

# Investigation of the Neutral Axis in the Positive Moment Region of Composite Beams

Su-Young Jeong, Won-Kee Hong, Seon-Chee Park, Gyun-Taek Lim, Eric Kim

**Abstract**—Researchers investigate various strategies to develop composite beams and maximize the structural advantages. This study attempted to conduct experiments and analysis of changes in the neutral axis of positive moments of a Green Beam. Strain compatibility analysis was used, and its efficiency was demonstrated by comparing experimental and analytical values. In the comparison of neutral axis, the difference between experimental and analytical values was found to range from 8.8~26.2%. It was determined that strain compatibility analysis can be useful for predicting the behaviors of composite beams, with the ability to predict the behavior of not only the elastic location of the composite member, but also of the plastic location.

**Keywords**—Composite beam, Strain compatibility, Neutral axis, Green Beam

## I. INTRODUCTION

It is becoming increasingly difficult for either steel frame or reinforced concrete to satisfy required structural performance and cost-effectiveness. Because of this, researchers are accelerating the development of composite members and maximizing structural advantages. However, structural design standards that are applicable to composite members have not yet been formulated. In addition, the absence of accurate analytical methods that can be used to evaluate composite members with more complex behaviors than those of single members makes it difficult to predict the behaviors of such composite members. This study analyzes the behaviors of composite members via experimentation and analysis of changes in neutral axis based on the positive moment region of Green Beams.

As this study attempted to analyze the behaviors of composite beams only in relation to the positive moments, the scope of the analysis is limited to the central regions of the beams.

## II. GREEN BEAM

The reinforced concrete structures have a shortcoming in that the quantity of structural members increases significantly, as the height of building rises.

The steel frame structures require reinforcement to prevent

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buckling and fire protection. Composite members can produce the same stiffness and strength as reinforced concrete members through the use of with smaller cross-sections. Because of this, the amount of construction materials used in a project can be reduced. Compared with steel frame structures, composite beams supplement the slenderness of steel frames and concrete can do the work of a fire protection that eliminates the need for buckling reinforcement and an additional fire-protection.

The Green Beam is a composite beam consisting of a structural tee section and reinforced concrete. The lower part of a Green Beam is made of precast concrete. The flange and web of the structural tee section embedded in the precast concrete. Then, the slab is cast in place along the edge of the precast concrete. Therefore, it can reduce story height in comparison with a steel frame structure where the slab is cast over the flange of the steel frame. The column of the Green Frame is fabricated as a three-section precast column, making it possible to connect beam and column via steel frames. In this manner, the use of the Green Beam can reduce construction time [1]-[4].

## III. STRAIN COMPATIBILITY

This study used strain compatibility analysis to analyze the behaviors of composite Green Beams [5]. Strain compatibility analysis linearizes the strain in composite beams and calculates the strains of all components in proportion to the strain of the top surface of the compressed concrete and in reference to the assumed neutral axis in order to predict the behavior of the composite member. An equilibrium equation using proportion for all components is arranged into an equation relative to the neutral axis, and the value of neutral axis can be calculated.

When strain compatibility analysis is used, the behaviors of composite beams are separated into three categories: pre-yield limit state, where the lower tensile reinforcement does not reach yield strain; yield limit state, where it reaches yield strain, and maximum load limit state, where the upper concrete strain reaches the extreme strain of 0.003.

## IV. EXPERIMENT AND ANALYSIS

### A. Detailed Specimen Analysis

Fig. 1 shows the basic cross-section of a Green Beam specimen. The concrete compressive strength of the specimen was 27 MPa, the tensile strength of the steel frame was 240 MPa and the tensile strength of reinforcement was 400 MPa. As for the stirrup, 10mm diameter reinforcements were placed with a 400mm spacing.

Two 22mm diameter reinforcements and five 25mm diameter reinforcements were placed in the upper and lower regions respectively.

Steel frames measuring 248mm×199mm×9mm×14mm were used, and 16mmdiameter stud bolts were attached to the lower flange and web of the steel frame at a 210 mm spacing. To ensure the accuracy of the experiment, two specimens with identical cross-sections were fabricated and tested.

