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A New Approach to Image Segmentation via Fuzzification of Rènyi Entropy of Generalized Distributions

Samy Sadek, Ayoub Al-Hamadi, Axel Panning, Bernd Michaelis, Usama Sayed

***Abstract*—**In this paper, we propose a novel approach for image segmentation via fuzzification of Rènyi Entropy of Generalized Distributions (REGD). The fuzzy REGD is used to precisely measure the structural information of image and to locate the optimal threshold desired by segmentation. The proposed approach draws upon the postulation that the optimal threshold concurs with maximum information content of the distribution. The contributions in the paper are as follow: Initially, the fuzzy REGD as a measure of the spatial structure of image is introduced. Then, we propose an efficient entropic segmentation approach using fuzzy REGD. However the proposed approach belongs to entropic segmentation approaches (i.e. these approaches are commonly applied to grayscale images), it is adapted to be viable for segmenting color images. Lastly, diverse experiments on real images that show the superior performance of the proposed method are carried out.

***Keywords*—**Entropy of generalized distributions, entropy fuzzification, entropic image segmentation.

I. Introduction

IMAGE segmentation is an elementary and significant component in many applications such as image analysis, pattern recognition, medical diagnosis and currently in robotic vision. However, it is one of the most difficult and challenging tasks in image processing, and determines the quality of the final results of the image analysis. Instinctively, image segmentation is the process of dividing an image into different regions such that each region is homogeneous while not a union of any two adjacent regions. An additional requirement would be that these regions have a correspondence to real homogeneous regions belonging to objects in the scene.

Various algorithms using different approaches have been proposed for image segmentation. These approaches include local edge detection (e.g. [1]), deformable curves (e.g. [2]), morphological region-based approaches (e.g. [3-5]), global optimization approaches on energy functions and stochastic model-based methods (e.g. [6-8]).

Recent developments of statistical mechanics based on a concept of nonextensive entropy have intensified the interest of investigating a possible extension of Shannon's entropy to Information Theory [9]. This interest appears mainly due to similarities between Shannon and Boltzmann/Gibbs entropy functions. The nonextensive entropy is a new proposal in order to generalize the Boltzmann/Gibbs's traditional entropy to nonextensive systems (i.e. strong correlated systems are good candidates to be nonextensive). In this theory a new parameter α is introduced as a real number associated with the nonextensivity of the system.

In this paper we propose a new approach for image segmentation which applies for the first time fuzzy conception on the Rènyi entropy of generalized distributions. Our work for image segmentation does better in comparison to the most recent entropic methods [10].

The remainder of the paper is organized as follows. In the next section, the essential concepts of Rènyi entropy of generalized distributions and nonextensive systems are addressed. Then, the proposed approach is elaborately described in section 3. Section 4 presents the experimental results that show the performance of the proposed approach. Finally, section 5 is dedicated for outlining the benefits of the proposed approach and concluding the paper.

II. Entropy of Generalized Distributions

In 1948 Shannon [11] redefined the entropy of Boltzmann/Gibbs as a measure of uncertainty regarding the information content of a system. He defined an expression for measuring the amount of information produced by a process. Let $P = (p_1, p_2, \ldots, p_n)$ be a finite discrete probability distribution, that is, suppose $p_k \geqq 0,\ k = 1,2,\ldots,n$ and $\sum_{k=1}^{n} p_k = 1$. The amount of uncertainty of the distribution P, that is, the amount of uncertainty concerning the outcome of an experiment, the possible results of which have the probabilities $p_1, p_2, \ldots, p_n$ is called the entropy of the distribution and is usually measured by the quantity $H(P) = H(p_1, p_2, \ldots, p_n)$, introduced by Shannon and defined by

$$H(p_1, p_2, \ldots, p_n) = \sum_{k=1}^{n} p_k log_2 \frac{1}{p_k} \tag{1}$$

It is easy to see that the Shannon entropy for the conjunction of two distributions P and Q satisfies

Samy Sadek is currently with the Institute for Electronics, Signal Processing and Communications, Otto-von-Guericke University Magdeburg, Magdeburg, CO 39106 Germany(corresponding author to provide phone: +493916711472; fax: +49-3916711231; e-mail: Samy.Bakheet@ ovgu.de).

