

# A hybrid Tabu Search Algorithm to Cell Formation Problem and its Variants

Tai-Hsi Wu, Jinn-Yi Yeh, Chin-Chih Chang

**Abstract**—Cell formation is the first step in the design of cellular manufacturing systems. In this study, a general purpose computational scheme employing a hybrid tabu search algorithm as the core is proposed to solve the cell formation problem and its variants. In the proposed scheme, great flexibilities are left to the users. The core solution searching algorithm embedded in the scheme can be easily changed to any other meta-heuristic algorithms, such as the simulated annealing, genetic algorithm, etc., based on the characteristics of the problems to be solved or the preferences the users might have. In addition, several counters are designed to control the timing of conducting intensified solution searching and diversified solution searching strategies interactively.

**Keywords**—Cell formation problem, Tabu search

## I. INTRODUCTION

TO make manufacturing systems more efficient and productive, group technology (GT) has been applied within manufacturing environments. GT groups parts with similar design characteristics or manufacturing characteristics into part families. One application of GT is cellular manufacturing system (CMS). A number of benefits arise from adopting CMS, such as: reduced inventory; reduced capacity; reduced labor and overtime costs; shorter manufacturing lead times; faster response to internal and external changes such as machine failures, product mix and demand changes [1]. CMS design requires information, such as parts to be produced, process plans, and machine assignments. The entire production system is decomposed into production cells. Machines are then assigned to these cells to process one or more part families so that each cell is operated independently and so that the intercellular movements are minimized or the number of parts processed within cells is maximized, i.e., parts do not have to move from one cell to the other for processing.

This cell formation process is one of the most important steps in CMS. It becomes difficult to obtain optimal solutions in an acceptable amount of time, especially for problems with large sizes. Extensive research has been devoted to cell formation (CF) problems, with many methods having been proposed for

identifying machine cells and part families. Many of them are developed on the basis of heuristic clustering techniques to obtain approximate solutions, but some of them may be far from optimal. The research of Moon and Kim [2] takes into account the process plans for parts and manufacturing factors such as production volume and cell size. Their process of forming manufacturing cells starts by collecting the above problem data and then converting it into a weighted graph representation in which the nodes represent machines and arcs represent their relationships defined as the value of total part flow between machines.

Although cellular manufacturing may provide great benefits, the design of CMS is complex for real life problems. It has been known that the cell formation problem in CMS is one of the NP-hard combinatorial problems [3]. Many models and solution approaches have been developed to identify machine cells and part families, as it becomes difficult to obtain optimal solutions in an acceptable amount of time, especially for large-sized problems. These approaches can be classified into three main categories: mathematical programming (MP) models [4, 5], heuristic/meta-heuristic solution algorithms [6, 7, 8], and similarity coefficient methods (SCM) [9, 10].

Although the CF has been the focus of researches for several decades, there are several variants of the CF deserve equal attentions on them, such as the CF allowing alternative process routings (APR), CF considering machine reliability issues, and CF considering production volume and operation sequence. Most of the above CF researches assume that each part has a unique process routing. However, it is well known that alternatives may exist in any level of a process plan. When each part has alternative process routings, the CF becomes the generalized CF [11]. Explicit consideration of APR may result in additional flexibility in the CMS design.

Similarly, only a limited amount of research in the context of the CF has dealt with machine breakdowns or reliability issues [12, 13, 14]. Traditionally, CF and work allocation are performed, assuming that all the machines are 100% reliable. However, this is not always the case. Their breakdowns can dramatically affect system performance measures and bring about detrimental effects on the due date performance. Machine failures should hence be taken into account during the design of CMS to improve the overall performance of the system.

As to the issue of incorporating operation sequence into the CF, Harhalakis et al. [15] considered the sequence of operation and suggested three evaluation criteria for judging the

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performances of the solutions obtained. Nair & Narendran [16] accounted the sequence of operation and within-cell compactness (defined as the ratio of the number of operations within it to the maximum number of operations possible in it), and proposed a new solution measure called “bond efficiency.”