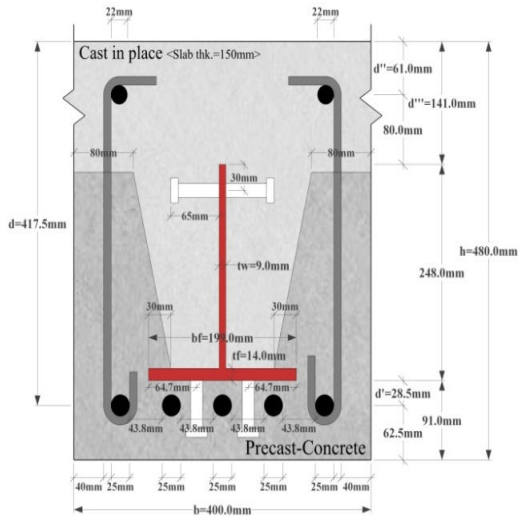


Fig. 1 Detailed Cross-Section of a Specimen

In this experiment, Green Beams fabricated in the factory were tested with a 1000 kN actuator. The specimen was a simple beam that is 4,500mm long, with a net span spacing of 4,000 mm. The central region of the beam was intensively loaded during the experiment. Fig. 2 shows how the loading was applied. Fig. 3 shows the specimen in the experimental site.

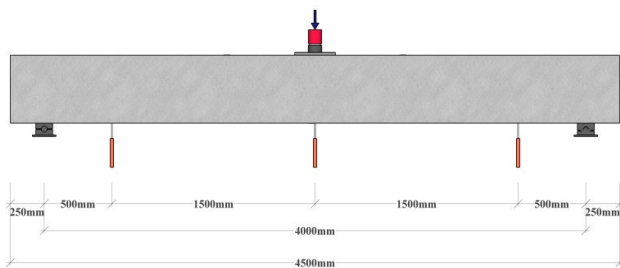


Fig. 2 Loading Method



Fig. 3 Specimen

### B. Experiment Results

The flexural moment strengths of the GreenBeams at the yield limit state were observed to be 637.7kN and 628.7kN for composite beams No. 1 and No. 2, respectively.

The flexural moment strengths of the GreenBeams at the maximum load limit state were 726.3kN and 733.4kN for composite beams No. 1 and No. 2, respectively.

Fig.4 and 5 illustrate the fore-strain shapes of composite beams No. 1 and No. 2 resulting from downward loading.

