

Mean square stability of impulsive stochastic delay differential equations with markovian switching and poisson jumps

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Abstract—In the paper, based on stochastic analysis theory and Lyapunov functional method, we discuss the mean square stability of impulsive stochastic delay differential equations with markovian switching and poisson jumps, and the sufficient conditions of mean square stability have been obtained. One example illustrates the main results. Furthermore, some well-known results are improved and generalized in the remarks.

Keywords—impulsive, stochastic, delay, Markovian switching, Poisson jumps, mean square stability.

I. INTRODUCTION

MANY evolution processes which are changed at certain moments are always affected by impulsive, such as, medicine, economics, biology, mechanics and so on. In recent years, the stability and other properties of impulsive differential equations have been investigated and many criteria of stability for these systems have been obtained [see[1]-[4]]. Stochastic effects are often taken into account, which is very necessary for good results, and some results of stability for impulsive stochastic delay differential equations (SDDE) have been gotten [see[10]-[13]]. However, the results of impulsive SDDE with jumps are very few, so the investigation is very necessary and valuable.

To the best of author's knowledge, the stability of impulsive SDDE have been studied. But the investigation of these equations which are embedded markov chains and poisson jumps are blank. In this paper, we will have a try to study them to fill the gap.

The markov chain and poisson jumps become very popular in recent years, because they are extensively used to model on many phenomena emerging in a lot of areas. So the first attempt that we investigate the mean square stability of impulsive SDDE with markovian switching and poisson jumps is very necessary.

This paper is organized as follows: In section II, we present some basic preliminaries; In section III, the main result of mean square stability and the proof have been given; In section IV, some well-known results are generalized in the remarks and an example is given to illustrate our conclusion.

II. PRELIMINARIES

Let $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbf{P}\}$ be a probability space with a filtration satisfying the usual conditions, i.e., the filtration

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is continuous on the right and $\mathcal{F}_0$ -contains all $\mathbf{P}$ -zero sets. Let $B(t) = (B_1(t), B_2(t), \dots, B_m(t))^T$ be an m -dimensional Brownian motion defined on the probability space. $\|\bullet\|$ is the Euelidean norm in R^n and $\|x(t)\|_\tau = \sup_{-\tau \leq \theta \leq 0} \|x(t+\theta)\|$.

Let $PC(I, R^n) = \{\phi : I \rightarrow R^n | \phi(t^+) = \phi(t) \text{ for } t \in I; \phi(t^-) \text{ exists for } t \in (t_0, \infty), \phi(t^-) = \phi(t) \text{ for all but points } t_k \in (t_0, \infty)\}$, where $I \subset R$ is an interval, $\phi(t^-)$ and $\phi(t^+)$ denote the left-hand and right-hand limits of function. Let $PC(\delta) = \{\phi : \phi \in PC([-\tau, 0], R^n) \text{ and } \|\phi\|_\tau \leq \delta\}$ and $PC_{\mathcal{F}_0}([-\tau, 0], R^n)$ denote the family of all $\mathcal{F}_0$ -measurable $PC([-\tau, 0], R^n)$ -valued stochastic process $\varphi = \{\varphi(s) : -\tau \leq s \leq 0\}$ such that $\sup_{-\tau \leq s \leq 0} E\|\varphi(s)\|^2 < \infty$, and $PC_{\mathcal{F}_0}^b(\delta) = \{\varphi : \varphi \in PC_{\mathcal{F}_0}^b([-\tau, 0], R^n), \text{ and } E\|\varphi(s)\|^2 < \delta\}$.

Let $\{r(t), t \in R_{t_0} = [t_0, +\infty)\}$ be a right-continuous Markov chain on the probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbf{P}\}$ taking values in a finite state space $S = \{1, 2, \dots, N\}$ with generator $\Gamma = (\gamma_{ij})_{N \times N}$ given by

$$P(r(t+\Delta) = j | r(t) = i) = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & \text{if } i \neq j \\ 1 + \gamma_{ii}\Delta + o(\Delta), & \text{if } i = j \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition rate from i to j , if $i \neq j$. while

$$\gamma_{ii} = -\sum_{j \neq i} \gamma_{ij}.$$

We assume that Markov chain $r(\cdot)$ is independent of the Brownian motion $B(\cdot)$. It is known that almost every sample path of $r(t)$ is right continuous step function with a finite number of simple jumps in any finite sub-interval of R_{t_0} .