## II. PROBLEM DEFINITION

### A. Simple CF Problem

Cell formation in a given 0-1 machine-part incidence matrix involves rearrangement of rows and columns of the matrix to create part families and machine cells. In this research, we attempt to determine a rearrangement so that the inter-cellular movement can be minimized and the utilization of the machines within a cell can be maximized. Two matrices shown in Fig. 1 are used to illustrate the concept. Fig. 1(a) is an initial matrix where no blocks can be observed directly. After rearrangement of rows and columns, two blocks can be obtained along the diagonal of the solution matrix in Fig. 1(b). After the rearrangement, blocks can be observed along the diagonal of the matrix. In the matrix, any 1s outside the diagonal blocks are called “exceptional elements”; any 0s inside the diagonal blocks are called “voids”.

### B. CF Problem with Alternative Process Routings

Cases in which each part may have more than one process routings are more complicated than the simple CF. A process routing for a given part is a set of machines that have passed by this part. It is assumed that the sequence of machines in each

process routing is identical with the operation sequence of the corresponding part. When parts are allowed to have more than one process routing, such as the case shown in Fig. 2, the CF becomes generalized, wherein cases are more complicated than the simple cell formation problem. Under this circumstance, the formation of part families, machine cells, and selection of routings for each part need to be determined to achieve the decision objectives, such as the minimization of intercellular movement or the maximization of grouping efficiency or grouping efficacy to be defined in the next section.

### C. Performance Measures

There have been several measures of goodness of machine-part groups in cellular manufacturing in the literature. Two measures frequently used are the grouping efficiency [17] and the grouping efficacy [18].

Grouping efficiency  $\eta$  is defined as follows:

$$\eta = q\eta_1 + (1-q)\eta_2$$

where  $\eta_1$  is the ratio of the number of 1s in the diagonal blocks to the total number of elements in the diagonal blocks of the final matrix,  $\eta_2$  is number of 0s in the off-diagonal blocks to the total number of elements in the off-diagonal blocks of the final matrix, and  $q$  is a weight factor.

Machines	Parts				
	P	P	P	P	P
	1	2	3	4	5
M1	1	0	0	1	0
M2	0	1	1	0	1
M3	1	0	0	1	0
M4	0	1	1	0	1
M5	1	0	0	1	0

(a) initial matrix

Machines	Parts				
	P	P	P	P	P
	2	3	5	1	4
M2	1	1	1	0	0
M4	1	1	1	0	0
M1	0	0	0	1	1
M3	0	0	0	1	1
M5	0	0	0	1	1

(b) matrix after rearrangement

Fig.1 rearrangement of rows and columns of matrix to create cells

	P1			P2			P3			P4			P5			P6			P7				P8	
	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3	R4	R1	R2			
M2		1	1		1	1					1		1			1	1	2		1				
M6		2															2		3	2	2			
M7											2	3	2	2										
M1	1			1			1	1	1	1		1		1	1					1	1			
M3							2	2	2	2														
M4	2			2				3	3	3		2			2			2						
M5	3		2	3	2	2	3		4						3			3						
M8		3		4		3	4	4	4	5				3	3					4				
M9	4		3		3						3	4				4	3							

Fig.2 Machine-part matrix involving alternative process

Although grouping efficiency has been used widely, critics argued that in some cases the size of the matrix impaired its discrimination ability. Kumar and Chandrasekharan [18] hence proposed another measure to overcome this problem, the grouping efficacy  $\Gamma$ , and can be defined as:

$$\Gamma = \frac{e - e_0}{e + e_v}$$

where  $e$  is the total number of 1s in the matrix;  $e_0$  is the total number of exceptional elements; and  $e_v$  is the total number of voids. Grouping efficacy ranges from 1 to 0, with 1 being the perfect grouping.

In addition to the grouping efficiency and grouping efficacy, the number of inter-cell flow has been another commonly used performance measure.

## III. PROPOSED SOLUTION SCHEME

Tabu Search (TS) is a meta-heuristic algorithm developed by Glover which has been successfully used to generate solutions for a wide variety of combinatorial problems. The main ideas of TS are to avoid recently visited area of the solution space and

to guide the search towards new and promising areas. Non-improving moves are allowed to escape from the local optima, and attributes of recently performed moves are declared tabu or forbidden for a number of iterations to avoid cycling [19, 20].