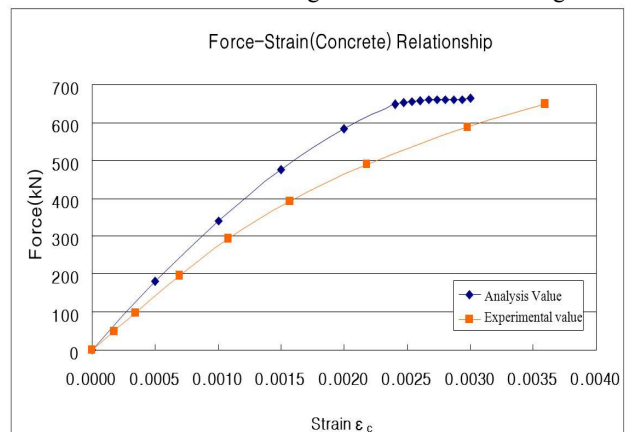


Fig. 4 Force-Strain Relationship for Specimen No. 1

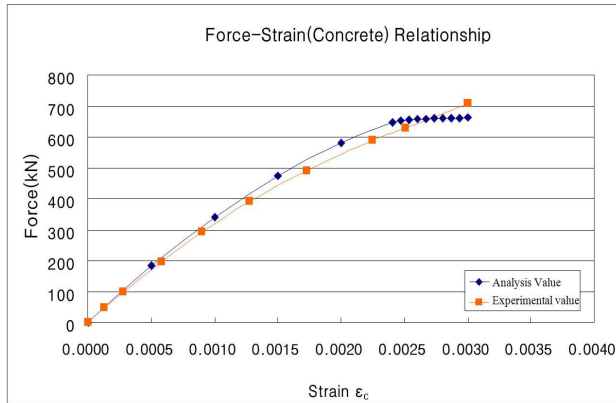


Fig. 5 Force-Strain Relationship for Specimen No. 2

To identify the neutral axis from the strain of the specimen, strain gauges were attached to the specimen. A total of 30 strain gauges were attached to the steel frame, upper and lower reinforcements, concrete and stirrup to measure strain at various elevations along the cross-section of the specimen.

To locate the neutral axis of the specimen, strains measured at each elevation were expressed in a linear proportional graph to determine the neutral axis at the point where the strain was zero. For purposes of comparison with the results of strain compatibility analysis, the neutral axis was identified in each of the three limit states including pre-yield limit state, yield limit state and maximum load limit state. Data analysis was conducted to identify points where the reading of the strain gauge attached to the lower tensile reinforcement reached yield strain and the reading of the strain gauge attached to the upper concrete reached extreme strain, and the neutral axis was located in the graph. Figs.6~13 are the graphs indicating strains measured by strain gauges installed at different elevations in Specimen No. 1 and 2 in each limit state.

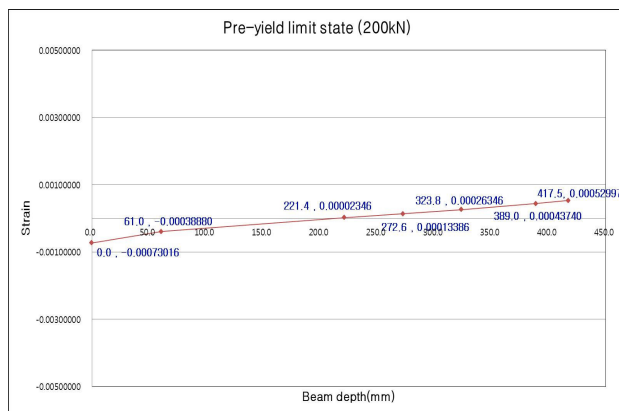


Fig. 6 Transition of Strain in Pre-Yield Limit State of Specimen No. 1(200kN)

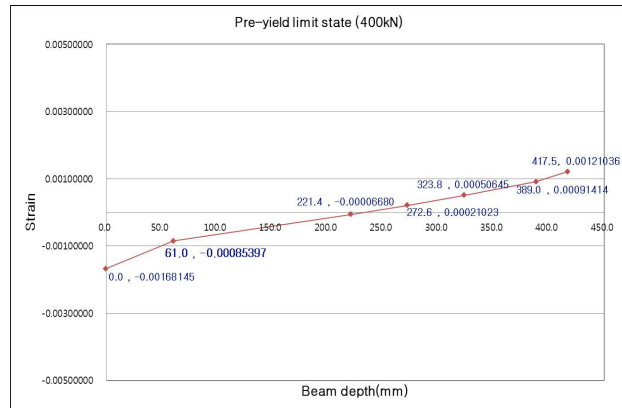


Fig. 7 Transition of Strain in Pre-Yield Limit State of Specimen No. 1(400kN)