Ayoub Al-Hamadi, Axel Panning ,and Bernd Michaelis are also with the Institute for Electronics, Signal Processing and Communications, Otto-von-Guericke University, Magdeburg, CO 39106 Germany (e-mail: Ayoub.Al-Hamadi@ovg.de).

Usama Sayed is with the Electrical Engineering Department, Assuit University, CO 34565 Egypt (e-mail: usama @aun.edu.eg).

$$H(P \oplus Q) = H(P) + H(Q) \quad (2)$$

Which states one of the most important properties of entropy, namely, its additivity: the entropy of a combined experiment consisting of the performance of two independent experiments is equal to the sum of the entropies of these two experiments. The formalism defined by Eq. (1) has been shown to be restricted to the Boltzmann-Gibbs-Shannon (BGS) statistics.

However, for nonextensive systems, some kind of extension appears to become necessary. Rènyi in 1959 [12] proposed a wider class of entropies which are useful for describing the properties of nonextensive systems and defined as

$$H_\alpha(p_1, p_2, \dots, p_n) = \frac{1}{\alpha-1} log_2 \sum_{k=1}^{n} {p_k}^\alpha \quad (3)$$

where $\alpha \geq 0$ and $\alpha \neq 1$. The real number α is called an entropic order that characterizes the degree of nonextensivity. This expression reduces to Shannon entropy in the limit $\alpha \rightarrow 1$. Thus Shannon's measure of entropy is the limiting of the measure of entropy H_α and it is called the measure of entropy of order 1 of the distribution.

It is worth mentioning that the parameter α in Rènyi entropy is typically interpreted as a quantity characterizing the degree of nonextensivity of a physical system [13]. In some cases the parameter α has no physical meaning, but it gives new possibilities in the agreement of theoretical models and experimental data [14]. In other cases, α is solely determined by constraints of the problem and by this means α may have a physical meaning [15].

We shall see that in order to get the fine characterization of Rènyi entropy, it is advantageous to extend the notion of a probability distribution, and define entropy for the generalized distributions. The characterization of measures of entropy (and information) becomes much simpler if we consider these quantities as defined on the set of generalized probability distributions.

Let $[\Omega, \mathrm{P}]$ be a probability space that is, Ω an arbitrary nonempty set, called the set of elementary events, and P a probability measure, that is, a nonnegative and additive set function for which $\mathrm{P}(\Omega) = 1$. Let us call a function $\xi = \xi(\omega)$ which is defined for $\omega \in \Omega_1$, where $\Omega_1 \subset \Omega$. If $P(\Omega_1) = 1$ we call ξ an ordinary (or complete) random variable, while if $0 < P(\Omega_1) \leq 1$ we call ξ an incomplete random variable. Evidently, an incomplete random variable can be interpreted as a quantity describing the result of an experiment depending on chance which is not always observable, only with probability $P(\Omega_1) < 1$. The distribution of a generalized random variable will be called a generalized probability distribution. Thus a finite discrete generalized probability distribution is simply a sequence $p_1, p_2, \dots, p_n$ of nonnegative numbers such that setting $P = (p_1, p_2, \dots, p_n)$ and taking

$$\varpi(P) = \sum_{k=1}^{n} p_k \quad (4)$$

where $\varpi(P)$ is the weight of the distribution and $0 < \varpi(P) \leq 1$. A distribution that has a weight less than 1 will be called an incomplete distribution. Now, using Eq. (3) and Eq. (4), the Rènyi entropy for the generalized distribution can be written as follows

$$H_\alpha(p_1, p_2, \dots, p_n) = \frac{1}{\alpha-1} log_2 \left[\frac{\sum_{k=1}^{n} {p_k}^\alpha}{\varpi(P)}\right] \quad (5)$$

Rènyi entropy has a nonextensive property for statistical independent systems, defined by the following pseudo additivity entropic formula

$$H_\alpha(A \oplus B) = H_\alpha(A) + H_\alpha(B) + (\alpha - 1) \cdot H_\alpha(A) \cdot H_\alpha(B) \quad (6)$$

III. Proposed Segmentation Method

In the occurrence of too much noise in the image the process of segmentation becomes a tricky. While there are much more techniques for image segmentation, some of them are time-consuming and the others call for huge storage space. The proposed technique achieves the task of segmentation in a novel way. This technique not only surmounts the noise in image but also it calls for neither more time nor massive storage space. This happens by the advantage of using fuzzy Rènyi entropy of generalized distributions to measure the structural information of image and then locate the optimal threshold depending on the postulation that the optimal threshold corresponds to the segmentation with maximum structure (i.e., maximum information content of the distribution). The proposed technique methodologically comprises the following main steps:

1. Pre-processing

Firstly, Gaussian smoothing is performed by convolving an image with a Gaussian operator in order to suppress of image noise. As an additional result of applying this step, isolated noise points and small structures are filtered out.