Although tabu search, together with other meta-heuristic algorithms such as genetic algorithm (GA), simulated annealing (SA), neural network (NN), have had excellent performances in solving combinatorial optimization problems through various well designed solution mechanisms, it is inevitable that solutions generated by these algorithms get trapped on local optima. A function or procedure guiding the solution to escape from the local optima and proceed toward more diversified solution space is hence strongly needed to search for the global optima. The mutation operator of the GA plays this role.

In GA, some initial solutions are selected to be parents to generate offspring via the crossover operator. All the solutions are then evaluated and selected based on Darwin's concept of survival of the fittest. The process of reproduction, evaluation, and selection is repeated until the stopping criterion is met. In GA, the mutation operator is usually applied to solutions at hand with a certain probability to escape from local solutions and/or to prevent premature convergence. This special feature of the mutation operator provides a higher degree of diversification in the solution searching process.

The proposal of this hybrid TS is to expect that the synergy effects from both the TS and the GA can be appreciated through a proper collocation of both techniques to solve the cell formation problem and its variants. Fig.1 shows the proposed general purpose computational scheme.

It can be observed from Fig. 1 that the proposed computational scheme is actually consists of the following seven steps:

- Step 1.* Initialization including number of cells and computational parameters;
- Step 2.* Construction of initial solution;
- Step 3.* Searching of improving neighborhood solutions;
- Step 4.* Update of Tabu List;
- Step 5.* Update of better solutions found;
- Step 6.* Check of timing for directing searching toward diversified solution space by applying mutation operator;
- Step 7.* Check of solution stagnancy.

Note that the first five steps are the same as most of the solution searching algorithm, while *Step 6* generates new solutions with higher degree of diversification in order to increase the probability of finding the global optima, and *Step 7* avoids spending too much computational efforts in order to have a balance between the computational effectiveness and efficiency.

#### IV. DISCUSSION

The proposed general purpose computational scheme shown in Fig. 1 can be used to solve the CF problem and its variants. This section provides discussions on the design of each step of the scheme in detail, when dealing with various types of CF problems.

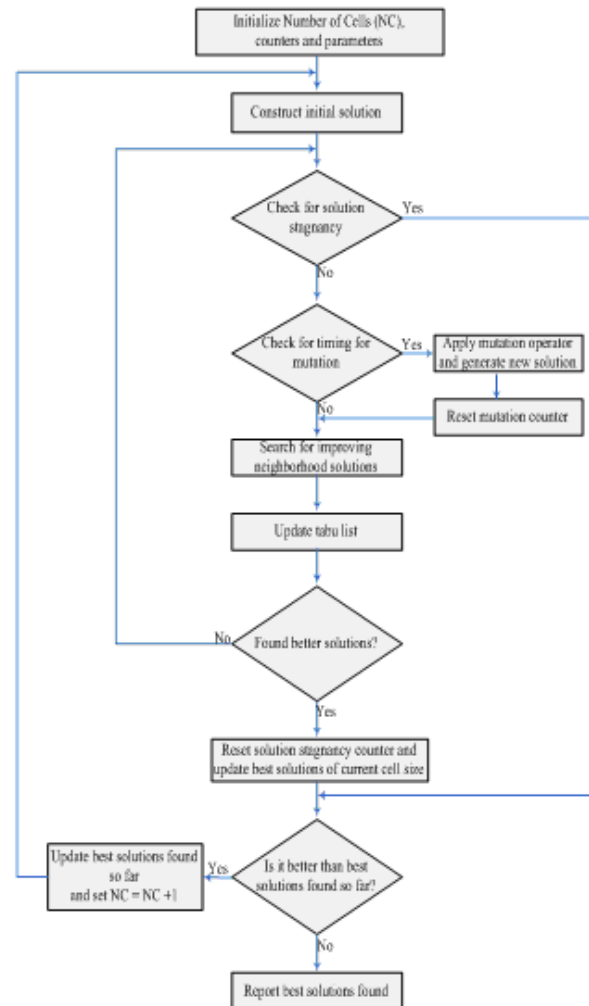


Fig. 1 The proposed general purpose computational scheme

##### A. Initial Solutions

###### A.1 Simple CF Problem

A large number of similarity coefficients methods (SCM) and rank order clustering (ROC) techniques have been proposed for the CF problem to generate quick initial solutions for later improvement. Since the CF problem considers the grouping of parts and machines, an intuitive solution approach is to decompose the entire problem into two subproblems dealing with the assignment of parts and machines, respectively. The SCM methods determine the assignment of parts and machines in two separate stages, while the ROC method forms the initial part families and machine cells in one

procedure. From the literature, it can be seen that the rules of assigning parts and machines have been very rich in variety. The users can adopt any of them or create their own when constructing the initial solutions.

