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Research on the Predict Method of Random Vibration Cumulative Fatigue Damage Life Based on the Finite Element Analysis

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Abstract**—**Aiming at most of the aviation products are facing the problem of fatigue fracture in vibration environment, we makes use of the testing result of a bracket, analysis for the structure with ANSYS-Workbench, predict the life of the bracket by different ways, and compared with the testing result. With the research on analysis methods, make an organic combination of simulation analysis and testing, Not only ensure the accuracy of simulation analysis and life predict, but also make a dynamic supervision of product life process, promote the application of finite element simulation analysis in engineering practice.

Keywords**—**Random vibration, finite element simulation, fatigue, frequency domain.

I. Introduction

WITH modern weapons and equipment industry development, the reliability demand is also constantly improve, especially the aerospace onboard equipment, Because most of the aviation equipment experience severe vibration environment during life time, the research for vibration fatigue caused great engineer attention. The structure vibration fatigue involving structure dynamics, structural fatigue, random vibration and the multidiscipline, research is still in its beginning stage, the technology is not mature, Especially for random vibration fatigue, which is different with general circulation fatigue, ordinary fatigue life estimation method can't be applied in structure random vibration fatigue life [6], how to solve this problem, is worth to the further research.

Face to the random vibration fatigue, the traditional way is using accelerating test method to study the random vibration fatigue, but this method waste long time, have a high cost, and the repeatability is poor. There is certain difficulty carrying out in the aerospace industry. The finite element software development and cumulative fatigue damage theory are the perfect solution to the problem of the new thinking, this expected methods make time period short, no damage to the product and the cost advantages, compared with the traditional method of test methods, not only can expected the fatigue life of the products, but also can be used as a traditional accelerated testing accuracy reference in the aerospace industry widely used.

II. Random Vibration Finite Element Simulation Test

Random vibration finite element simulation process as shown in Fig. 1:

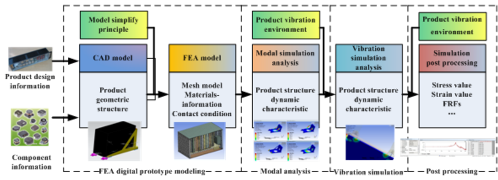

Fig. 1 Random vibration finite element simulation flow chart

The implementation process of random vibration finite element simulation is divided into four steps:

The first step is FEA digital prototype modeling. Using ANSYS-Workbench finite element analysis platform make product vibration simulation analysis. First need to collect information on products, including product long, wide, high, thickness, quality, etc, and internal circuit boards, the component model, the quality and other detailed information, with special CAD software for the platform, entity structure modeling, get the CAD digital prototype, the step modeling in not affect to the results of the finite element analysis, meanwhile, digital prototype was investigated, simplify properly, delete chamfering, round horn, thread etc unnecessary structure; The finite element software CAD file through the connection, import the digital prototype CAD into finite element simulation software platform; Finally established digital prototype model of FEA in the finite element simulation platform, including product information base material properties, finite element mesh model, contact conditions and constraint conditions, etc. Digital prototype FEA modeling process as shown in Fig. 2:

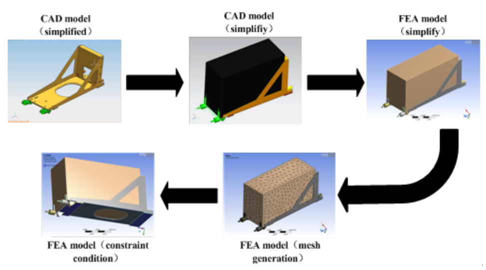

Fig. 2 Digital prototype FEA modeling process

The second step, finite element modal simulation analysis. The main purpose of this step is to get the simulation dynamic

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characteristics of the product, it is necessary to set the methods for reconciliation calculate conditions correctly, through the finite element simulation software for the solution, get tested structure vibration modal, including natural frequencies and modal shape, as the conditions of random vibration simulation. In order to verify the correctness of the simulation test, still need to take product modal test, with the results, we can make simulation test parameters settings fixed. A product bracket modal test and simulation test results contrast as shown in Fig. 3:

