An Adaptive ARQ – HARQ Method with Two RS Codes

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Abstract—In this paper we proposed multistage adaptive ARQ/HARQ/HARQ scheme. This method combines pure ARQ (Automatic Repeat reQuest) mode in low channel bit error rate and hybrid ARQ method using two different Reed-Solomon codes in middle and high error rate conditions. It follows, that our scheme has three stages. The main goal is to increase number of states in adaptive HARQ methods and be able to achieve maximum throughput for every channel bit error rate. We will prove the proposal by calculation and then with simulations in land mobile satellite channel environment. Optimization of scheme system parameters is described in order to maximize the throughput in the whole defined Signal-to-Noise Ratio (SNR) range in selected channel environment.

Keywords—Signal-to-noise ratio, throughput, forward error correction (FEC), pure and hybrid automatic repeat request (ARQ).

I. INTRODUCTION

THE wireless communication systems are recorded in huge boom in recent decades. Also is clearly obvious that technology progress in recent years accelerated the introduction of previous knowledge of wireless technologies into practice.

We live in time, where everyone has mobile phone and also everyone want to get an access to information, no matter, where is our location. This is the motivation for mobile communications. The main disadvantages of wireless systems are their capacity and unreliability. Due to the propagation of radio waves, it is clear, that capacity and transfer rate are very limited. Unreliability is the consequence, why are transferred data exposed to distortion and loss.

Two different basic methods are generally used to suppress errors: Automatic Repeat reQuest (ARQ) where error detection scheme is combined with request for retransmission of erroneous data. Forward Error Correction (FEC) adds redundant (parity) data to a message, such that it can be recovered by a receiver even when more errors (up to the capability of the code) were detected, either during the process of transmission, or on storage.ARQ and FEC are basic error control techniques and both mentioned methods have limits in throughput or reliability of transmission. Improvement of ARQ schemes are adaptive ARQ schemes, because they estimate channel state and change transmission mode. This change depends on channel state. Second possibility to remove restrictions is hybrid ARQ, which combines ARQ and FEC.

In this paper we deal with analysis, design and modification of multistage adaptive scheme. Final proposal scheme uses pure ARQ method in low channel error rate and it is Go-Back-N scheme [1]. GBN scheme is very popular. Firstly, scheme provides higher throughput than Send-and-Wait ARQ scheme. Secondly, implementation of GBN is quite simpler than Selective-Repeat ARQ scheme (it does not require buffer at receiver side). In middle and high channel error rate we use two HARQ schemes type 1 with different code rate.

Basis of our proposal is in Modify Yao adaptive method [1], [2]. This scheme is based on the original Yao adaptive scheme with difference in HARQ mode, where is used Reed-Solomon (RS) code [5]. The idea of dynamically changing ARQ algorithm between two operation modes is proposed in [3], [6]. Scheme uses two ARQ operation modes (classic GBN and n-copy GBN). Another modification is shown in [4], [5]. In this method is n-copy GBN replaced by hybrid ARQ scheme in H channel state (high error rate). This paper proposes new multistage method. It provides higher throughput than other mentioned schemes.

II. THEORETICAL BASIS

Every mentioned scheme and also schemes in [7], [8], [9] contained 2 states. There is a free space for multistage schemes. In this paper for the first time we deal with 3-stage proposal with sliding window. It is because we want to use more effective all error rate range. For state L of low channel error rate transmitter follows pure ARQ method. This case is without any change. On the other hand, to the comparison with Yao adaptive scheme [1], [2] mentioned in Introduction, our modification depends on replacing r-copy scheme by hybrid ARQ. And change also occurred that state of high channel error rate is now divided into middle (HARQ₁) H_1 and high error state (HARQ₂) H_2 states. Both states use RS code, but with different code rate of course.

A. Throughput Analysis

Corresponding to the three channel states, there are three operating modes: pure ARQ (GBN), HARQ₁ with *RS code (511, 383, 64)* and HARQ₂ with *RS code (511, 255, 128)*. If the channel is in the L state, the transmitter will follow the classic GBN ARQ method and throughput is expressed in (1). It is a throughput of pure GBN ARQ scheme multiplied with weight. It is because of detection:

