International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:6, No:7, 2012

Ride Control of Passenger Cars with Semi-active Suspension System Using a Linear Quadratic Regulator and Hybrid Optimization Algorithm

Ali Fellah Jahromi, Wen Fang Xie, Rama B. Bhat

Abstract**—**A semi-active control strategy for suspension systems of passenger cars is presented employing Magnetorheological (MR) dampers. The vehicle is modeled with seven DOFs including the, roll pitch and bounce of car body, and the vertical motion of the four tires. In order to design an optimal controller based on the actuator constraints, a Linear-Quadratic Regulator (LQR) is designed. The design procedure of the LQR consists of selecting two weighting matrices to minimize the energy of the control system. This paper presents a hybrid optimization procedure which is a combination of gradient-based and evolutionary algorithms to choose the weighting matrices with regards to the actuator constraint. The optimization algorithm is defined based on maximum comfort and actuator constraints. It is noted that utilizing the present control algorithm may significantly reduce the vibration response of the passenger car, thus, providing a comfortable ride.

Keywords**—**Full car model, Linear Quadratic Regulator, Sequential Quadratic Programming, Genetic Algorithm

I. INTRODUCTION

REDUCING the ride vibration in passenger cars with effective suspension systems has been one of the main challenges in the automobile industry since it has a direct influence on the safety and comfort of passengers. The suspension system in a passenger car consists of tires, springs and dampers which may be modeled as linear elements. One of the strategies to improve ride quality is to control the damping force in suspension systems [1], as in semi-active systems. Many schemes have been proposed in order to increase the damping and in turn to increase the amount of dissipated energy. The aim of ride control is to reduce the vehicle vibrations produced by the bumpy road and consequently to minimize the energy per each cycle of vehicle body oscillations. Ride control can be defined as full-active (active) or semi-active depending on whether an actuator is used in the suspension system or not.

A. F. Jahromi is with the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, QC, H3G 2W1, Canada, phone:+1(514) 848-2424 Ext. 7268, (e-mail: al_fel@encs.concordia.ca).

W. F. Xie is with the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, QC, H3G 2W1, Canada, (e-mail: wfxie@encs.concordia.ca).

R. B. Bhat is with the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, QC, H3G 2W1, Canada, (e-mail: rbhat@alcor.concordia.ca).

In other words, if the suspension system consists of only an active part (no passive elements such as damper and spring) ride control is called full-active [2]. But if the suspension system consists of an actuator, which can continuously vary the rate of the energy dissipation (without adding energy) using a controllable damper, it is called semi-active system [3]. The semi-active suspension system was developed by Karnopp [1], who applied it to a quarter-car model with a variable damping force and a combination of variable damping force and load leveling. Karnopp [1] derived the state space equations of the system using the bond graph method. The quarter car model has been used by other researchers who continued the study of Karnopp [1]. The present paper focuses on the research on semi-active models with the effect of the actuator (MR damper) dynamics. Magnetorheological (MR) oil is a smart material whose viscosity changes with applied magnetic field. This property of MR oil makes it favorable in smart shock absorbers in industrial applications. There are many analytical and experimental studies on the application of MR dampers. For instance, Du et al. [4] studied the application of MR damper as an actuator in the quarter-car models both theoretically and experimentally. Based on their experimental results, they suggested a polynomial force for the MR damper as a function of current. This research shows the margin of desired force for a medium size passenger car (weight: 2000 kg, wheelbase: 4.6 m, and track: 1.8 m) is around 2000 N for a current around 1 A. Fallah et al. [5] suggested a force control strategy for a quarter-car model including the kinematics of the Macpherson suspension system. They studied the hysteretic behavior of the MR damper theoretically and conducted a series of experiments similar to those of Du et al. [4]. In this research, the authors studied the response of the system by applying the velocity to one end of the MR damper based on the simulation analysis of the damper. They examined the response of the simulation (state variables and the input current of MR damper). Fallah et al. [5] used an inverse method to find the force of the MR damper (around 800 N) and compared it with the desired force. The results obtained from the simulation showed that the damper can approximately follow the desired force.

Studies of Du et al. [4] and Fallah et al. [5] are a combination of experiment and theory, which are based on the quarter-car model.