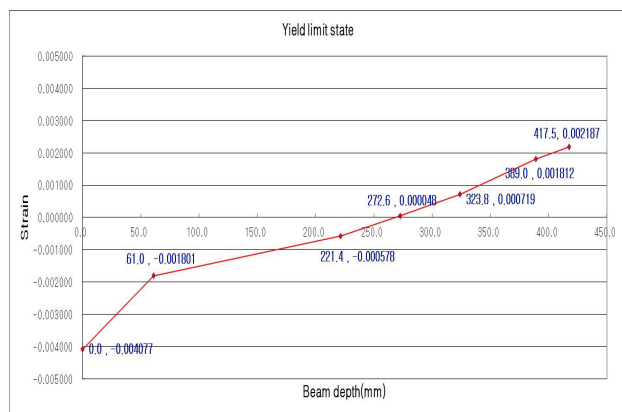


Fig. 8 Transition of Strain in Yield Limit State of Specimen No. 1

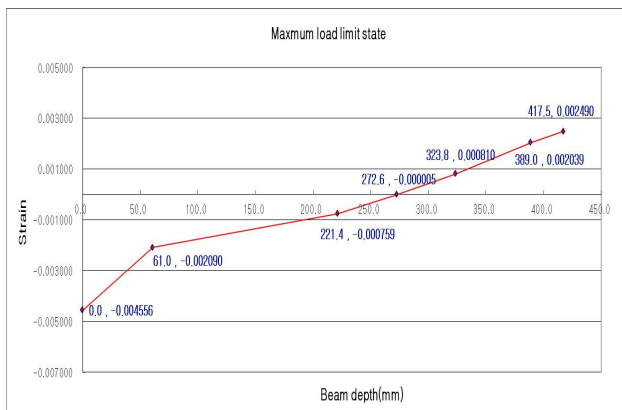


Fig. 9 Transition of Strain in Maximum Load Limit State of Specimen No. 1

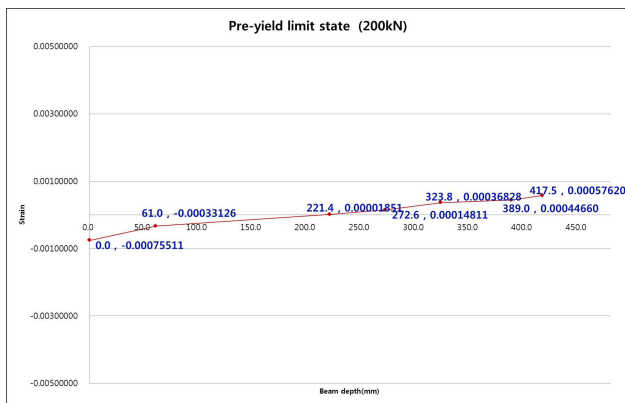


Fig. 10 Transition of Strain in Pre-Yield Limit State of Specimen No. 2 (200kN)

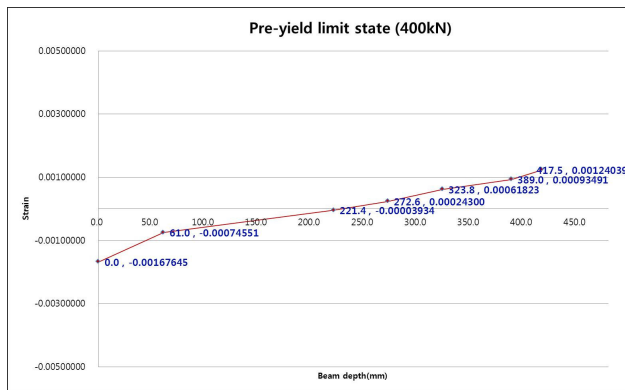


Fig. 11 Transition of Strain in Pre-Yield Limit State of Specimen No. 2 (400kN)