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$$\eta_L = \frac{1-P_e}{1+S \cdot P_e} \cdot \frac{N \cdot b - CRC}{N \cdot b} \qquad (1)$$

where P_e is packet error probability of pure ARQ mode, CRC (Cyclic Redundancy Check) is error-detecting code, N is a packet size (length in symbols), b is a number of bits over the symbol:

$$b = \log_2(N+1) \qquad (2)$$

Throughput of HARQ modes (in channel states H_1 and H_2) can be expressed:

$$\eta_H = \frac{1 - \sum_{i=t+1}^{N} \binom{N}{i} P_S^i (1-P_S)^{N-i}}{1 + S \cdot \sum_{i=t+1}^{N} \binom{N}{i} P_S^i (1-P_S)^{N-i}} \cdot \frac{K}{N} \qquad (3)$$

where K is a number of information symbols, S is a delay in blocks (number of block, that the transmitter could potentially send until there is confirmation), P_S is a symbol error probability expressed in [4],[5].Equipment (3) is a formula where we insert a packet error rate (decoding error probability of RS code) into formula of pure ARQ method [4], [5].

B. Switching Logic

In our proposal we use 3 confirmations:
- ACK
- ACK$^+$
- NAK

NAK is sent when packet contains an error (pure ARQ) or *RS code* is not able to correct packet (HARQ$_1$, HARQ$_2$). Otherwise, ACK is sent when packet is error-free (pure ARQ) or *RS code* is able to correct packet (HARQ$_1$, HARQ$_2$). And finally, we use special confirmation ACK$^+$. It is sent when in last W packets there is no error. It means that there are only *"0's"* in register. *"0"* is sent to the register only if packet is error-free. *"1"* is sent to register even if RS code can correct error(s). As you can see in fig. 2 respectively fig. 3, it is a new idea of transitions from HARQ modes to ARQ.

Switching logic works as follows:
- Transitions from ARQ to HARQ modes or HARQ$_1$ to HARQ$_2$ is handled by transmitter
- Transitions from HARQ modes to ARQ is handled by receiver

Principle of receiver is shown in fig. 1. Receiver role in this solution is delivered information about switching to transmitter.
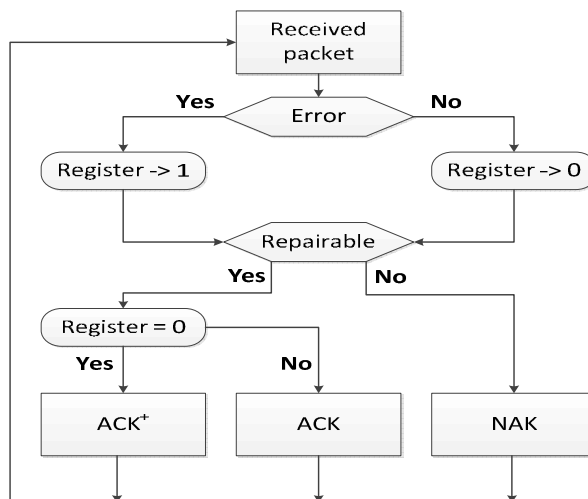

Fig. 1 Logic of receiver in our proposal

Fig. 2 shows function of the transmitter. It is also clearly obvious, that our presented adaptive method has three states as we mentioned before.
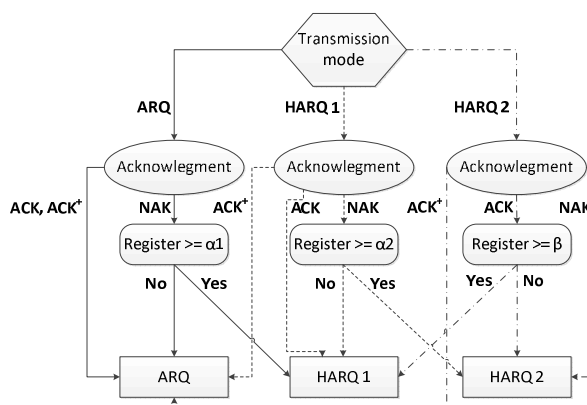

Fig. 2 Logic of transmitter in our proposal

C. Calculation of Thresholds

In this paper is important to understand that when we are talking about thresholds, it means switching points. Switching point is a transition, where our adaptive method transit from one to other scheme. In other words, it is a point, where two schemes have the same value of throughput at the same SNR. If SNR value is bigger or less than this SNR, one of two schemes under consideration will have always bigger value of relative throughput. Exceptions are cases of low SNE values, when both methods have throughput equal to zero.We gradually present calculation all possible transitions.