Let $\{v(dt, du), t \in R_{t_0}, u \in R\}$ be a centered Poisson random measure with parameter $\pi(du)dt$.

Consider the following impulsive stochastic delay differential equations with markovian switching and poisson jumps:

$$\begin{aligned} dx(t) &= f(t, x(t), x_t, r(t))dt + g(t, x(t), x_t, r(t))dB(t) \\ &\quad + \int_{-\infty}^{+\infty} h(t, x(t), u)v(dt, du) \quad t \geq t_0, t \neq t_k \quad (1) \\ x(t_k) &= H_k(x(t_k^-)) \quad k = 1, 2, 3, \dots \end{aligned}$$

with the initial condition $x_0 = x(t_0 + s) = \varphi(s) \in PC_{\mathcal{F}_0}^b(\delta)$, where $s \in [-\tau, 0]$ and $H_k(x(t_k^-)) = (H_{1k}(x(t_k^-)), H_{2k}(x(t_k^-)), \dots, H_{nk}(x(t_k^-)))^T$ represents the

impulsive perturbation and satisfies the global Lipschitz condition as follows:

$$\|H_k(x(t_k^-))\| \leq M_k \|x(t_k^-)\| \quad M_k \geq 0, k = 1, 2, \dots, \quad (2)$$

the fixed moments of time t_k satisfies $0 \leq t_1 \leq t_2 \leq \dots \leq t_k \leq \dots, \lim_{k \rightarrow \infty} t_k = \infty$.

In the paper, we always assume that under some conditions the system (1) has a unique solution $x(t) = (x_1(t), \dots, x_n(t))^T$ and $x_t = (x_{1t}, \dots, x_{nt})^T, x_{it} = x_i(t - \tau_i), i = \{1, 2, \dots, n\}$, and $\tau = \max_{0 \leq i \leq n} \{\tau_i\}$.

Assume that:

$$f : R \times R^n \times R^n \times S \rightarrow R^n;$$

$$g : R \times R^n \times R^n \times S \rightarrow R^{n \times m};$$

$$h : R \times R^n \times R \rightarrow R^n.$$

Further, assume that $f(t, 0, 0, i) \equiv 0$ and $g(t, 0, 0, i) \equiv 0$ for all $i \in S$, and $h(t, 0, \cdot) \equiv 0$, then system (1) has a trivial solution $x(t) \equiv 0$.

Denote by $C^{2,1}(R^n \times [t_0, \infty) \times S; R_+)$ the family of all non-negative function $V(x, t, i)$ on $R^n \times [t_0, \infty) \times S$ which are continuously twice differential with respect to x and once differential with respect to t .

For any $(x, t, i) \in R^n \times [t_0, \infty) \times S$, define an operator L by

$$\begin{aligned} LV(x, y, t, i) &= V_t(x, t, i) + V_x(x, t, i)f(t, x, y, i) \\ &+ \frac{1}{2} \text{trace}[g^T(t, x, y, i)V_{xx}(x, t, i)g(t, x, y, i)] \\ &+ \sum_{j=1}^N \gamma_{ij} V(x, t, j) + \int_{-\infty}^{+\infty} [V(x + h(t, x, u), t, i) \\ &- V(x, t, i) - V_x(x, t, i)h(t, x, u)]\pi(du), \end{aligned} \quad (3)$$

where

$$\begin{aligned} V_t(x, t, i) &= \frac{\partial V(x, t, i)}{\partial t}; \\ V_x(x, t, i) &= \left(\frac{\partial V(x, t, i)}{\partial x_1}, \dots, \frac{\partial V(x, t, i)}{\partial x_n} \right); \\ V_{xx}(x, t, i) &= \left(\frac{\partial^2 V(x, t, i)}{\partial x_i \partial x_j} \right)_{n \times n}. \end{aligned}$$

The generalized Itô formula reads as follows:

$$\begin{aligned} EV(x(t+h), t+h, r(t+h)) \\ = EV(x(t), t, r(t)) + E \int_t^{t+h} LV(x(s), x_s, s, r(s))ds. \end{aligned} \quad (4)$$

