

# Investigation of Self-Similarity Solution for Wake Flow of a Cylinder

A.B. Khoshnevis, F. Zeydabadi, F. Sokhanvar

**Abstract**— The data measurement of mean velocity has been taken for the wake of single circular cylinder with three different diameters for two different velocities. The effects of change in diameter and in velocity are studied in self-similar coordinate system. The spatial variations of velocity defect and that of the half-width have been investigated. The results are compared with those published by H.Schlichting. In the normalized coordinates, it is also observed that all cases except for the first station are self-similar. By attention to self-similarity profiles of mean velocity, it is observed for all the cases at the each station curves tend to zero at a same point.

**Keywords**—Self-similarity, wake of single circular cylinder

## I. INTRODUCTION

A turbulent flow is said to be self-similar when some or all of its statistical properties depend only on certain combinations of the independent variables rather than on each independent variable individually. The consequence of this is that the number of independent variables in the problem is reduced, thus greatly facilitating its solution. Geometrically, a self-similar flow possesses a certain symmetry; for example the flow pattern on any two cross sections perpendicular to a given axes may be identical except for a scale factor. The property of self-similarity has been used on many occasions in fluid dynamics to derive elegant solutions to otherwise very difficult problems (such as the structure of turbulent boundary layers, jets, and wakes). Self-similarity in the wake flow is realized if the normalized mean velocity and turbulent statistics are independent of the stream wise location. Many studies for self-similarity solution in the wake flow and other shear flows have been confined to flat plate flow.

Self-similarity of boundary layers is a very useful phenomenon: It allows easy solution, better understanding of the boundary layer, and a better organized classification and comprehension of experimental results.

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## II. REVIEW

One of the most important studies about the wake of flow by Townsend is investigation on a symmetrical cylinder and comparison with its diameter [1].

A study from wynanski, champagne, and marasli showed that wakes with asymmetric bodies, normal velocities, scales of length and distribution of Reynolds stresses are not self similar and depends on their geometrical shapes [2].

The existence of self-similar solution of incompressible two-dimensional turbulent wall flows with injection was examined analytically by Micha Wolfshtein [3]. It was shown that such solutions are possible for wall jets and boundary layers under adverse pressure gradient and suction.

Partially wetting drops sliding down an inclined plane develop a “corner singularity” at the rear, consisting of two dynamic contact lines that intersect. Snoeijer, Rio, Le Grand and Limat analyzed the three-dimensional flow in the vicinity of this singularity by exploring similarity solutions of the lubrication equations [4].

Mikhail V. Medvedev presented the analytical self-similar solution describing the boundary Layer, which forms in the vicinity of a spinning neutron star [5]. His solution is hot, highly subsonic and contains no shocks.

Takhar and Nath investigated a self-similar solution of the unsteady flow in the stagnation point region of a rotating sphere with a magnetic field [6].

The probability density function (pdf) of a streamwise velocity component is studied in zero-pressure gradient boundary layers. From analyzing the data up to  $R_{\theta} = 13,000$ , Tsuji, Lindgren, and Jahansson found that pdfs have self-similar profiles ranging from  $y^+ = 180$  to  $0.049_{\theta}^+$ , where  $\theta$  is Rotta–Clauser boundary layer thickness [7].

Turbulent wakes are known to develop self-similarly sufficiently far downstream from obstacles that generate them. It has long been assumed that the spreading rate of the wake in the self-similar regime is independent of the details of the body generating the wake, being dependent only on the total drag (or momentum deficit). This assumption seems to be in contradiction with some recent experiments. Sandip Ghosal and Michael M.Rogers attempted to complement these experimental investigations through a numerical study of a time-developing wake [8].