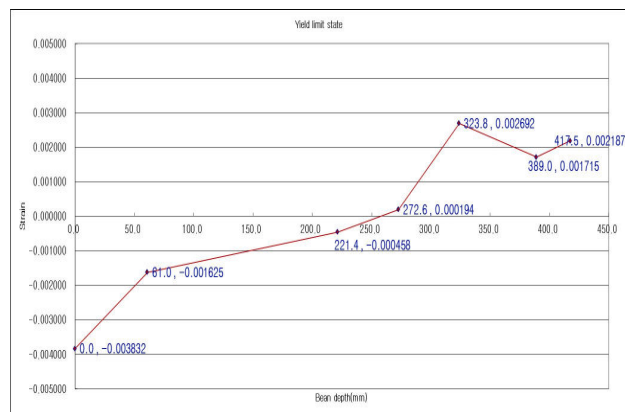


Fig. 12 Transition of Strain in Yield Limit State of Specimen No. 2

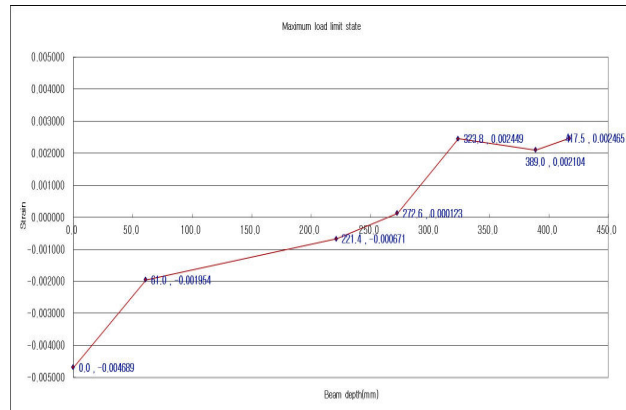


Fig. 13 Transition of Strain in Maximum Load Limit State of Specimen No. 2

A linear interpolation method was used to calculate the value of the neutral axis located between the gauges. Table I shows the location of the neutral axis in each specimen for each limit state. The values shown in the table refer to the distance to the neutral axis from the upper concrete of the beam.

TABLE I  
NEUTRAL AXIS IN THE EXPERIMENT

	Specimen No. 1	Specimen No. 2
Pre-Yield Limit State(200kN)	212.27mm	212.91mm
Pre-Yield Limit State(400kN)	233.75mm	228.53mm
Yield Limit State	268.67mm	257.37mm
Maximum Load Limit State	272.91mm	264.67mm

The analysis of the neutral axis located in the experiment confirmed that the distance of the neutral axis from the upper concrete increased as increased load was applied and as the composite beam moved from pre-yield limit state to the maximum load limit state.

### C. Analysis of Neutral Axis by Strain Compatibility

The composite beam with the identical cross-section as the specimen was analyzed by the strain compatibility analysis method. The compressive and tensile forces applied to the loaded composite member were the same and are expressed as follows (1).

$$F_c + F'_t + F'_{flange} + F'_{web} = F_t + F_{flange} + F_{web} \quad (1)$$

Where,  $F_c$  = Compressive force of concrete

$F'_t$  = Compressive force of compressive reinforcement

$F'_{flange}$  = Compressive force of flange

$F'_{web}$  = Compressive force of web

$F_t$  = Tensile force of tensile reinforcement

$F_{flange}$  = Tensile force of flange

$F_{web}$  = Tensile force of web

Equilibrium Equation (1) can be rendered as a linear or quadratic equation against the neutral axis. Equation (2) shows Equilibrium Equation (1) for the composite member in the maximum load limit state rendered as an equation of the neutral axis.

$$\left( \alpha f'_c b + 2 t_w F_y \right) c - \left\{ A_s f_y + A_f F_y + A'_s f_y + t_w F_y (d - d' + d'' - t_f) \right\} = 0 \quad (2)$$

$$c = 201.46 \text{ mm}$$

Where,  $\alpha$  = Stress factor [6]

$F'_c$  = Compressive strength of concrete

$F_y$  = Tensile strength of steel frame

$A_s$  = Cross-section of tensile reinforcement

$f_y$  = Tensile strength of reinforcement

$A_f$  = Cross-section of flange

$A'_s$  = Cross-section of compressive reinforcement

The values of the neutral axis of the composite member specimen from analysis using the above-mentioned process in each limit state are shown in Table II.

TABLE II  
NEUTRAL AXIS FROM ANALYSIS

Analysis Values	
Pre-Yield Limit State (200kN)	193.49mm
Pre-Yield Limit State (400kN)	199.53mm
Yield Limit State	213.53mm
Maximum Load Limit State	201.46mm

This analysis showed that the value of the neutral axis increased up to the yield limit state but decreased in the maximum load limit state.