First, we explain switching point from ARQ to HARQ$_1$ (parameter α_1). In this case it is about counting NAKs, which receiver sent to transmitter. When the amount of NAKs in last W (W - windowing is a size of sliding window) packets overreach a parameter α_1, transmission is switched from pure

ARQ mode to the HARQ1.Probability of P_L transition is equal to probability of receiving NAK, if there is already $\alpha_1 - 1$ NAKs in last $W - 1$ confirmations:

$$P_L = \binom{W-1}{\alpha_1} P_e^{\alpha_1+1} (1 - P_e)^{W-\alpha_1-1} \qquad (4)$$

At first it is important to calculate a value of P_b where throughput of ARQ scheme is equal to throughput of HARQ1 (equality (1) and (3)). Where P_e [4],[5] in (1) is replaced by:

$$P_e = 1 - (1 - P_b)^K \qquad (5)$$

Respectively P_S [4],[5] in (3) is replaced by:

$$P_S = 1 - (1 - P_b)^b \qquad (6)$$

Mentioned equality has only one unknown variable and it is P_b. Next step is back calculation of P_e by (5) followed by final expression of parameter α_1, where packet error probability is weighted by window size:

$$\alpha_1 = P_e \cdot W \qquad (7)$$

The similar process described in previous paragraph we can also use to express transition from ARQ to HARQ2.

Other parameter is transition from HARQ1 to HARQ2. It means that left and right side of equipment has the same formula and it is (3). One for *RS code (511, 383, 64)* and other one for *RS code (511, 255, 128)*. Responsibility is to express unknown P_S. It is about number of uncorrectable symbols HARQ1 scheme specifically we have to count NAKs again (the same case as in previous two paragraphs). Final result is α_2:

$$\alpha_2 = P_S \cdot W \qquad (8)$$

Based on research in [6], [10], transition from H_1 or H_2 to L is the best to do when there is no error detection in last W packets. P_H is the probability, that the last W received packets are without any symbol error. It means that probability of transition from state H_j to L is equal to:

$$P_{H_j} = (1 - P_{e_j})^W \qquad (9)$$

where j can take value 1 or 2.

Reciprocally to the previous parameter α_2, transition from HARQ2 to HARQ1 is used the same (3) on both sides of equation), but parameter β monitors amount of ACKs (packet is correctable or without errors). Similarly to ACK$^+$ and knowledge from [6], [10], the transition is best to do only when there is W ACKs in last W packets, i.e. $\beta = W$.

D.Evaluation Parameters

We use throughput and modify Mean Square Error (MSE) as parameters for comparing ideal throughput with our solution. Definition of ideal throughput can be expressed as the maximum of three modes in each compared sample:

$$\eta_{IDEAL} = \max(\eta_{ARQ}, \eta_{HARQ1}, \eta_{HARQ2}) \qquad (10)$$

where η_{ARQ} - throughput of pure ARQ, η_{HARQ1} - throughput of HARQ with RS code (511,383,64) , η_{HARQ2} - throughput of HARQ with RS code (511,255,128).

For complete imagination, fig. 3 is shown ideal throughput of combination three different schemes.
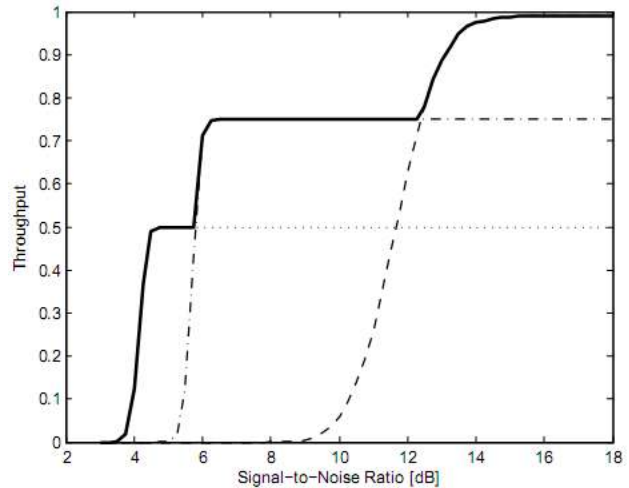

Fig. 3 Ideal throughput composed of three used schemes dashed line – pure GBN method, dash-dotted line – HARQ scheme with RS code (511, 383, 64), dotted line - HARQ scheme with RS code (511, 255, 128), solid line – ideal throughput

For complete understanding of calculation progress, here is very helpful parameter Sample Square Error, i.e. SSE is shown how it changes the square difference between the ideal and the simulated curve at each step of simulation (SNR sample):

$$SSE_i = (\eta_{IDEAL_i} - \eta_{SIM_i})^2 \qquad (11)$$

where η_{SIM} - throughput of simulated adaptive

ARQ/HARQ/HARQ method i is serial number of sample.