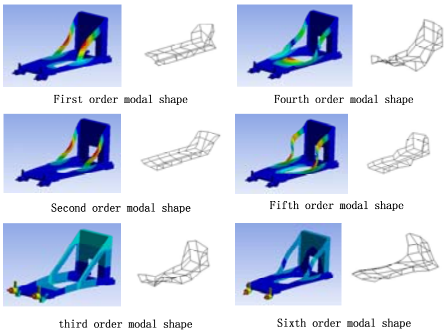
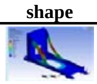
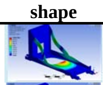
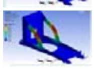
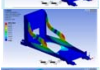
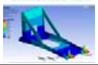
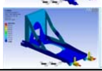


Fig. 3 Results of modal test and simulation test

Product bracket modal analysis results as is shown in Table I:

TABLE I
PRODUCT BRACKET MODAL ANALYSIS RESULTS

Modal	Frequency /Hz	Mode shape	Modal	Frequency /Hz	Mode shape
1	298.7		4	544.7	
2	313.6		5	707.6	
3	333.3		6	779	

The third step, vibration simulation analysis. The software can make simulation conditions according to the actual products vibration environment, in view of aerospace products always working in the random vibration environment, this paper mainly make product random vibration finite element simulation analysis. In the solution process, it is necessary to correctly set product input incentive, fixed supported way and the damping, etc. A product bracket random vibration simulation analysis results as shown in Fig. 4:

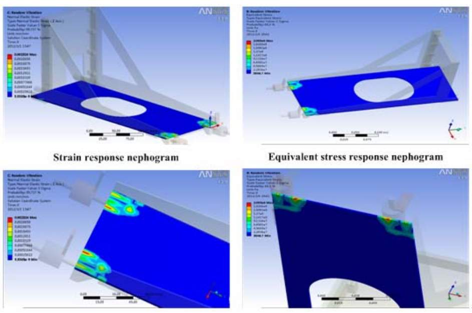


Fig. 4 Brackets random vibration simulation analysis results

The Fourth step, vibration simulation post-processing. This step is mainly according to the already solution of FEA prototype, generates vibrations simulation results, and obtain the cumulative fatigue damage data, mainly has the structure of the tested Frequency Response function FRFs (Frequency Response Functions), stress and strain distribution cloud images, and weak link unit power spectral density PSD (power spectral density) and stress value, as shown in Fig. 5. In addition, we still need to invoke classic ANSYS program, take frequency domain data into interpolation reconstruction processing, access to equal-interval frequency signal data value for later calculation.

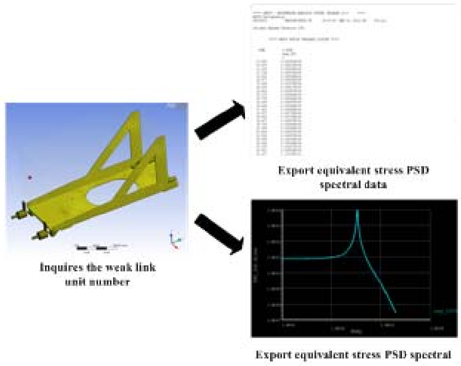


Fig. 5 Vibration simulation post-processing results

III. CUMULATIVE FATIGUE DAMAGE RESULT ANALYSIS

A product bracket as Fig. 6 shows, its main function is to fix products. Mainly by brackets subject, bracket fixed pin, lock the component and so on. Through the fixed pin and lock link products and bracket, then screwed the brackets fixed.

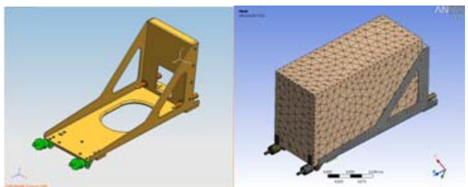


Fig. 6 A Product bracket CAD drawing

Exerting 4G to 22G random vibration stress to fatigue fracture, and recording the incentive time, as shown in Fig. 7, the fixed bracket delivering near pin occurs fatigue fracture in 22G random vibration at 600 seconds, location follows and simulation analysis results. loading time in the Test as shown in Fig. 7:

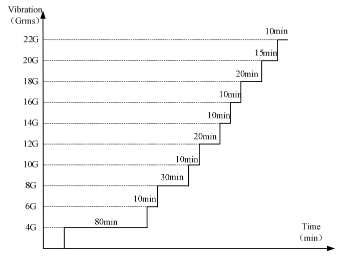


Fig. 7 Testing situation and bracket loading vibration spectrum

A. Frequency Domain Method Result Analysis

Fatigue life frequency domain method, is in the frequency domain response with power spectral density (PSD) spectrum parameters describing the stress response information, and then combining with the corresponding rain flow amplitude distribution model and the fatigue life curve to confirm fatigue life of structure. When choose the frequency domain method to expect life, we must first define the response PSD spectrum is broadband process or narrowband process. For different stress response process, its rain flow amplitude distribution is different. For different response to stress process, and its rain flow in the form of amplitude distribution is different. Rain flow amplitude distribution of narrowband Gaussian process is simple, general are approximate Rayleigh distribution; Rain flow amplitude distribution broadband Gaussian process is very complex, and have different approximation probability density function (PDF) model.

According to the Miner and Manson, it studies the judge material structure as dyadic D cumulative fatigue damage of the measure, when actual damage cycle number and fatigue damage cycle number are equal (namely D = 1), material fatigue failure happened [1][2], as follows:

$$D = \frac{n}{N} \tag{1}$$

n is actual damage cycle number, resulted in the actual process of vibration of analysis, N is cycle number of material fatigue damage, get from the stress life curve (S-N curve).

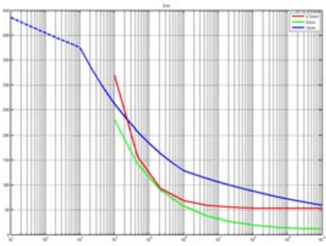


Fig. 8 Different thickness 2A12 plank s-n curve

For the random vibration response process, the stress state is continuation distribution, so in time period T and stress range ($S_i, S_i + \Delta S_i$), the stress cycling number [3] is:

$$n_i = vTp(S_i)\Delta S_i \tag{2}$$

V is stress cycling number in unit time, $p(S)$ is the stress amplitude probability density function.

Simultaneous on two type can get the fatigue damage D of continuous distribution stress state within the time T:

$$D = \frac{n}{N} = \frac{vT}{N}\int_0^\infty p(S)\,dS \tag{3}$$

When D = 1, material structure occurs fatigue failure, material fault starting time (product life) for:

$$T = \frac{N}{v\int_0^\infty p(S)\,dS} \tag{4}$$

For narrowband random process, the stress response of the PSD spectrum to obey the Rayleigh distribution [4], the damage ratio D can be expressed as:

$$\begin{aligned} D &= \frac{n}{N} = \frac{vT}{N}\int_0^\infty p(S)\,dS \\ &= vT \cdot C^{\frac{1}{b}} \cdot \left(\sqrt{2}\sigma_{RMS}\right)^m \Gamma\left(\frac{1}{2}m+1\right) \end{aligned} \tag{5}$$

$$T = \frac{1}{v \cdot C^{\frac{1}{b}} \cdot \left(\sqrt{2}\sigma_{RMS}\right)^m \cdot \Gamma\left(\frac{1}{2}m+1\right)} \tag{6}$$

Related parameters as follows:

v: stochastic process stress cycling number within unit time, namely the equivalent frequency

σ_{RMS}: The root mean square value of stress response (RMS values) in weak link, get from the ANSYS weak link PSD spectrum analysis, $\sigma_{RMS} = \sqrt{\int_{f_s}^{f_e} W_s(f)df}$

$\Gamma(x)$: x obey gamma distribution, the value can use the following approximate equation: $\Gamma(x) = \sqrt{2\pi}x^{x-\frac{1}{2}}e^{-x}\left[1+r(x)\right]$, the approximate estimates for $r(x)$ is $\left|r(x)\right| \le e^{\frac{1}{12x}} - 1$

m is material S-N curve parameters, N is material fatigue cycle number of failure, can get according to σ_{RMS} through the materials S-N curve.

The irregular factor r of PSD spectrum is 0.952, equivalent frequency v^+ is 334.59Hz, test material limit tensile strength for 275MPa, apply to narrowband stochastic process spectrum calculation.