2. Fuzzy entropic thresholding

This step takes in the subsequent sub steps:

2.1 Entropies calculation

For an n graylevels image let $p_i = (p_1, p_2, \dots, p_n)$ be the probability distribution. From this distribution, we could derive two sub probability distributions, one for the foreground (class A) and the other for the background (class B) given by $p_A = \{p_k\}_{k=1}^{t}$ and $p_B = \{p_k\}_{k=t+1}^{n}$ respectively; where t is the threshold value. Subsequently the priori Rènyi entropy of generalized distributions for each distribution can be defined as follows

$$H_\alpha{}^A(t) = \frac{1}{\alpha-1} log_2 \left[\frac{\sum_{k=1}^{t} (p_A)^\alpha}{\sum_{k=1}^{t} p_A}\right] \quad (7)$$

$$H_\alpha{}^B(t) = \frac{1}{\alpha-1} log_2 \left[\frac{\sum_{k=t+1}^{n} (p_B)^\alpha}{\sum_{k=t+1}^{n} p_B}\right] \quad (8)$$

2.2 Entropy fuzzification

Fuzzy sets have been introduced by L.A. Zadeh (1965) as an extension of the classical notion of a set [16]. Fuzzy sets are sets whose elements have degrees of membership. Mathematically, a fuzzy set, A is defined as set whose elements characterized by a one-to-one function called member function, $\mu_A(x_i)$ where x_i refers to the i-th element in the set. This membership function assigns a membership value to every element in the fuzzy set, which is suggestive of the amount of vagueness in the fuzzy set. The membership value of an element in a fuzzy set lies in [0, 1]. A higher membership value refers to stark containment of the element in the set, while a lower value indicates weak containment.

The fuzzification of entropy at this juncture comprises the process of incorporating the fuzzy membership into the relations of entropy described by (7) and (8). Hence fuzzy segmentation deems the fuzzy memberships as an indication of how strongly a pixel value belongs to the background or to the foreground. Really, the farther away a value of pixel is from a presumed threshold (the deeper in its region), the greater becomes its probability to belong to a specific class. As a result, for any foreground and background pixel, which is i levels below or i levels above a given threshold t, the membership values are determined by

$$\mu_A(t-i) = 0.5 + \frac{\sum_{k=0}^{i} p(t-k)}{2p(t)} \qquad (9)$$

that is, its measure of belonging to the foreground (class A), and by

$$\mu_B(t+i) = 0.5 + \frac{\sum_{k=1}^{i} p(t+k)}{2[1-p(t)]} \qquad (10)$$

respectively (see Fig. 1).

Evidently on the value corresponding to the threshold, one should have the maximum ambiguity, such that $\mu_A(t) = \mu_B(t) = 0.5$. Now, considering the two equations (9), (10), the fuzzy form of entropic equations (7), (8) can be written as

$$H_\alpha{}^A(t) = \frac{1}{\alpha-1} log_2 \left[\frac{\sum_{k=1}^{t} (\mu_A(k))^\alpha}{\sum_{k=1}^{t} \mu_A(k)}\right] \qquad (11)$$

$$H_\alpha{}^B(t) = \frac{1}{\alpha-1} log_2 \left[\frac{\sum_{k=t+1}^{n} (\mu_B(k))^\alpha}{\sum_{k=t+1}^{n} \mu_B(k)}\right] \qquad (12)$$

2.3 Getting the optimum threshold

In image processing, thresholding is the most regularly used method to distinguish objects from background. In this step the optimum threshold value $t^\star$ is automatically determined from maximizing the total entropy, $H_\alpha^{A+B}(t)$. This value will be used for preliminary segmentation (thresholding). When total entropy is maximized, the value of parameter t that maximizes the function is believed to be the optimum threshold value [17]. Mathematically, the problem can be formulated as shown below:

$$t^\star = argmax\left[H_\alpha^{A+B}(t)\right] = argmax\begin{bmatrix} H_\alpha{}^A(t) + H_\alpha{}^B(t) \\ +(\alpha-1)\cdot H_\alpha{}^A(t) \cdot H_\alpha{}^B(t)\end{bmatrix} \qquad (13)$$