ISSN: 2517-9950
Vol:6, No:7, 2012

Hence, they could study only the bounce motion of the vehicle, and neglected the effect of roll and pitch. However, the real dynamic motion of the vehicle consists of roll, pitch and bounce which are coupled. Therefore, by neglecting the effect of roll and pitch motion in the quarter-car model the bounce motion will be unrealistic and possibly overestimated.

Full-car model of a suspension system will have at least seven DOFs: roll, pitch and bounce of the body, and four DOFs for the tires. Cheng et al. [6], and Jahromi and Zabihollah [7] studied the semi-active suspension system by using a linear full-car model with seven DOFs. They investigated the effectiveness of the MR damper on reducing the vibrations of the vehicle body. The results include the effect of semi-active suspension system on the ride quality and the reduction of the roll and pitch motions due to the bumpy road in straight motion of the vehicle.

Guclu [8] studied the semi-active suspension system considering the effect of passenger seat motion by using a full-car model with eight DOFs. The effect of dry friction in the suspension mechanism was considered in this work, and an electric motor was used under the passenger seat as an extra actuator. The dry friction makes the dynamic equations of the full-car model nonlinear; therefore, Guclu [8] selected a non-model based control strategy (PID) to improve the ride quality. The results show that the controller totally absorbs the vibration of all the DOFs of the system. However, the study did not report the required force and therefore it is not clear whether the force is in the feasible region.

Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian [10] (LQG) methods are optimal control strategies which can be used in the linear Multi-Input Multi-Output (MIMO) control system. The suspension system in most of the studies was modeled using a linear formulation; therefore, LQR controller as a state feedback control strategy was utilized to improve the ride quality of the passenger car [7], [9], [10]. The LQR control gain is an optimal pole placement gain which is achieved based on the minimization of the actuation energy, and guarantees the stability of the system. The mathematical procedure to find the LQR gain strongly depends on two weighting matrices Q and R whose size is related to the number of inputs and state variables; therefore, the matrices can have any elements to satisfy the positive semi definite condition for Q and positive definite condition for R, [7], [9], [10]. Although the Q and R matrices are important in the efficiency of the controller, none of the above studies in [7], [9], [10] present a method to identify these matrices.

The Q and R matrices can be found using the Sequential Quadratic Programming (SQP) method and linear equality and inequality constraints [11], [12]. However, the SQP method cannot guarantee finding the global optimum point since the optimization procedure depends on the initial point. In order to select the initial point near the global optimum point, the Genetic Algorithm (GA) [13] is utilized.

In the present study the hybrid optimization algorithm is developed in order to identify the optimal Q and R matrices for LQR controller for ride control of passenger cars with MR damper. This algorithm is a combination of the gradient-based (SQP) and evolutionary methods (GA) which uses the advantages of both algorithms.

In the following sections of this paper, first, the dynamic modeling of the passenger car is presented in the dynamic analysis of semi-active suspension system. Then, the control algorithm is discussed in the optimal control section, and the optimization methods are presented with respect to the required parameters for optimal control design. The response of the control system using a hybrid optimization method is presented and discussed in the results. Finally, this study is summarized in the conclusion.

II. Dynamic Analysis of Semi-active Suspension System

The full-car model consists of springs, passive and variable damping components, tires with coupling dynamic effect of the passenger car suspension system. The effect of roll, pitch and bounce of the vehicle model have seven DOFs: four for the tires and three for the body (two for roll and pitch and one for the bounce).The MR damper of the present model has a static viscosity and variable force, and therefore, the damping force of the car dampers is divided into passive and variable forces. The vehicle is schematically shown in Figs.(1- 2).