Mean square error is one of many ways to quantify the difference between values implied by an estimator and the true

values of the quantity being estimated. MSE is a risk function, corresponding to the expected value of the squared error loss or quadratic loss. MSE measures the average of the squares of the "errors." MSE can be express as:

$$MSE = E\left[(\bar{X} - X)^2 \right] = \frac{1}{N}\sum_{i=1}^{N}\left(\bar{X} - X_i \right)^2 \qquad (12)$$

For our needs we edited (12) to:

$$MSE = \frac{\sum_{i=1}^{N}\left(\eta_{IDEAL\,i} - \eta_{SIM\,i}\right)^2}{N} = \frac{\sum_{i=1}^{N}SSE_i}{N} \qquad (13)$$

III. RESULTS

In this section we will examine at what is the impact of thresholds calculation to throughput. Results of trial and error method will be compared with optimal settings parameters of adaptive method.

Our simulations are done in conditions of Land-Mobile-Satellite (LMS) channel specifically in RURAL area. As we mentioned in theoretical part of paper, for ARQ mode is used GBN ARQ scheme and hybrid ARQ scheme is type 1 with *RS code*. Every packet of all three schemes has constant packet size and it is 511 symbols. Every symbol is equal to 9 bits. Difference is in number of information and security symbols respectively bits:

TABLE I
NUMBER OF INFORMATION AND REDUNDANCY BITS OF PACKET FOR VARIOUS CODING TECHNIQUES

Modes	Information bits	Redundancy bits
ARQ	4567	32*
HARQ 1	3447	1152
HARQ 2	2295	2304

*Redundancy for Cyclic Redundancy Check

Outputs of simulations are processed in two kinds of graphs:
- Dependence of relative throughput on Signal-to-Noise Ratio
- Dependence of Sample Error Square on SNR

In fig. 4 are results from trial and error method. There is not used calculation of thresholds. We can see the problem in transaction, because there is significant decrease of throughput. On the other hand, when the method is in steady state (there is no transition), then the adaptive method is copied the ideal curves. It means that the proposal of adaptive method is right, but we need calculation to eliminate noticeable drops in transitions.
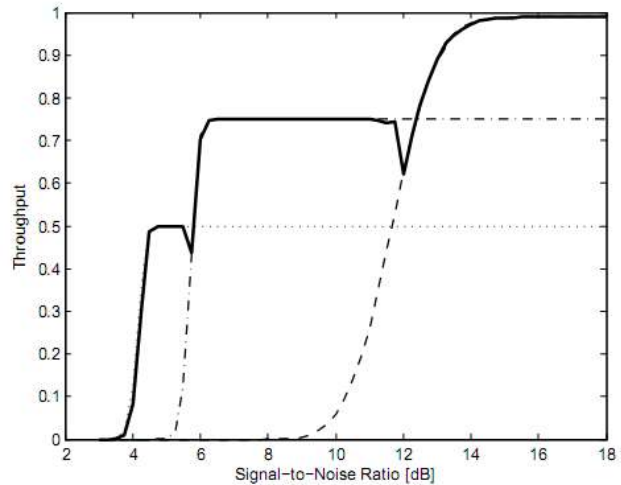

Fig. 4 Dependence of relative throughput on SNR

Parameters: $\alpha_1 = 10$, $\beta = \alpha_2 = W = 30$; dashed line – pure GBN method, dash-dotted line – HARQ scheme with RS code (511, 383, 64), dotted line - HARQ scheme with RS code (511, 255, 128), solid line – 3-stage adaptive method

Compare with previous fig. 4, simulation results of adaptive scheme viewed in next fig. 5 are clearly showed improve in throughput of problematic transitions. This time for all simulation SNR range our adaptive scheme is followed one of ideal curves.
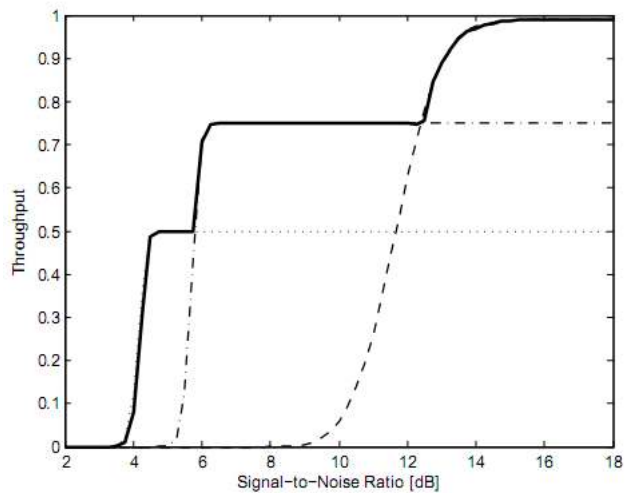

Fig. 5 Dependence of relative throughput on SNR

Parameters: $\alpha_1 = 10$, $\beta = \alpha_2 = W = 64$; dashed line – pure GBN method, dash-dotted line – HARQ scheme with RS code (511, 383, 64), dotted line - HARQ scheme with RS code (511, 255, 128), solid line – 3-stage adaptive method

When we compare fig. 3 and fig. 5, it is obvious, that one is close enough to other. In other words, our method is approximated to ideal throughput consisting of the highest throughput of three schemes. We used only two values to do

it.