Results are as follows:

TABLE II
A PRODUCT BRACKET ACCUMULATED FATIGUE DAMAGE FREQUENCY DOMAIN ANALYSIS RESULTS

G value	Excitation time/s	D（frequency domain）	LDR（D）
4	4800	0.0000586	5.864E-05
6	600	0.0001	0.000158
8	1800	0.002	0.0021
10	600	0.0027	0.0048
12	1200	0.0175	0.0223
14	600	0.024	0.0463
16	600	0.056	0.102
18	1200	0.24	0.342
20	900	0.354	0.696
22	417	0.304	1

It can be seen from the table, using the frequency domain method for the results, products in 22 G random vibration inspire, fatigue fracture happened at the 417 seconds, which have 1.42% of the error with the actual test results. Error of generation, is mainly because LDR method in calculating the cumulative fatigue damage process, do not consider the effect of load order, ignore the effect between incentives, lead to its expected results relatively conservative, but because its benefit for calculation, it is widely used in aerospace engineering.

B. Time Domain Method Result Analysis

From ANSYS random vibration simulation analysis, we can obtain the weak link equivalent stress response spectrum density, it describes the stochastic process on frequency change characteristics, is random process in frequency domain of important numerical characteristic features, and characterization of the energy distribution of stochastic process, in this paper, the signal frequency domain and time domain signal reconstruction simulation signal, get the response time process, and then rain flow count method for the cycle count processing, according to the material fatigue performance curve and cumulative fatigue damage theory, we can predict product vibration cumulative fatigue life. The main process as shown in Fig. 9:

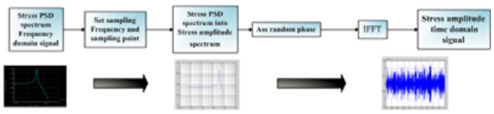


Fig. 9 Time domain method of fatigue damage accumulation

Using interpolation method on the stress response PSD spectrum, making signal reconstruction and time-frequency transition, we get the random vibration time signal in 100s, then through the rain flow count processing, obtain the corresponding cycle of stress amplitude and stress mean (shown in Fig. 10), make average stress fixed, get the stress amplitude whose average stress is zero, combined with the material S-N curve for cumulative fatigue calculation:

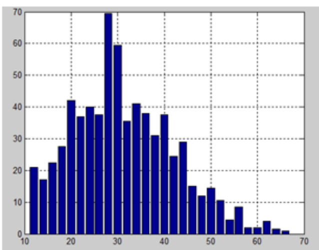


Fig. 10 Rain flow count method and time signal results

The most widely rule used in engineering cumulative fatigue damage is Miner rule which is proposed in 1945, the Linear damage standards (LDT), the criterion predict damage in different loading level respectively, when sum of damage in different level is 1 fatigue failure occurred. As follows:

$$\sum \frac{n_i}{N_i} = 1 \tag{7}$$

n_i is cycle number at a loading level, N_i is the material life at a certain loading level.

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{8}$$

D is fatigue damage at vibration stress time in 100s, n_i is the rain flow count cycle number for a particular stress amplitude, N_i is the corresponding fatigue life cycles according to the stress amplitude material under the S-N curve, make fatigue damage in 100s spread to the actual test time, we can obtain the value of excitation fatigue damage.

The calculate results by time domain method as Table III shows:

TABLE III
A PRODUCT BRACKET ACCUMULATED FATIGUE DAMAGE TIME DOMAIN ANALYSIS RESULTS

G value	Excitation time/s	Damage	LDR(D)
4	**4800**	**0**	**0**
6	**600**	**0.000768**	**0.000768**
8	**1800**	**0.0015**	**0.002268**
10	**600**	**0.00366**	**0.005928**
12	**1200**	**0.0192**	**0.025128**
14	**600**	**0.03**	**0.055128**
16	**600**	**0.057**	**0.112128**
18	**1200**	**0.396**	**0.508128**
20	**900**	**0.396**	**0.904128**
22	**79.2**	**0.096**	**1**

From the calculation results, it is known that the results of cumulative fatigue damage is 79.2s, product fatigue fracture, the error is 4.04%, among which besides the error of linear fatigue damage, signal distortion from conversion between frequency signal and time signal, also can affect the results.