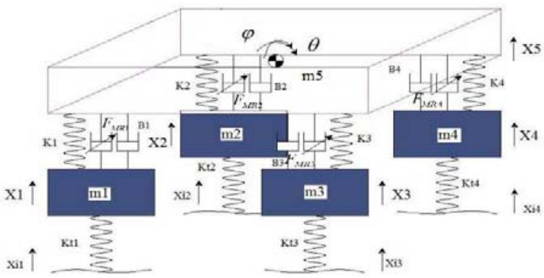

Fig. 1 Full-car model with 7 DOFs

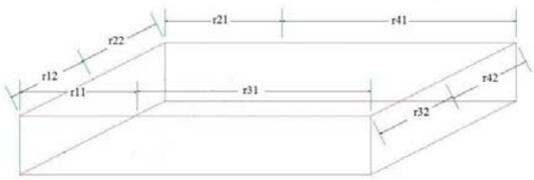

Fig. 2 Distance between wheels and the center of gravity of the body

The kinematic relations among the DOFs are defined in (1)-(4). While (5) defines the tire dynamic motion, (6) describes bounce of the vehicle body and the equations for the pitch and roll motions are presented in (7)-(8). The roll center is defined based on the type of the suspension system, and is usually different from the center of gravity of the vehicle. The kinematics of the suspension system is considered in the modeling of the vehicle by (8) which presents the effect of the difference between the position of the roll center and center of gravity on body motion.

$$V_1 = x_1 - x_5 - \varphi r_{11} - \theta r_{12} \tag{1}$$

$$V_2 = x_2 - x_5 - \varphi r_{21} + \theta r_{22} \tag{2}$$

$$V_3 = x_3 - x_5 + \varphi r_{31} - \theta r_{32} \tag{3}$$

$$V_4 = x_4 - x_5 + \varphi r_{41} + \theta r_{42} \tag{4}$$

$$m_j \ddot{x}_j + K_j V_j + B_j \dot{V}_j + K_{tj} x_j = K_{tj} x_{ij} - F_j \quad {}_{j=1-4} \tag{5}$$

$$m_5 \ddot{x}_5 - \sum_{j=1}^{4} V_j K_j - \sum_{j=1}^{4} \dot{V}_j B_j = \sum_{j=1}^{4} F_{MR_j} \quad {}_{j=1-4} \tag{6}$$

$$\begin{aligned} & I_z \ddot{\varphi} - \sum_{j=1}^{2} r_{j1} V_j K_j - \sum_{j=1}^{2} r_{j1} \dot{V}_j B_j \\ & + \sum_{j=3}^{4} r_{j1} V_j K_j - \sum_{j=3}^{4} r_{j1} \dot{V}_j B_j = \sum_{j=1}^{4} r_{j1} F_{MR_j} \quad {}_{j=1-4} \end{aligned} \tag{7}$$

$$\begin{aligned} & \left(I_y - \frac{h_s m_5}{m_5 + \sum_{j=1}^{4} m_j} \right) \ddot{\theta} - r_{12} K_1 V_1 - r_{32} K_3 V_3 + \\ & r_{22} K_2 V_2 + r_{42} K_4 V_4 - r_{12} B_1 \dot{V}_1 - r_{32} B_3 \dot{V}_3 + r_{22} B_2 \dot{V}_2 \\ & + r_{42} B_4 \dot{V}_4 + M g h_s \theta = \sum_{j=1}^{4} r_{j2} F_{MR_j} \quad {}_{j=1-4} \end{aligned} \tag{8}$$

where M, m_1, m_2, m_3 and m_4 stand for the mass of the car body and the four wheels, respectively. I_x and I_y are the moments of inertia of the car body about the roll and pitch axes, respectively. The terms k_1, k_2, k_3 and k_4 are the stiffnesses of the front and rear springs of the suspension system. The terms k_{t1}, k_{t2}, k_{t3} and k_{t4} are the stiffnesses of the tires. B_1, B_2, B_3 and B_4 are the damper coefficients. While x_1, x_2, x_3 and x_4 indicate the DOFs of the tires, x_5, θ and φ are the DOFs of the car body, and x_{i1}, x_{i2}, x_{i3} and x_{i4} denote the road disturbance (displacement). The numerical values of these variables are presented in Table I.