Definition 2.1 The solution of system (1) is mean square stability if for any $\varepsilon > 0$, there exists a scalar $\delta > 0$ and the initial function $\varphi \in PC_{\mathcal{F}_0}^b(\delta)$, such that

$$E\|x(t)\|^2 < \varepsilon, \quad t \geq t_0.$$

III. MAIN RESULTS

Theorem 3.1 Assume that there exist $\lambda_1 > 0, \lambda_2 > 0, \lambda_4 > 0, \lambda_3 \in R$ and a Lyapunov function $V(x, t, i) \in C^{2,1}(R^n \times [t_0, \infty) \times S; R_+)$, such that

$$\begin{aligned} (i) \lambda_1 \|x(t)\|^2 &\leq v(x(t), t, i) \leq \lambda_2 \|x_t\|_\tau^2; \\ (ii) LV(x(t), x_t, t, i) &\leq \lambda_3 V(x(t), t, i) + \lambda_4 V(x_t, t, i) \\ t &\in [t_{k-1}, t_k), \quad k = 1, 2, \dots; \\ (iii) 0 < \lambda < 1, \quad \text{where } \lambda &= \sup\{\lambda_k | \lambda_k = \frac{\lambda_2}{\lambda_1} M_k^2, \\ k &= 1, 2, \dots\}; \\ (iv) (\lambda_3 + \frac{\lambda_4}{\lambda})(t_k - t_{k-1}) &< -\ln \lambda, \quad k = 1, 2, \dots \end{aligned}$$

where $M_k, k = 1, 2, 3, \dots$ have been defined in (2). Then the trivial solution of system (1) is mean square stability.

Proof For any $\varepsilon > 0$, there exists a scalar $\delta = \delta(\varepsilon) > 0$, such that $\delta < \frac{\lambda_1}{\lambda_2} \varepsilon$. For any $t_0 \geq 0$ and $x_0 = \varphi \in PC_{\mathcal{F}_0}^b(\delta)$, let $x(t) = x(t, t_0, \varphi)$ be the solution of system (1).

Due to (4), we obtain that

$$\begin{aligned} EV(x(t), t, r(t)) \\ = EV(x(t_k), t_k, r(t_k)) + E \int_{t_k}^t LV(x(s), x_s, s, r(s))ds, \\ t \in [t_k, t_{k+1}) \end{aligned} \quad (5)$$

For sufficiently small $\Delta t > 0$, such that $t + \Delta t \in [t_k, t_{k+1})$. We get

$$\begin{aligned} EV(x(t + \Delta t), t + \Delta t, r(t + \Delta t)) \\ = EV(x(t_k), t_k, r(t_k)) \\ + E \int_{t_k}^{t + \Delta t} LV(x(s), x_s, s, r(s))ds, \quad t \in [t_k, t_{k+1}] \end{aligned} \quad (6)$$

Using (5), (6) and condition (ii), we observe that

$$\begin{aligned} EV(x(t + \Delta t), t + \Delta t, r(t + \Delta t)) - EV(x(t), t, r(t)) \\ = E \int_t^{t + \Delta t} LV(x(s), x_s, s, r(s))ds, \\ \leq \int_t^{t + \Delta t} (\lambda_3 EV(x(s), s, r(s)) + \lambda_4 EV(x_s, s, r(s)))ds, \\ t \in [t_k, t_{k+1}) \end{aligned}$$

therefore,

$$\begin{aligned} D^+ EV(x(t), t, r(t)) &\leq \lambda_3 EV(x(t), t, r(t)) \\ &+ \lambda_4 EV(x_t, t, r(t)), \quad t \in [t_k, t_{k+1}). \end{aligned}$$

Now we claim that

$$EV(x(t), t, r(t)) \leq \frac{\lambda_2}{\lambda} \delta, \quad t_0 \leq t \leq t_1. \quad (7)$$

Due to $x_0 \in PC_{\mathcal{F}_0}^b(\delta)$ and condition (i), it's obvious that

$$\begin{aligned} EV(x(t), t, r(t)) \\ = EV(x(t_0 + \theta), t_0 + \theta, r(t_0 + \theta)) \\ \leq \lambda_2 E\|x_0\|_\tau^2 \leq \lambda_2 \delta \leq \frac{\lambda_2}{\lambda} \delta, \quad t_0 - \tau \leq t \leq t_0. \end{aligned}$$

If (7) does not hold, then there exists some $s \in (t_0, t_1)$, such that

$$EV(x(s), s, r(s)) > \frac{\lambda_2}{\lambda} \delta > \lambda_2 \delta \geq EV(x(t_0), t_0, r(t_0)).$$

Let

$$s_1 = \inf\{s \in [t_0, t_1] | EV(x(s), s, r(s)) > \frac{\lambda_2}{\lambda} \delta\}.$$