Next two figures show comparison our method with different thresholds settings, respectively one curve is with and other is without using thresholds settings. As we mentioned before, SSE is the difference between simulation and ideal values. It is clear, that they are the same graphs (difference is in axe y: fig. 6 has linear both axes, on the other hand, fig.7 has also linear axe x, but axe y is logarithmic). Both figures have common, that the biggest difference between curves is in places of transitions.
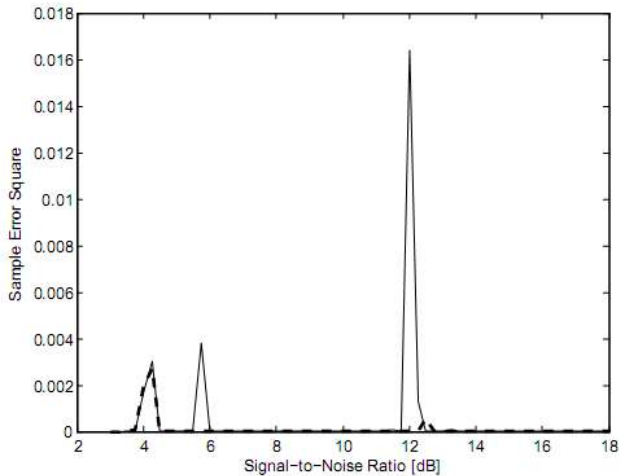

Fig. 6 Sample Square Error comparison with linear axes
Solid line – trial and error variant, dashed line – variant with thresholds calculation

For complete imagination we declare graph with the same values, but axe y is not linear this time, but it is logarithmic.
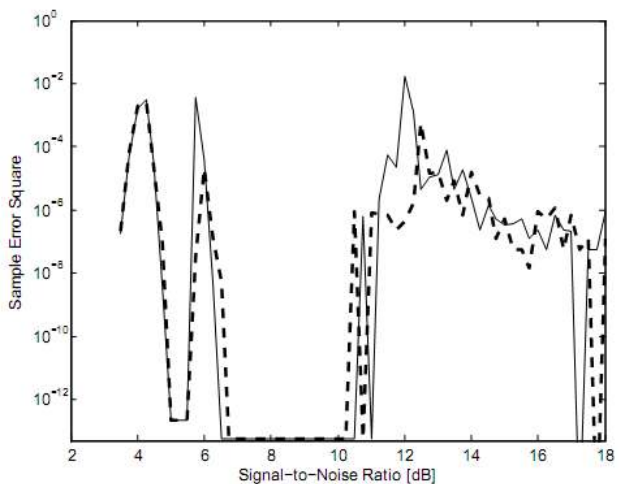

Fig. 7 Sample Square Error comparison with logarithmic SSE axe
Solid line – trial and error variant, dashed line – variant with thresholds calculation

Previous figures suggest the impact of our proposal. This assertion is confirmed by the following table, where you can observe a significant reduction in MSE:

TABLE II
COMPARISON OF *MSE* FOR VARIOUS PARAMETER SETTINGS

Result for	*MSE*
Fig. 4	$435,42.10^{-6}$
Fig. 5	$89,326.10^{-6}$

IV. CONCLUSION

In this paper we presented three stage adaptive method and comparison of our proposal with classic pure ARQ and HARQ methods. Main idea of method is approach to a maximum throughput from three different schemes in the entire studied area of SNR. The overall results showed that we met this goal.

We introduced theoretical analysis of transitions. Mostly, it is about calculation of values, when one scheme has to switch to other one. Subsequently based on calculations, we realized simulations. An important fact is that despite the relatively more thresholds in the calculation, we need only two values in simulations to obtain the ideal curve.

Proof of the correctness of our proposal shows a comparison with the so-called trial and error method. Better throughput is seen visual and also in numeric way, because with thresholds calculation we reduced MSE to one-fifth.

In further research we can focus on the expansion of the number of states of adaptive methods, i.e. propose multistage method. This would be especially useful in the channel with a high fluctuation of the signal-to-noise ratio.

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