TABLE I
NUMERICAL VALUES FOR THE FULL-CAR MODEL

Symbol	Quantity	Value
m_5	Mass of car body	1400 (kg)
I_z	Inertia around pitch axis	1271.1 (kg.m^2)
I_y	Inertia around roll axis	2745.6 (kg.m^2)
$m_{1\text{-}4}$	Mass of one wheel	40 (kg)
$K_{1\text{-}4}$	Stiffness of the car springs	18000 (N/m)
$K_{t1\text{-}4}$	Stiffness of the tires	150000 (N/m)
$B_{1\text{-}4}$	Viscosity of the shock absorber	1100 (Ns/m)
$R_{1\text{-}2\,1}$	The distance between the front wheels and center of gravity	1.1 (m)
$R_{3\text{-}4\,1}$	The distance between the rear wheels and center of gravity	1.6 (m)
$R_{1\text{-}2\,2}$	The distance between the right wheels and center of gravity	0.85 (m)
$R_{3\text{-}4\,2}$	The distance between the left wheels and center of gravity	0.85 (m)
h_s	The distance between the center of gravity of the body and ground	0.505 (m)

III. Optimal Control

The LQR is employed as an optimal pole placement method which uses a state feedback control strategy.

The Control objective is to minimize the bounce velocity, and roll and pitch rates of the body. This method can be implemented in the state space representation of the system. The full-car model is a mass-spring system with seven DOFs. The (9) presents the dynamic equations of motion in matrix form, and the state space equations are shown in (10).

$$M\ddot{x}_s + B\dot{x}_s + Kx_s = F_{MR} + F_{Road} \tag{9}$$

$$\begin{aligned} \dot{z}_s &= Az_s + B_{MR} u + Wv \\ Y &= Cz_s + Du \end{aligned} \tag{10}$$

where matrices *M*, *B*, *K*, F_{MR} and F_{Road} show mass, damping, stiffness, actuating force and disturbance force due to the road profile, respectively, matrices *A*, B_{MR} and *W* represent the dynamic properties of the system, the effect of the actuator, and the effect of road profile, respectively, matrix *C* is identity matrix of size 14×14 which defines the output of the system, and matrix *D* is the static effect which is zero in this model. The vectors *u* and *v* are input of the system and disturbance of the road and parameter *Y* represents the output of the system. The vector x_s presents the variables for each DOFs. The matrices and state variables are given by:

$$A = \begin{bmatrix} 0_{7\times7} & I_7 \\ -M^{-1}K & -M^{-1}B \end{bmatrix}_{14\times14} \tag{11}$$

$$B_{MR} = \begin{bmatrix} 0_{7\times4} \\ M^{-1}F_{MR} \end{bmatrix}_{14\times14} \tag{12}$$

$$W = \begin{bmatrix} 0_{7\times4} \\ M^{-1}F_{Road} \end{bmatrix}_{14\times14} \tag{13}$$

$$\begin{aligned} z_s = [&x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad \varphi \quad \theta \ldots. \\ &\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3 \quad \dot{x}_4 \quad \dot{x}_5 \quad \dot{\varphi} \quad \dot{\theta}]^T \end{aligned} \tag{14}$$

The LQR responds by changing the location of poles of the system to the optimal place. Time response, overshoot and steady-state depend on the location of the system poles. The LQR controls the system by a matrix gain, which is obtained from (15). To solve the power equation of the system, Riccati equation, shown in (16), was used. In this equation, *Q* is a symmetric positive semi-definite matrix and *R* is a symmetric positive definite matrix. The matrices *Q* and *R* are very effective in the controller performance and actuator energy. Therefore, these parameters should be selected based on the controller desired and the maximum energy of the actuator. Finally, the LQR gain can be achieved from Eq. 17 to change the location of system poles by changing *A* matrix as shown in (18). The mathematical procedure of the LQR design guarantees the stability of the control system. However, it cannot minimize the amount of the required actuator energy by selecting the *Q* and *R* matrices as arbitrary matrices.

$$J = \frac{1}{2} \int_0^{\infty} (x^T(t) Q x(t) + u^T(t) R u(t)) dt \tag{15}$$

$$A^T S + SA + SB_{MR} R^{-1} B_{MR}^T S + Q = 0 \quad (16)$$

$$K_{LQR} = R^{-1} B_{MR}^T S \quad (17)$$

$$A_{LQR} = A - B_{MR} K_{LQR} \quad (18)$$

IV. Optimization Algorithms

In order to achieve optimal values of the Q and R matrices, the objective function is chosen as the bounce velocity, roll and pitch rate. This problem is a multi-input multi-output (MIMO) control problem. The bounce velocity, roll rate and pitch rate must be minimized based on the effect of each MR damper on them. As a result, the system has twelve objective functions which are mathematically shown in (19) and (20). The objective is to minimize the surrounding area of the response of the control system to bounce velocity, pitch and roll rates with respect to the actuation of each MR damper. Fig. 3 shows the objective function of bounce velocity due to the actuation of the MR damper in the front left of vehicle body.

$$Area = \int_0^t |\dot{x}|\, dt \quad (19)$$

$$\dot{x} = (A - B_{MR_i} K_{LQR_j}) x + B_{MR_i} K_{LQR_j} u \quad (20)$$

where j is the number of the LQR gain matrix rows and i is the number of the input matrix columns which show the effect of each MR damper.

