementation to the problem studied. As can be observed in Fig. 4, forecast values are close to the actual values, thus providing an advantageous insight for the future actions of the MPC module of the framework.

Fig. 5 depicts production orders, while Fig. 6 shows the course of product inventory over the time period studied. It must be pointed out that the inventory gradually approaches the set point over the course of time and shows small and decreasing variance from its set point, especially towards the end of the period studied.

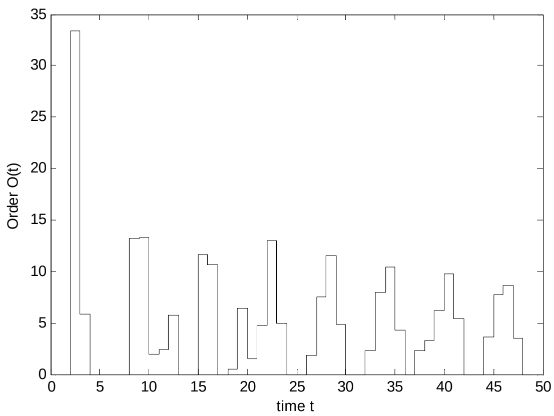

Fig. 5 Production orders (ANN forecasting case)

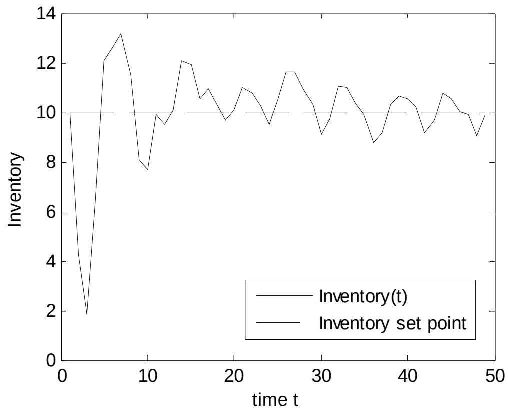

Fig. 6 Product inventory (ANN forecasting case)

IV. Conclusion

A framework for supply chain management based on Model Predictive Control combined with a forecasting module was presented. Various linear and nonlinear forecasting methodologies were evaluated in order to investigate the existence of possible nonlinearity in the sales time series. The nonlinear method used, namely RBF neural networks, exhibited superior forecasting performance, showing that the series had mostly nonlinear character. The simulation results demonstrated that forecast accuracy leads to improved control performance, thus leading to more efficient management of the supply chain.

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