#### A.2 CF Problem with Alternative Process Routings

A large number of SCM methods have been proposed for grouping entities such as parts or machines in the simple CF problem, far fewer SCMs have been designed especially for the CF problem with alternate routings. The works by Kusiak and Cho [21], and by Won and Kim [22] can be considered as the two most widely used approaches. Kusiak and Cho's method is a part-based approach which uses a similarity coefficient defined between part routings, while Won and Kim's study defined a machine-based similarity coefficient in their algorithm. Due to the fact that machine-based similarity coefficient method suffers less computational burden, it is usually used to generate the similarity matrix for forming machine cells when constructing the initial solution in this research.

After the machine cells have been obtained, the next task is to assign a process routing for each part to the machine cells. Won and Kim [22] performed the work based on the maximum density rule, while Wu et al. [23] assigned the part routings to machine cells that will result in the least number of exceptional elements. Again, the users can adopt any of them or create their own when constructing the initial solutions for the CF with APR.

#### B. Searching of Improving Neighborhood Solutions

The tabu search has been chosen in this work as the core algorithm embedded in the proposed computational scheme to search for better neighborhood solutions. Other meta-heuristics, such as the SA, GA, NN or local searching algorithms, can be used to gear to the proposed computational scheme as well. However, the above-mentioned solution searching algorithms would not be effective without an elaborate design for finding improving neighborhood solutions. The design greatly depends upon the full understanding of the characteristics of problems under study.

#### C. Searching Toward Diversified Solution Space

No matter how well the solution searching algorithms may have performed in solving combinatorial optimization problems through various well designed solution mechanisms, it is inevitable that solutions get trapped on local optima. A function or procedure guiding the solution to escape from the local optima and proceed toward more diversified solution space is strongly needed to search for the global optima. The mutation operator of the GA plays this role. SA provides this capability through the usage of Boltzmann function.

After generating an initial solution, SA attempts to move from the current solution to one of the neighborhood solutions. The changes in the objective function values ( $\Delta E$ ) are calculated. If the new solution results in a better objective

value, it is accepted. However, if the new solution yields a worse value, it can still be accepted according to a probability function, i.e., the Boltzmann function,  $P(\Delta E) = \exp(-\Delta E/k_B T)$ , where  $k_B$  is Boltzmann's constant and  $T$  is the current temperature. This check is done by selecting a random number from (0, 1). If the random number chosen is less than or equal to the probability value, the new solution is accepted; otherwise, it is rejected. By accepting worse solutions, SA can avoid being trapped on local optima. The parameter  $T$  is gradually decreased by a cooling function as SA proceeds until the stopping criterion is met.

Apart from the mutation operator of the GA and the Boltzmann function of the SA, several diversification strategies can be found in Rolland et al. [24] and Sun et al. [25].

### V. CONCLUDING REMARKS

In this study, a general purpose computational scheme employing a hybrid tabu search algorithm as the core has been proposed to solve the cell formation problem and its variants. In the proposed scheme, great flexibilities are left to the users. The core solution searching algorithm embedded in the scheme can be easily changed to any other meta-heuristic algorithms, such as the simulated annealing, genetic algorithm, etc., based on the characteristics of the problems to be solved or the preferences the users might have. In addition, several counters have been used in the algorithm to speed up the solution search process and to direct the searching toward diversified solution space to escape from the local optima.

Issues regarding the design of each step of the proposed scheme in detail have been discussed, including the construction of initial solutions, searching for improving neighborhood solutions, and searching toward diversified solution space, when solving the simple CF and its variants.

Having this general purpose computational scheme on hand, the next task is to design corresponding solution algorithm for each type of CF problems. Further analyses of the resulting computational performances and comparisons with benchmark results from the literature follow in order to validate the proposed scheme and algorithms.

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