For any $t \in [t_0 - \tau, t_0]$, $EV(x(t), t, r(t)) < \frac{\lambda_2}{\lambda} \delta$, note that $EV(x(t), t, r(t))$ is continuous for variable on $[t_0, t_1]$, then

$$\begin{aligned} EV(x(s_1), s_1, r(s_1)) &= \frac{\lambda_2}{\lambda} \delta; \\ EV(x(t), t, r(t)) &\leq \frac{\lambda_2}{\lambda} \delta, \quad t_0 - \tau \leq t \leq s_1; \\ D^+ EV(x(s_1), s_1, r(s_1)) &\geq 0. \end{aligned} \quad (8)$$

From the inequalities $\frac{\lambda_2}{\lambda} \delta > \lambda_2 \delta$, then there exists $s_2 \in [t_0, s_1]$, such that

$$\begin{aligned} EV(x(s_2), s_2, r(s_2)) &= \lambda_2 \delta; \\ EV(x(t), t, r(t)) &\geq \lambda_2 \delta, \quad s_2 \leq t \leq s_1; \\ D^+ EV(x(s_2), s_2, r(s_2)) &\geq 0. \end{aligned} \quad (9)$$

Combing (8) and (9), we get

$$EV(X_t, t, r(t)) \leq \frac{\lambda_2}{\lambda} \delta \leq \frac{1}{\lambda} EV(x(t), t, r(t)), \quad t \in [s_2, s_1],$$

and

$$\begin{aligned} D^+ EV(x(t), t, r(t)) &\leq \lambda_3 EV(x(t), t, r(t)) + \lambda_4 EV(x_t, t, r(t)) \\ &\leq (\lambda_3 + \frac{\lambda_4}{\lambda}) EV(x(t), t, r(t)) \end{aligned} \quad (10)$$

Therefore, for any $t \in [s_2, s_1]$

$$\int_{s_2}^{s_1} \frac{D^+ EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds \leq \int_{s_2}^{s_1} (\lambda_3 + \frac{\lambda_4}{\lambda}) ds.$$

Applying condition (iii) and (iv), we have

$$\begin{aligned} \int_{s_2}^{s_1} (\lambda_3 + \frac{\lambda_4}{\lambda}) ds &\leq \int_{t_0}^{t_1} (\lambda_3 + \frac{\lambda_4}{\lambda}) ds \\ &= (\lambda_3 + \frac{\lambda_4}{\lambda})(t_1 - t_0) < -\ln \lambda. \end{aligned}$$

So

$$\int_{s_2}^{s_1} \frac{D^+ EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds < -\ln \lambda.$$

At the same time,

$$\begin{aligned} \int_{s_2}^{s_1} \frac{D^+ EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds &= \int_{EV(x(s_2), s_2, r(s_2))}^{EV(x(s_1), s_1, r(s_1))} \frac{du}{u} \\ &= \int_{\lambda_2 \delta}^{\frac{\lambda_2}{\lambda} \delta} \frac{du}{u} \\ &= \ln(\frac{\lambda_2}{\lambda} \delta) - \ln(\lambda_2 \delta) \\ &= -\ln \lambda, \end{aligned}$$

which is a contradiction, so (7) holds.

Combing (2),(7) and condition (i), we get

$$\begin{aligned} EV(x(t_1), t_1, r(t_1)) &= EV(H_1(x(t_1^-)), t_1, r(t_1)) \\ &\leq \lambda_2 E \|H_1(x(t_1^-))\|_\tau^2 \\ &\leq \lambda_2 M_1^2 E \|x(t_1^-)\|_\tau^2 \\ &\leq \frac{\lambda_2 M_1^2}{\lambda_1} \sup_{-\tau \leq \theta \leq 0} EV(x(t_1^- + \theta), \\ &\quad t_1^- + \theta, r(t_1^- + \theta)) \\ &\leq \lambda \frac{\lambda_2}{\lambda} \delta \leq \lambda_2 \delta \leq \frac{\lambda_2}{\lambda} \delta \end{aligned}$$

Now we assume that for $m = 1, 2, \dots, k$, the following inequalities hold,

$$\begin{aligned} EV(x(t), t, r(t)) &\leq \frac{\lambda_2}{\lambda} \delta, \quad t_{m-1} \leq t \leq t_m; \\ EV(x(t_k), t_k, r(t_k)) &\leq \frac{\lambda_2}{\lambda} \delta, \quad k = 1, 2, \dots, \end{aligned} \quad (11)$$

for $m = k + 1$, we claim that

$$EV(x(t), t, r(t)) \leq \frac{\lambda_2}{\lambda} \delta, \quad t_k \leq t \leq t_{k+1}. \quad (12)$$