III. SELF-SIMILARITY STUDY FOR THE WAKE OF A CYLINDER

In the far distances from the point that wake occurs, behavior of the wake is special importance. Thus it needs a self-similarity solution. Difference between mean velocity profile and other amounts in two position of x should be attributed with changes in scale not in their function forms. So in a wake, we deal with two scales  $\Delta U(x)$  and  $l(x)$ .

$$U_e - \bar{U} = \Delta U f(\eta) \tag{1}$$

In the self-similar coordinates, we use  $(U_e - \bar{U}) / \Delta U$  and  $\eta$  function for normalized axes. Where U is mean velocity,  $U_e$  is max of mean velocity (potential velocity) in stations and  $\Delta U$  is velocity defect. After doing self-similarity solution for Navier-Stokes equations, we can obtain similarly function  $f(\eta)$  Whereas:

$$f(\eta) = e^{-U_e \alpha / 4 v_i \eta^2} \tag{2}$$

When we study wake of cylinder with diameter d we have:

$$\alpha = d \tag{3}$$

$$f(\eta) = e^{-R \eta^2 / 4} \tag{4}$$

Where

$$R = \frac{d U_e}{v_i} \tag{5}$$

Thus

$$\eta = \frac{y}{\sqrt{d(x - x_0)}} \tag{6}$$

For  $\eta < 0.3$  these equations are true but when effect of intermittency is important these equations are not convenient. Here, d is the cylinder diameter,  $x_0 = 0$  and x is the distance from the cylinder (1, 2, 3, and 4) [9].

IV. MEAN VELOCITY PROFILES IN THE WAKE

Fig. 1 and Fig. 2 show the profiles of normalized mean velocity ( $U/U_{ref}$ ) in the wake of a cylinder along the centerline of the test section. Note that the ordinate  $+y/L$  and  $-y/L$  are directed towards up and down of the centerline of the cylinder, respectively. It is observed that the mean velocity profiles for all the cases are roughly not similar at station  $x/d=1$ , due to high vorticity produced behind of the cylinder which is considerable for all bluff bodies. In these figures we show the effect of difference in cylinder diameter on mean velocity profiles. It is observed in Fig. 1 for lower velocity at far away from the cylinder (means increase  $x/d$ )

profile velocities are more similar. Although at high velocity Fig. 2 it cannot be seen similar mean velocity profile at  $x/d=4$  for high diameter  $d=25$ mm. It is concluded for higher velocity bigger diameter wake will decay rapidly. We can see potential flow region are same for all cases at all stations. Fig. 3 and Fig. 4 show these profiles for one diameter and different velocities. We study the effect of different velocity in these figures. It is shown at stations  $x/d=2, 3,$  and  $4$  for both diameter velocity defects for higher velocity is less than for lower velocity.