In the pre-yield limit state, as strain on the concrete increased, the area of concrete subject to compressive force became larger, pushing the neutral axis downward. However, in the yield limit state where the lower tensile reinforcement yields, the tensile reinforcement cannot support stress greater than the yield stress. Thus, the area of concrete subject to compressive force begins to decrease, with the neutral axis rising beyond that in the yield limit state.

#### V.COMPARATIVE ANALYSIS

Table III and Fig.14 provide a comparison between neutral axis found in the experiment and the analysis. The comparison demonstrated that the neutral axis of the specimen was located slightly farther from the upper concrete than that in the analysis value. In the pre-yield limit state, the values from the experiment and the analysis were relatively similar to each other, with an error of 8.8%. However, when additional load was applied, the error was found to increase as the limit state progressed. For the analytical value, the neutral axis tended to rise, while the neutral axis value in the experiment continued to

fall, which increased the error between the analysis and experimental value.

TABLE III  
COMPARISON OF VALUES BETWEEN EXPERIMENT AND ANALYSIS

	Pre-yield limit state (200kN)	Pre-yield limit state (400kN)	Yield limit state	Maximum load limit state
Analysis value	193.49mm	199.53mm	213.53mm	201.46mm
Specimen No. 1	212.27mm	233.75mm	268.67mm	272.91mm
Error	18.78mm (8.8%)	34.22mm (14.6%)	55.14mm (20.5%)	71.45mm (26.2%)
Specimen No. 2	212.91mm	228.53mm	257.37mm	264.67mm
Error	18.78mm (8.8%)	34.22mm (14.6%)	43.84mm (17%)	63.21mm (23.9%)

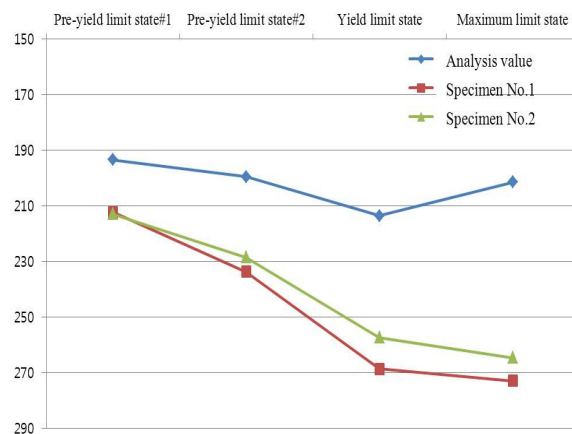


Fig. 14 Transition of Neutral Axis in Experiment and Analysis

The experiment and the analysis resulted in different neutral axis because of the different properties of the components of the specimen.

In the experiment, the ratio of neutral axis length declined significantly beyond the maximum load limit state. Therefore, the neutral axis length was expected to decrease beyond the maximum load limit state, similar to the value from the analysis.

In addition, given that the neutral axis was found to be located lower in the experiment than in the analysis, it was supposed that the tensile strength of the reinforcement in the specimen was actually higher than the value estimated by strain compatibility analysis.

Although some error was found between the values from the experiment and the analysis, it was decided that not only the elastic behavior of the composite member, but also its plastic behavior can be predicted by strain compatibility analysis.

## VI. CONCLUSION

This study conducted a comparative study of the experimental and analytical behaviors of neutral axis in Green Beams, which are composite beams consisting of a steel frame and reinforced concrete. The strain compatibility analysis is used to theoretical analysis.

In the prediction of the analytical behavior of the neutral axis, the neutral axis was located farther from the upper concrete up to the yield limit state and was positioned closer to the upper concrete in the maximum load limit state. In the experimental analysis, the neutral axis continued to drift away from the upper concrete during the transition in limit state.

In the comparison of neutral axis found in the analysis and the experiment, the error rate increased as the limit state progressed from the pre-yield limit state to the maximum load limit state in the range of 8.8~26.2%. Based on this finding, it is believed that the actual strength of tensile reinforcement is greater than the estimated analytical value.

It is expected that strain compatibility analysis can provide academic references for the estimation of composite beams not only in the elastic location, but also in the plastic location.

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