In the case of RGB color images, the preceding scalar equation is replaced with the following vector equation

$$\vec{t}^\star = argmax\begin{bmatrix} H_\alpha{}^A(\vec{t}) + H_\alpha{}^B(\vec{t}) \\ +(\alpha-1)\cdot H_\alpha{}^A(\vec{t}) \cdot H_\alpha{}^B(\vec{t})\end{bmatrix} \qquad (14)$$

where $\vec{t} = (t_R, t_G, t_B)$ and the optimum threshold vector satisfies

$$\|\vec{t}^\star\| = \sqrt{(\omega_R t_R)^2 + (\omega_G t_G)^2 + (\omega_B t_B)^2} \qquad (15)$$

where ω_R, ω_G, ω_B are the normalized energies of the channels $R, G, and\ B$ respectively, that is,

$$\omega_R + \omega_G + \omega_B = 1 \qquad (16)$$

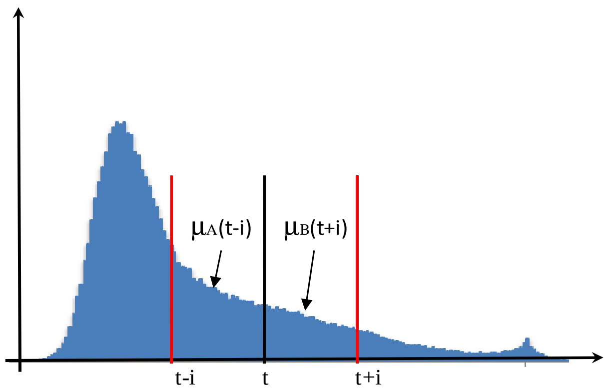

Fig.1 Fuzzy membership as an indication of how strongly a pixel belongs to its region.

3. Post-processing

This step consists of the following sub steps:

3.1 Morphological filtering

This step aims to enhance the results of the previous thresholding step. Because of the inconsistency within the color of objects, the binary image maybe contains some holes inside. The process of filling holes attempts to get rid of the holes form the binary image. This problem can be overcome by the filling holes process. Opening with small structure element is used to separate some objects that are still connected in small number of pixels [18]. In image processing, dilation, erosion, filling holes and opening are all identified as morphological operations.

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3.2 Watershding

In this step, watershed algorithm [19] is applied on the Euclidean Distance Transform (EDT) of the image. The EDT of a binary image works as: for each pixel in the binary image, the transform assigns a number that is the distance between that pixel and the nearest nonzero pixel of the image. The distance is calculated using the Euclidean distance metric. The peaks of the distance transform lay in the middle of each object. The idea is to run watershed using these peaks as markers. For this, we invert the distance transform so that the peaks become the regional minima the objects are correctly separated by watershed.

3.3 Wrong objects removals

This step contributes to remove incorrect objects according to the range of size of the object. Consequently tiny noise objects of sizes that are less than the minimum predefined threshold can be discarded. Also objects whose size greater than the maximum threshold size can be removed as well. It is worth to say that those thresholds are user-defined data and dependent on the application. The preceding steps of the proposed fuzzy entropic segmentation method can be depicted by the block diagram in Fig. 2.

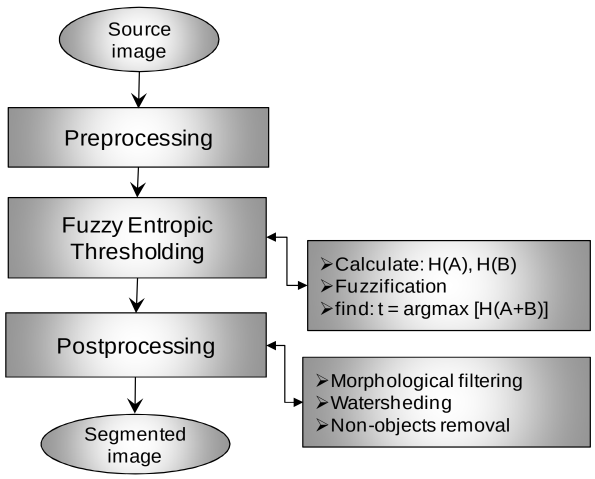

Fig.2 Block diagram of the proposed segmentation method.