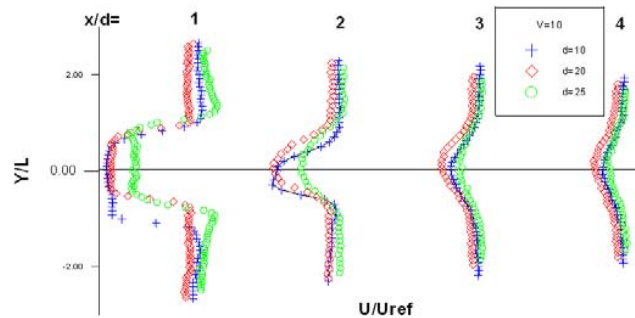


Fig. 1 Mean velocity profile  $U/U_{ref}$  in the wake

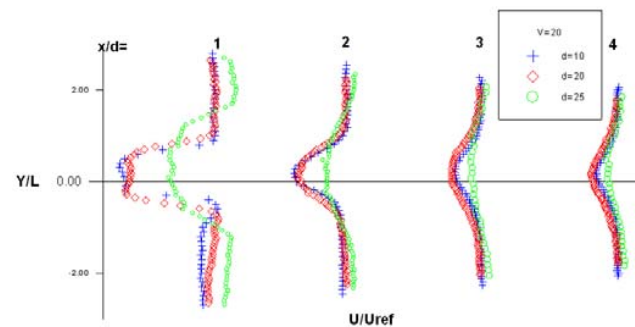


Fig. 2 Mean velocity profile  $U/U_{ref}$  in the wake

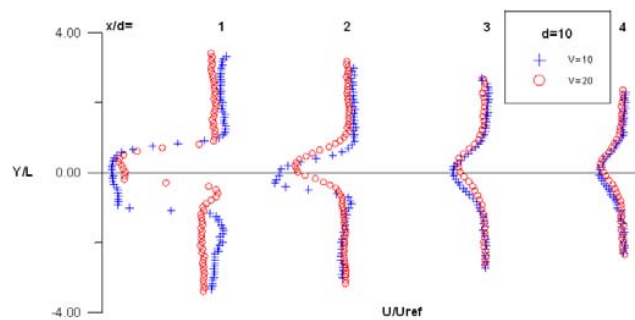


Fig. 3 Effect of velocity on self-similarity of mean velocity profiles in the wake

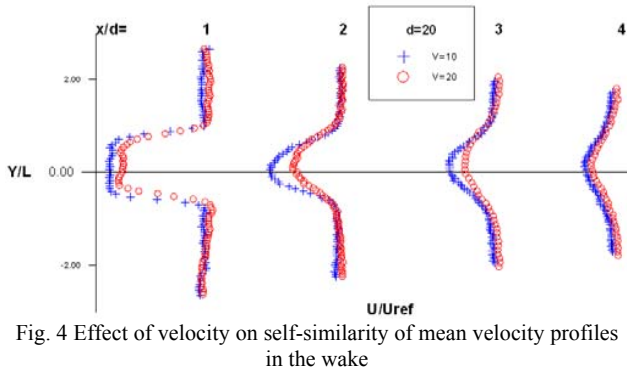


Fig. 4 Effect of velocity on self-similarity of mean velocity profiles in the wake

V. VARIATION OF THE WAKE PARAMETERS

Fig. 5 (Note: we use "velocity defect" for  $\Delta U$ ) shows that velocity defect, with  $\Delta U$  occurs near the centerline. Similar behavior curve for all diameters at 10m/s is seen in Fig. 5 also the same behavior curve is observed for 20m/s however differences exist in magnitude of  $\Delta U$ . The spatial variations of L (Half-width) are shown in Fig. 6.

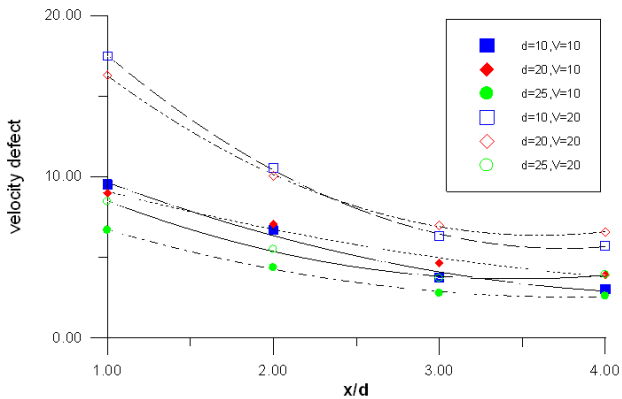


Fig. 5 Velocity effect

From Fig. 5 and Fig. 6 it is revealed that  $\Delta U$  and L are proportional to  $x^{-1/2}$  and  $x^{1/2}$ , respectively. The results support the idea of H.Schlichting [10] for the spatial development of velocity defect and the wake half-width.