If (12) does not hold, then there exists some $p \in (t_k, t_{k+1})$, such that

$$EV(x(p), p, r(p)) > \frac{\lambda_2}{\lambda} \delta > \lambda_2 \delta \geq EV(x(t_k), t_k, r(t_k)).$$

Let

$$p_1 = \inf\{p \in [t_0, t_1] | EV(x(p), p, r(p)) > \frac{\lambda_2}{\lambda} \delta\}.$$

For any $t \in [t_{k-1}, t_k]$, $EV(x(t), t, r(t)) < \frac{\lambda_2}{\lambda} \delta$, note that $EV(x(t), t, r(t))$ is continuous for variable on $[t_k, t_{k+1}]$, then

$$\begin{aligned} EV(x(p_1), p_1, r(p_1)) &= \frac{\lambda_2}{\lambda} \delta; \\ EV(x(t), t, r(t)) &\leq \frac{\lambda_2}{\lambda} \delta, \quad t_0 - \tau \leq t \leq p_1; \\ D^+ EV(x(p_1), p_1, r(p_1)) &\geq 0. \end{aligned} \quad (13)$$

From the inequalities $\frac{\lambda_2}{\lambda} \delta > \lambda_2 \delta$, then there exists $p_2 \in [t_k, p_1]$, such that

$$\begin{aligned} EV(x(p_2), p_2, r(p_2)) &= \lambda_2 \delta; \\ EV(x(t), t, r(t)) &\geq \lambda_2 \delta, \quad p_2 \leq t \leq p_1; \\ D^+ EV(x(p_2), p_2, r(p_2)) &\geq 0. \end{aligned} \quad (14)$$

Combing (13) and (14), we get

$$EV(X_t, t, r(t)) \leq \frac{\lambda_2}{\lambda} \delta \leq \frac{1}{\lambda} EV(x(t), t, r(t)), \quad t \in [p_2, p_1],$$

and

$$\begin{aligned} D^+ EV(x(t), t, r(t)) &\leq \lambda_3 EV(x(t), t, r(t)) + \lambda_4 EV(x_t, t, r(t)) \\ &\leq (\lambda_3 + \frac{\lambda_4}{\lambda}) EV(x(t), t, r(t)) \end{aligned} \quad (15)$$

Therefore, for any $t \in [p_2, p_1]$

$$\int_{p_2}^{p_1} \frac{D^+ EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds \leq \int_{p_2}^{p_1} (\lambda_3 + \frac{\lambda_4}{\lambda}) ds.$$

Applying condition (iii) and (iv), we have

$$\begin{aligned} \int_{p_2}^{p_1} (\lambda_3 + \frac{\lambda_4}{\lambda}) ds &\leq \int_{t_0}^{t_1} (\lambda_3 + \frac{\lambda_4}{\lambda}) ds \\ &= (\lambda_3 + \frac{\lambda_4}{\lambda})(t_1 - t_0) < -\ln \lambda. \end{aligned}$$

So

$$\int_{p_2}^{p_1} \frac{D^+ EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds < -\ln \lambda.$$

At the same time,

$$\begin{aligned} \int_{p_2}^{p_1} \frac{D^+ EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds &= \int_{EV(x(p_2), p_2, r(p_2))}^{EV(x(p_1), p_1, r(p_1))} \frac{du}{u} \\ &= \int_{\lambda_2 \delta}^{\frac{\lambda_2}{\lambda} \delta} \frac{du}{u} \\ &= \ln\left(\frac{\lambda_2}{\lambda} \delta\right) - \ln(\lambda_2 \delta) \\ &= -\ln \lambda, \end{aligned}$$

which is a contradiction, so (12) holds.