IV. Experimental Results

In this section, we present the results of the proposed method and make a comparison with another segmentation method based on Tsallis entropy (see [15] for more details). Firstly, to investigate the proposed approach for image segmentation we have initiated by different image histograms. Each of these histograms describes the "foreground" and the "background". As demonstrated earlier, the segmentation procedure searches for a luminance value that separates the two regions "foreground" and "background" in the image. This process allows judging the quality of segmentation result as function of some parameters such as amplitude, position and width of the peaks in the histograms. All these parameters have a key role in the characterization of the image, as for instance: homogeneity of the scene illumination (graylevel contrast), image and object size, "foreground" and "background" texture, noisy images, etc.

To evaluate the proposed method, several real images have been utilized and many values of the parameter α are experimented. All the results presented here were obtained at the same value of parameter α (α=0.9). In figure 3 an image of flower with a heterogeneous distribution of light around it, leading to an irregular histogram of two peaks. The proposed entropic method could be very practical in such applications.

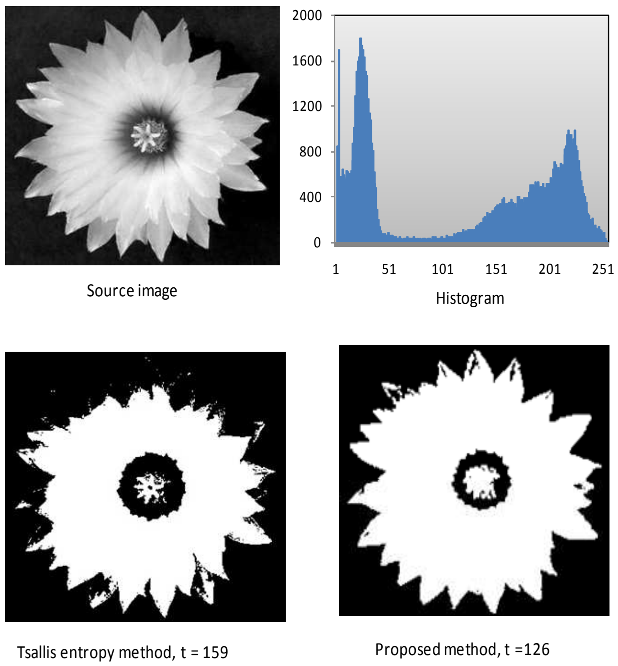

Fig.3 Entropic segmentation for image of a flower with a heterogeneous distribution of light around.

Fig. 4 shows a well-known image of "Lena" with a complicated background. In the figure, many regions on the face, hair and hat are interlaced. However, the proposed approach could successfully segment the image into desired regions, as shown below. The regions after segmentation are more consistent and are not affected by the inextricable background. In Fig. 5, an RGB color image of woman with a background of skin-like color around it, leading to a similarity between foreground color and background color. It is clear that the proposed fuzzy entropic approach could appropriately identify all regions of image especially when the approach takes into account the color information.

It is worth mentioning here that most of the previous proposed entropic segmentation methods related to our method (e.g. Tsallis method) have merely dealt with grayscale images. On the other hand the proposed method has successfully extended and adapted the concept of entropy via the fuzzification to be viable for segmenting color images. This is happening for the first time. Furthermore the proposed method rapidly did the task of segmentation (i.e. less than one second was sufficient for segmenting an image of size equal to 360 × 280 by using an ordinary Desktop PC (Intel Pentium 4

Processor 360, 1 GB RAM) running Microsoft Windows XP).