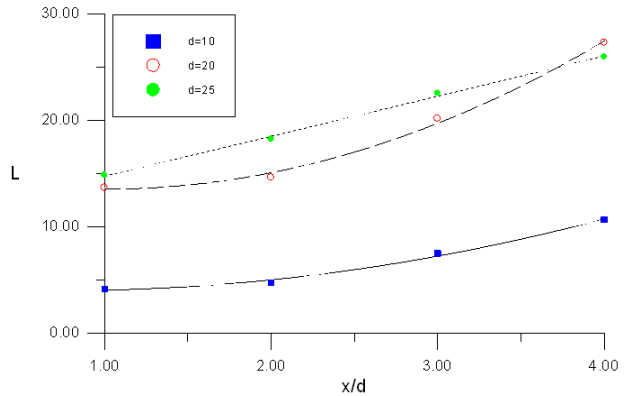


Fig. 6 Half width

VI. SELF-SIMILARITY CHARTS FOR MEAN VELOCITY

In Fig. 7 and Fig. 8 ( $\eta$  for  $\frac{U_e - \bar{U}}{\Delta U}$ ) chart we show  $\frac{U_e - \bar{U}}{\Delta U}$  respect  $\eta$  for the first station in different diameters and it is observed that there is no self-similarity in the first station which seems to be due to high vorticity behavior of the cylinder.

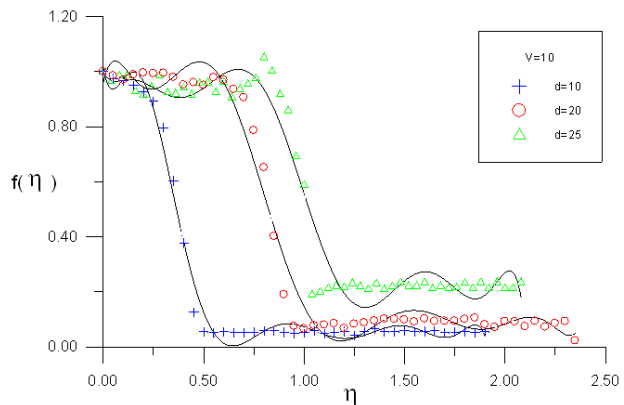


Fig. 7 Self-similarity in first station

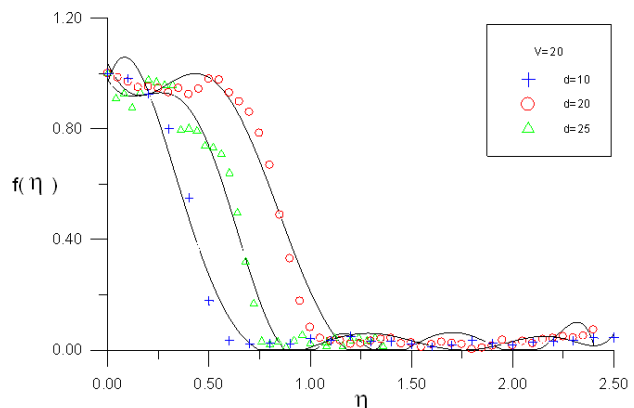


Fig. 8 Self-similarity in first station and V=20

In Fig. 9 and Fig. 10 we obtained self-similarity for wake of cylinder with diameter=10 for two different velocities. Fig. 11 and Fig. 12 show self-similarity of cylinder with d=20 and Fig. 13 and Fig. 14 for cylinder with d=25.