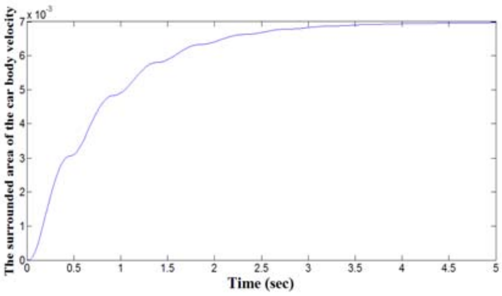

Fig. 3 The surrounded area for the objective function

The constraint involved in the optimization procedure should be defined based on the actuator specifications. The regular MR damper for a normal passenger car can produce a force up to 2200 N. Therefore, the main constraint for the optimization algorithm is to find Q and R matrices to obtain the force in the feasible region of the actuator. In order to define this constraint for the system, the abovementioned condition for each MR damper should be considered. The mathematical definitions of the inequality constraints are defined in (21) and (22).

$$F_{MR_j} = K_{LQR_i} \dot{x}_5 + K_{LQR_i} \dot{\varphi} + K_{LQR_i} \dot{\theta} \quad (21)$$

$$\begin{aligned} & h_k < 0 \quad {}_{k=1-4} \\ & h_k = F_{MR_k} - 2200 \end{aligned} \quad (22)$$

The other constraint for optimizing the Q and R matrices is the controller performance; these matrices have direct effect on the efficiency of the controller. When the coefficient of Q is greater than that of R, a larger force is achieved; however, the energy of the system is not minimized under these conditions. In order to find the best performance with minimum energy, the coefficients of the Q matrix should be always smaller than that of the R matrix (23). The goal of LQR in this study is to control the response of the system with the best performance in the feasible region of the actuator.

$$\begin{aligned} & h_5 < 0 \\ & h_5 = R_c - Q_c \end{aligned} \quad (23)$$

where Q_c and R_c are coefficients of the matrices Q and R.

The objective and constraint functions to optimize the Q and R matrices are nonlinear expressions (19)-(23). Therefore, Quadratic Programming (QP) as an optimization method for problems with nonlinearity in the objective or/and constraint functions is utilized to find the optimum value of the Q and R matrices. The QP changes the objective and constraint functions into linear observations. The mathematical procedure for these functions are shown in (24)-(26) [14].

$$Area(Q_c, R_c) = D_c^T d + \frac{1}{2} d^T d \quad (24)$$

$$N^T d = e \quad (25)$$

$$A_{in}^T d \le b \quad (26)$$

where the constant e is a value used in equality conditions which do not exist in the present problem. The constant b is a value that represents inequality constraints and is set equal to 2200 N in this study. The A_{in} and N matrices are coefficient matrices in the equality and inequality equations, respectively. The vector d is the direction vector and D_c is the derivative matrix defined as:

$$D_{Cn} = \frac{\partial Area(Q_c, R_c)}{\partial x_n} \quad x_n = Q_c, R_c \quad (27)$$

The SQP method models a nonlinear problem in the current point by defining a quadratic sub-problem. The solution of the sub-problem is then used to find a new point. The Karush-Kuhn-Tucker (KKT) necessity conditions are an optimization method which can be employed to solve the QP sub-problem (linear objective and constraints) by Newton's method [14]. The KKT conditions define the linear objective and constraint functions as multiple equations with multiple variables which should be solved together to satisfy the KKT conditions in order to reach to the optimal point.