C. Three-section Method Result Analysis

For simplified calculation, we can use the method which Steinberg put forward based on Gaussian distribution, three sections calculation method for fatigue damage, stress more than 3S only occurred in 0.27% of the time, and can't cause any damage [5]. So, using the Miner's law for fatigue calculation, we treat stress into three stress level, the overall damage calculation formula can write as:

$$D = \frac{n_{1S}}{N_{1S}} + \frac{n_{2S}}{N_{2S}} + \frac{n_{3S}}{N_{3S}} \tag{9}$$

n_{1S} : Actual cycle number which stress amplitude is equal to or less than 1S level ($0.683 T v_0^+$);

n_{2S} : Actual cycle number which stress amplitude is equal to or less than 2S level ($0.271 T v_0^+$);

n_{3S} : Actual cycle number which stress amplitude is equal to or less than 3S level ($0.0433 T v_0^+$);

v_0^+ : Statistical average frequency;

N_{1S} , N_{2S} , N_{3S} : corresponding cycles according to fatigue curve respectively for 1S, 2S and 3S stress level.

The result is as Table IV shows:

TABLE IV
BRACKETS ACCUMULATED FATIGUE DAMAGE WITH THREE-SECTION METHOD ANALYSIS RESULTS

G value	Stress/MPa	Excitation time/s	Damage	LDR(D)
4	7.9	4800	9.72E-08	9.72E-08
6	11.85	600	1.40E-06	1.49E-06
8	15.8	1800	0.0001213	0.000123
10	19.75	600	5.50E-04	0.00067
12	23.7	1200	0.0093	0.00997
14	27.65	600	0.0175	2.75E-02
16	31.6	600	0.045	7.25E-02
18	35.56	1200	0.219	2.92E-01
20	39.5	900	0.353	6.45E-01
22	43.45	490	0.435	1

The use of three interval method for accumulation of fatigue damage generated 0.08% of error, the results is close to frequency domain method, the three-section method is widely used as an engineering method, has the certain precision, but many article points out, the three-section method is not apply to all products for all injury, but only as a calculation reference, but when compared to the frequency domain method and the time domain method, three-section method for calculation is quick and easy when used in the engineering.

Results of three kinds of cumulative fatigue damage method are not identical, but its expected fatigue fracture time, are less than the actual test time that three methods are expected to tend to conservative. Among them, the time domain method produce a bigger error, this is due to the finite element simulation analysis results from the transformation between frequency domain time domain, cause the loss of signal data, and produced the further error. Therefore, in the actual process, we should try to avoid the conversion between signal time domain and frequency domain, causing unnecessary errors. At the same time, through the three different methods results can explain, the finite element method of fatigue damage, using the finite element method for product accumulated fatigue damage analysis is feasible.

Based on the above cumulative fatigue damage result, we can further expected the product life, the random vibration incentive time as Table V shows:

TABLE V
A PRODUCT BRACKET LIFE RESULT

Vibration value/Grms	life/h	Vibration value/Grms	life/h
4G	22771	14G	7.05
6G	1665	16G	2.98
8G	260.5	18G	1.39
10G	61.76	20G	0.71
12G	19.05	22G	0.382

IV. CONCLUSION

This paper make research on the cumulative fatigue damage analysis based on the finite element, forming the ANSYS finite element analysis cumulative fatigue damage method, and connecting with the results of testing, use the frequency domain, time domain method and three-section method to verified the validity and feasibility of cumulative fatigue damage analysis, so as to use this method for the product life predictions results, and draw the conclusion as follows:

(1) the material S-N curve, the important parameter for accumulated fatigue damage analysis, can't be used directly because the random vibration stress amplitude and mean stress are all changes randomly, we must use frequency domain or time domain analysis method.

(2) When products are in accumulation fatigue damage analysis, we should try to avoid the conversion between time signal and frequency signal, because in the process of transformation, it is easy to lose important signal data, causing the signal distortion. The finite element simulation analysis method tend to use frequency domain signals directly to calculate fatigue, case analysis also shows this result.

(3) From the result from three methods, we can see results of the random vibration cumulative fatigue damage based on the finite element analysis are close to the real test results, this method not only can replace physical experiments, which are high cost, difficult operation, but also can be used as guidance and verification for real test, can be widely used in the airline industry engineering field.

This paper combining theory with practice, confirmed feasibility of application for the finite element simulation analysis in the random vibration cumulative fatigue damage, as a valuable experience for combination between simulation analysis and product reliability, but study still has some problems, such as random vibration encourage order effect on the damage of product, applicability of linear damage standards (LDR), signal distortion in conversion between frequency domain and time domain, etc, these problems will be study in the further, in order to improve the accuracy of the cumulative fatigue damage calculation.

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