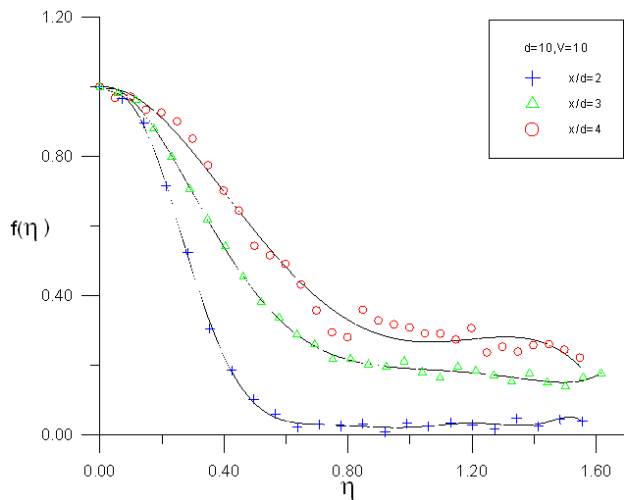


Fig. 9 Self-similar profile of wake for d=10 v=10

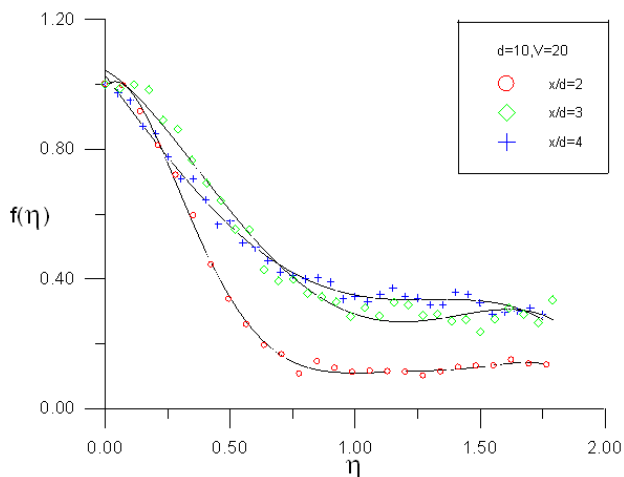


Fig. 10 Self-similar profile of wake for d=10 v=20

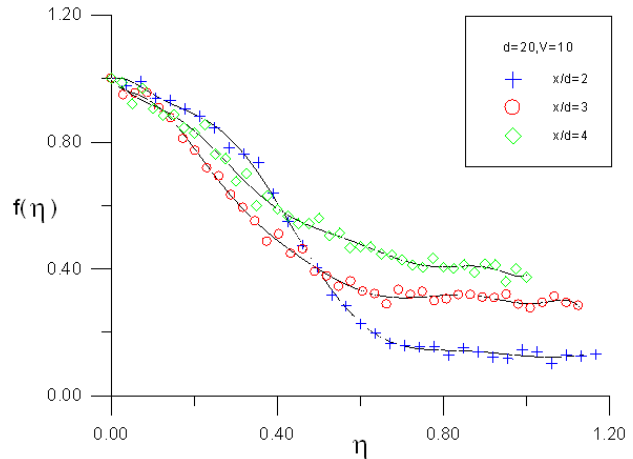


Fig.11 Self-similar profile of wake for d=20 v=10

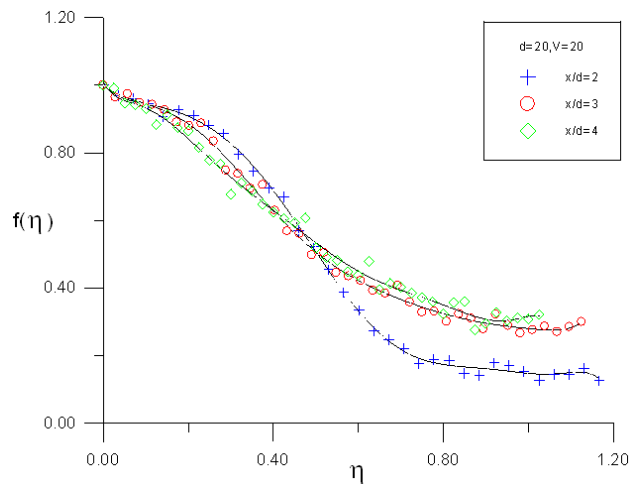


Fig. 12 Self-similar profile of wake for d=20 v=20

As shown in these figures in all cases self-similarity of mean velocity profiles of wake of a cylinder obtain for stations 2, 3, and 4. For all cases (different diameters and different velocity) at the each station charts tend to zero at a same point.

## VII. CONCLUSION

Mean velocity profiles were studied for three different diameters and two different velocities. The spatial development of half width and that of the velocity defect shows the power laws of dependencies  $x/c$ , which as predicted by Schlichting [10] are  $(x/c)^{1/2}$  and  $(x/c)^{-1/2}$ , respectively. Mean velocity profiles indicate that all cases are self-similar except for first station. This may be concluded from vorticity produced behind the cylinder. By attention to self-similarity profiles of mean velocity, it is observed for all the cases at each station curves tend to zero at a same point.

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