Mathematical procedure of the KKT is presented in (28)-(31). Equation (28) is the Lagrangian function, and (29)-(31) are gradient conditions [14].

$$L(Q_c,R_c,u,s)=Area(Q_c,R_c)+\sum_{k=1}^{p}v_k^*h_k(Q_c,R_c) \\ +\sum_{k=1}^{p}v_k^*(g_r(Q_c,R_c)+s_r^2)\quad x_n=Q_c,R_c \tag{28}$$

$$\frac{\partial L}{\partial x_n}=\frac{\partial Area(Q_c,R_c)}{\partial x_n}+\sum_{k=1}^{p}v_k^*\frac{\partial h_k(Q_c,R_c)}{\partial x_n} \\ +\sum_{k=1}^{p}v_k^*\frac{\partial g_r(Q_c,R_c)}{\partial x_n}\quad x_n=Q_c,R_c \tag{29}$$

$$\frac{\partial L}{\partial v_k}=0\Rightarrow h_k(x_n^*)=0\quad x_n^*=Q_c^*,R_c^* \tag{30}$$

$$\frac{\partial L}{\partial u_r}=0\Rightarrow g_r(x_n^*)+s_r^2=0\quad x_n^*=Q_c,R_c \tag{31}$$

where h and g are equality and inequality functions, and k and r are the number of equality and inequality equations. The u parameter is the Lagrangian multiplier for equality constraint, and s is an additional variable to make equality from inequality equations. The derivative function, v, is defined as:

$$v=\frac{\partial Area(Q_c^*,R_c^*)/\partial x_n}{\partial h(Q_c^*,R_c^*)/\partial x_n} \tag{32}$$

Where the Q^*_c and R^*_c are initial points to start the optimization procedure for the Q and R coefficient matrices.

In optimization problems, if the objective functions have only one optimum point with respect to the constraints, the optimization algorithm will be independent of the starting point. The present optimization problem can have infinite local optimum points corresponding to the constraints. The gradient based methods find the first optimum point near the initial point. As a result, the KKT method in this problem depends on the initial point to search for the optimal point, and the algorithm cannot guarantee finding a global optimum point. Therefore, in order to choose the initial point near the global optimum point, the GA is utilized.

The GA is an optimization method for multi variables, linear or nonlinear objective functions and unconstrained or constrained problems [14]. This optimization method is a popularity search algorithm which avoids local minimum area. Therefore, the results of this method are independent of the selected initial point. However, the result of this algorithm cannot guarantee finding a global optimum point. However, the results are usually found near the global optimum point. Therefore, the GA can be utilized to find the initial point near the global optimum point, and then the SQP can find the optimal point based on these initial points. The GA is designed based on identified objective and constraint functions (19), (20) and (21)-(23).

The results of the GA by using Matlab Optimization Toolbox and defining 30 populations are presented below.

$$Q=27464I_{14\times14} \tag{34}$$

$$R=0.029I_{4\times4} \tag{35}$$

In this study, the initial values for the SQP algorithm to find the optimum points, Q_c and R_c, are equal to 27464 and 0.029, respectively. The results of SQP based on the answers of the GA as initial points using Matlab Optimization Toolbox are shown in Table II.

TABLE II
THE SQP RESULTS BASED ON THE GA INITIAL POINTS

Symbol	Value
Q_c	28289
R_c	.000711

V. RESULTS OF THE LQR USING A HYBRID OPTIMIZATION ALGORITHM

In order to show the performance of the LQR with optimized Q and R matrices, white noise disturbance is applied to the system to simulate the effect of the road profile disturbance. Also a LQR with non-optimized Q_c and R_c (Q_c = 10000 and R_c = .001) is designed to show the effect optimization in the coefficient of the LQR. The white noise is applied for 5 sec and maximum amplitude of 7 cm under the passenger car tires. The response of the optimized and non-optimized LQR, and passive car for the linear velocity of the body, and pitch and roll rates are shown in Figs. (4-6).