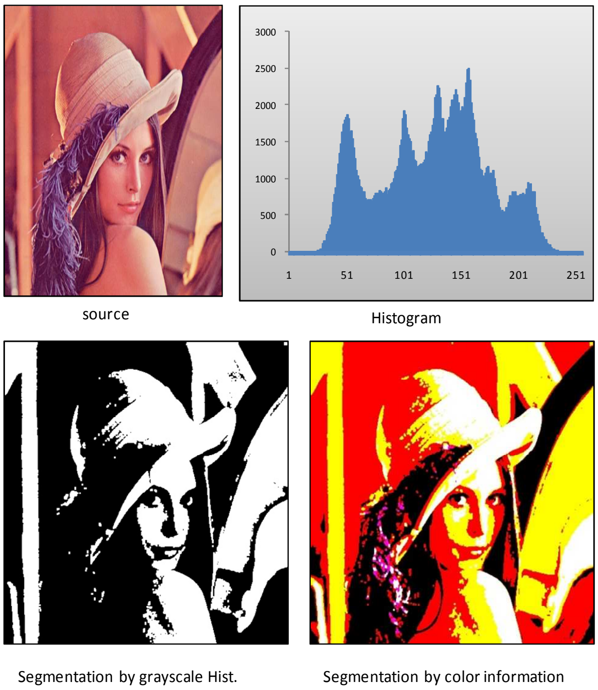

Fig.4. Fuzzy entropic segmentation for Lena image with a knotty background.

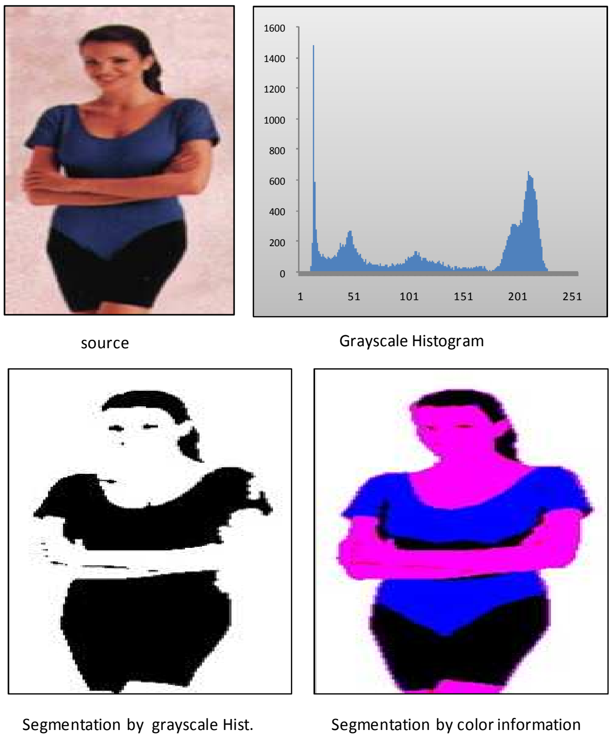

Fig.5. Fuzzy entropic segmentation for color image of a woman with a skin-like color distribution around.

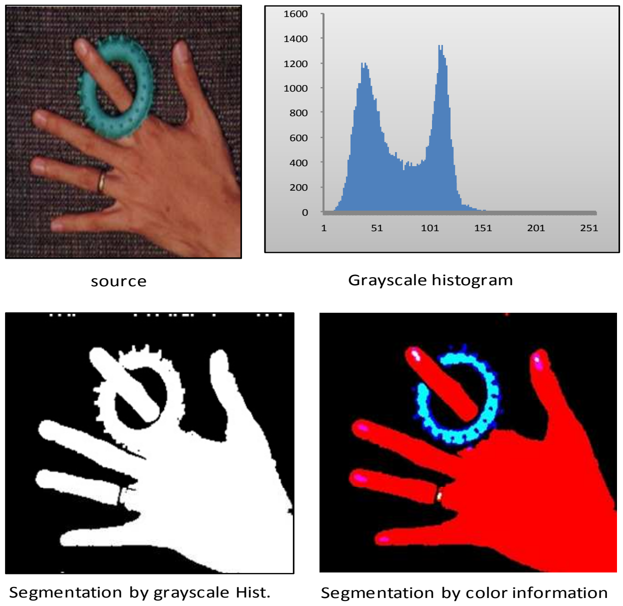

Fig.6. Fuzzy entropic segmentation for color image of a hand with a skin-like color distribution around.

V. Conclusion

In this paper, we have described a new approach for image segmentation via fuzzification of Rènyi entropy. The proposed approach could successfully segment both grayscale and color images. Furthermore it could do better when applied to noisy images or images of complicated backgrounds compared to other entropic segmentation approaches. The preliminary results obtained, show that using the formalism of fuzzy Rènyi entropy is more viable than using entropy alone in image segmentation task. An additional benefit of the approach comes from the rapidity and easiness of implementation. Although the proposed approach has been applied to still images, it can be straightforwardly applied to motion scenes due to its rapidity.

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