Combing (2),(12) and condition (i), we get

$$\begin{aligned} EV(x(t_{k+1}), t_{k+1}, r(t_{k+1})) &= EV(H_{k+1}(x(t_{k+1}^-)), t_{k+1}, r(t_{k+1})) \\ &\leq \lambda_2 E \|H_{k+1}(x(t_{k+1}^-))\|_{\tau}^2 \\ &\leq \lambda_2 M_{k+1}^2 E \|x(t_{k+1}^-)\|_{\tau}^2 \\ &\leq \frac{\lambda_2 M_{k+1}^2}{\lambda_1} \sup_{-\tau \leq \theta \leq 0} EV(x(t_{k+1}^- + \theta), t_{k+1}^- + \theta, r(t_{k+1}^- + \theta)) \\ &\leq \lambda \frac{\lambda_2}{\lambda} \delta \leq \lambda_2 \delta \leq \frac{\lambda_2}{\lambda} \delta \end{aligned}$$

By the mathematical induction, we can conclude that

$$\begin{aligned} EV(x(t), t, r(t)) &\leq \frac{\lambda_2}{\lambda} \delta, \quad t_{k-1} \leq t \leq t_k; \\ EV(x(t_k), t_k, r(t_k)) &\leq \frac{\lambda_2}{\lambda} \delta, \quad k = 1, 2, \dots \end{aligned}$$

Therefore

$$EV(x(t), t, r(t)) \leq \frac{\lambda_2}{\lambda} \delta, \quad t \geq t_0,$$

which yields

$$E \|x(t)\|^2 \leq \frac{\lambda_2}{\lambda_1 \lambda} < \varepsilon, \quad t \geq t_0.$$

Now, we can obtain that the solution of system (1) is mean square stability by definition 2.1.

IV. REMARKS AND AN EXAMPLE

Remark 4.1 When $r(t) \equiv 0$ and $h(t, x(t), \cdot) \equiv 0$, the system (1) reduces to

$$\begin{aligned} dx(t) &= f(t, x(t), x_t)dt + g(t, x(t), x_t)dB(t) \\ t &\geq t_0, t \neq t_k \\ x(t_k) &= H_k(x(t_k^-)) \quad k = 1, 2, 3 \dots \end{aligned} \quad (16)$$

with the initial condition $x_0 = x(t_0 + s) = \varphi(s) \in PC_{\mathcal{F}_0}^b(\delta)$, where $s \in [-\tau, 0]$, which is recently studied in the similar literatures. That is to say, we generalize the results of the similar literatures.

Example 4.1 Consider the following impulsive stochastic delay differential equations:

$$\begin{aligned} \begin{pmatrix} dx_1(t) \\ dx_2(t) \end{pmatrix} &= \left[\begin{pmatrix} -10.5 & 0 \\ 0 & -12.2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \right. \\ &\quad + \begin{pmatrix} 1.2 & -0.2 \\ 0.6 & 2.4 \end{pmatrix} \begin{pmatrix} \sin x_1(t) \\ \arctan x_2(t) \end{pmatrix} \\ &\quad + \begin{pmatrix} 1.6 & 0.3 \\ -0.5 & 1.8 \end{pmatrix} \begin{pmatrix} \sin x_1(t - \frac{1}{2}) \\ \arctan x_2(t - \frac{1}{3}) \end{pmatrix} \left. \right] dt \\ &\quad + \begin{pmatrix} 2x_1(t) & x_2(t - \frac{1}{3}) \\ x_1(t - \frac{1}{2}) & -x_2(t) \end{pmatrix} \begin{pmatrix} dB_1(t) \\ dB_2(t) \end{pmatrix}, \\ t &\geq t_0, t \neq t_k \\ \begin{pmatrix} x_1(t_k) \\ x_2(t_k) \end{pmatrix} &= e^{-0.1k} \begin{pmatrix} 0.5 & -0.15 \\ 0.12 & 0.6 \end{pmatrix} \begin{pmatrix} x_1(t_k^-) \\ x_2(t_k^-) \end{pmatrix} \\ k &= 1, 2, 3 \dots \end{aligned} \quad (17)$$

where $t_0 = 0$ and $t_k = t_{k-1} + 0.15$ ($k=1,2,\dots$).

Let $\lambda_1 = 0.5600$, $\lambda_2 = 0.6800$, $\lambda_3 = -1.8670$, $\lambda_4 = 2.1071$ and $M_k = 0.62e^{-0.1k}$, $0 < \lambda = 0.313 < 1$, then $(\lambda_3 + \frac{\lambda_4}{\lambda})(t_k - t_{k-1}) = 0.7293 < 1.1608 = -\ln \lambda$. So the solution of system (17) is mean square stability by our theory.

Remark 4.2 With the process of example 4.1, we can obtain that the conditions of mean square stability have become much easier to be satisfied than the similar literatures.

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