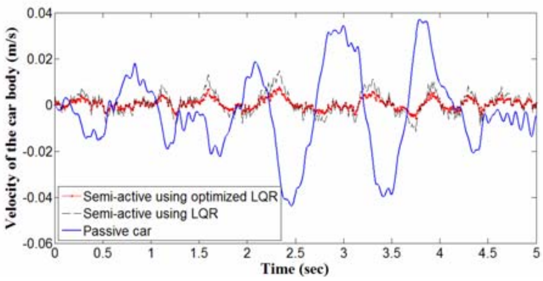

Fig. 4 The body velocity along the vertical vehicle axis excited by white noise

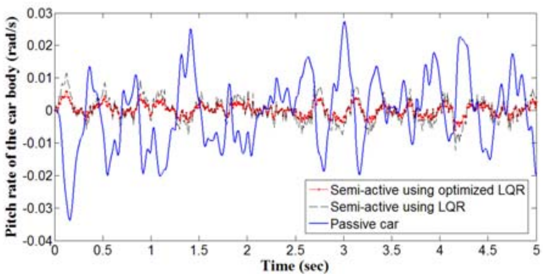

Fig. 5 The body pitch rate excited by white noise

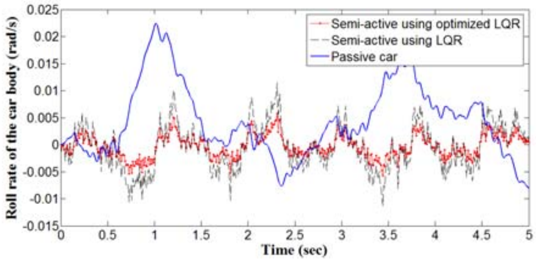

Fig. 6 The body roll rate excited by white noise

The aim of the LQR controller is to absorb the vibration of the vehicle to make the linear and angular velocity of the body equal to zero. Figs. (4-6) show the ability of the controller to absorb the vibration of the body. In order to numerically show the performance of the LQR optimized by a hybrid method, Root Mean Square Error (RMSE) for bounce velocity, and roll and pitch rates are presented in Table III. The results show that the RMSE values for the semi-active system are significantly less than those of the passive system and semi-active system with non-optimized LQR.

TABLE III
THE RMSE OF PASSIVE AND SEMI-ACTIVE SYSTEM

Parameters	RMSE		
	Passive system	Semi-active system with non-optimized LQR	Semi-active system with optimized LQR
Bounce	0.0185	0.0047	0.0024
Pitch	0.0114	0.0037	0.0019
Roll	0.08	0.042	0.0021

The aim of the present research is to achieve the best performance of the system with the minimum energy of the actuator in the working region. The applied forces to the suspension system by MR dampers are shown in Figs. (7-10). The figures show the maximum required force for vibration control of the vehicle is less than 2000 N, which satisfies the defined constraint for the system in the simulation.

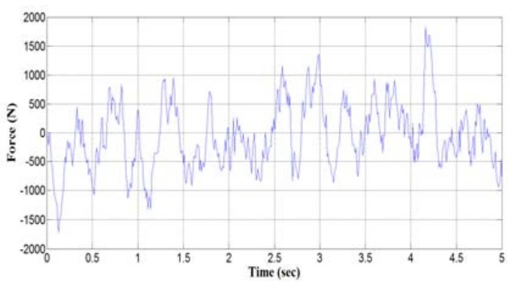

Fig. 7 The force of the MR damper located in the front left of the vehicle

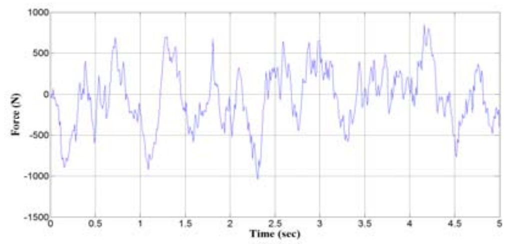

Fig. 8 The force of the MR damper located in the front right of the vehicle

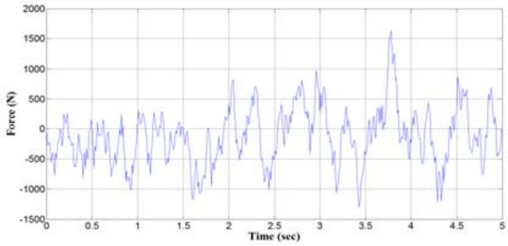

Fig. 9 The force of the MR damper located in the rear left of the vehicle

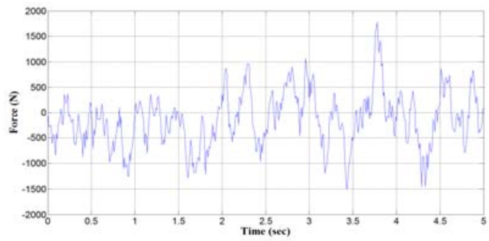

Fig. 10 The force of the MR damper located in the rear right of the vehicle

VI. CONCLUSION

In the present study, the suspension system of a passenger car was modeled by the full-car model with seven DOFs. Based on the dynamic equations of the vehicle, the LQR was implemented in the system simulation. Matrices Q and R in the design procedure of the LQR were found by a hybrid optimization algorithm which consists of the SQP method and GA based on the maximum performance of the controller and feasibility of the actuators force. The GA in the hybrid algorithm was utilized to find the initial point and avoid the local optimum point. Then, the response of this method was used by SQP to find the nearest optimum point to the initial point. The results of the suspension system simulation show that the implemented LQR optimized by the hybrid method can absorb the vehicle vibration by utilizing available industrial MR dampers.

REFERENCES

[1] Kamopp, "Active Damping in Road Vehicle Suspension Systems," Vehicle System Dynamics, vol. 12, pp. 291–311, 1983.
[2] H. Liu; K. Nonami; and T. Hagiwara, "Active Following Fuzzy Output Feedback Sliding Mode Control of Real-vehicle Semi-active Suspensions," Journal of Sound and Vibration, vol. 314, pp. 39–52, 2008.

[3] S.S. Sedeh, R.S. Sedeh and K. Navi, “Using Car Semi-active Suspension System to Decrease Undesirable Effects of Road Excitation on Human Health,” in proc. BIOCOMP, 2006, pp. 242–250.
[4] H. Du, K.Y. Sze; and J. Lam, “Semi-active H∞ Control of Vehicle Suspension with Magneto-rheological Dampers,” Journal of Sound and Vibration, vol. 283, pp. 981–996, 2005.
[5] M.S. Fallah; R.B. Bhat; and W.F. Xie, “Optimized Control of Semi-active Suspension Systems Using H∞ Robust Control Theory and Current Signal Estimation,” IEEE/ASME Transactions on Mechatronics, vol. 16, pp. 1–12, 2011.
[6] C.P. Cheng; C.H. Chao; Li T.H.S, “Design of Observer-based Fuzzy Sliding-mode Control for an Active Suspension System with Full-car Model,” in proc. Systems Man and Cybernetics (SMC), 2010 IEEE International Conference on, 2010, pp.1939–1944.
[7] A.F. Jahromi; and A. Zabihollah, “Linear Quadratic Regulator and Fuzzy Controller Application in Full-car Model of Suspension System with Magnetorheological Shock Absorber,” Mechatronics and Embedded Systems and Applications (MESA), 2010 IEEE/ASME International Conference, 2010, pp. 522–528.
[8] R. Guclu, “Active Control of Seat Vibrations of a Vehicle Model Using Various Suspension Alternatives,” Turkish Journal of Engineering and Environmental Sciences, vol. 27, pp. 361–373, 2003.
[9] M.M. Elmadany; and Z. Abduljabbar, “Linear Quadratic Gaussian Control of a Quarter-car Suspension,” Vehicle System Dynamics, vol. 32, pp. 479–497, 1999.
[10] L.V.V.G. Rao; and S.Narayanan, “Sky-hook Control of Nonlinear Quarter Car Model Traversing Rough Road Matching Performance of LQR Control,” Journal of Sound and Vibration, vol. 323, pp. 515–529, 2009.
[11] M.J.D. Powell “A Fast Algorithm for Nonlinearly Constrained Optimization Calculations,” Numerical Analysis Lecture Notes in Mathematics, vol. 630, pp. 144–157, 1978.
[12] M.J.D. Powell. The Convergence of Variable Metric Methods for Nonlinearly Constrained Optimization Calculations. Nonlinear Programming 3, Academic Press, 1978.
[13] J. H. Holland. Adaption in Natural and Artificial Systems. 1st edition, United States, MIT Press/Bradford Books Edition, 1992.
[14] J. S. Arora. Introduction to Optimum Design. 2nd edition, United States, Elsevier Academic Press, 2004, ch. 4, 16.