International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:4, No:8, 2010

# Robust BIBO stabilization analysis for discrete-time uncertain system

Zixin Liu, Shu Lü, Shouming Zhong, and Mao Ye,

*Abstract*—The discrete-time uncertain system with time delay is investigated for bounded input bounded output (BIBO). By constructing an augmented Lyapunov function, three different sufficient conditions are established for BIBO stabilization. These conditions are expressed in the form of linear matrix inequalities (LMIs), whose feasibility can be easily checked by using Matlab LMI Toolbox. Two numerical examples are provided to demonstrate the effectiveness of the derived results.

*Keywords*—Robust BIBO stabilization, delay-dependent stabilization, discrete-time system, time delay.

## I. Introduction

It is well-known that, because of the finite switching speed, memory effects and so on, time delay is unavoidable in technology and nature. It extensively exists in various mechanical, biological, physical, chemical engineering, economic systems, and can make important effects on the stability of dynamic systems. Thus, the studies on stability for delayed control system such as neural networks, switched system, bilinear system etc, are of great significance. There has been a growing research interest on the stability analysis problems for delayed system, and many excellent papers and monographs have been available (see [1]-[7]). On the other hand, during the design of control system and its hardware implementation, the convergence of a control system may often be destroyed by its unavoidable uncertainty due to the existence of modeling error, the deviation of vital data, and so on. Generally, these unavoidable uncertainties can be classified into two types: that is, stochastic disturbances and parameter uncertainties. As pointed out in [8] that, while modeling real control system, both of the stochastic disturbances and parameter uncertainties are probable the main resources of the performance degradations of the implemented control system. Therefore, the studies on robust convergence or mean square convergence of delayed control system have been a hot reach direction. As for parameter uncertain system, many sufficient conditions, either delay-dependent or delay-independent, have been proposed to guarantee the robust asymptotic or exponential stability for different class of delayed systems, such as neural networks, bilinear system, complex system and so on (see [9]-[14]).

It is worth pointing out that most of these previous issues are mainly in Lyapunov meaning. Recently, in order to track out the reference input signal in real word, many researchers have focused growing interest in the analysis of the BIBO stabilization (see [15]-[27]). In [15] and [16], by constructing appropriate Lyapunov functions, some delay-independent BIBO stabilization criteria for two classes of different time-delayed control systems were established. In [17], a class of linear delayed system with parameter uncertainty was considered, and some robust BIBO stabilization criteria were derived in terms of linear matrix technique. In [18], based on Riccati-equations, the authors considered the BIBO stabilization problem of piecewise switched linear systems. On the other hand, BIBO stabilization in mean square was also considered in [19] . However, these previous results have been assumed to be in continuous time, but seldom in discrete time (see [25],[26]). In practice, discrete-time control system is more applicable to problems that are inherently temporal in nature or related to biological realities. And it can ideally keep the dynamic characteristics, functional similarity, and even the physical or biological reality of the continuous-time systems under mild restriction. Thus, the BIBO stabilization analysis problems for discrete-time case are necessary.

Motivated by the above discussions, the objective of this paper is to study the BIBO stabilization and robust BIBO stabilization problems for a class of discrete-time control system with time delay and parameter uncertainties. Based on linear matrix inequalities (LMIs) technique, an augmented Lyapunov function is constructed, and three different sufficient conditions are established for BIBO or robust BIBO stabilization. Finally, two numerical examples are provided to demonstrate the effectiveness of the derived results.

*Notation*: The notations are used in our paper except where otherwise specified. $\|\cdot\|$ denotes a vector or a matrix norm; $\mathbb{R}, \mathbb{R}^n$ are real and n-dimension real number sets, respectively; $\mathbb{N}^+$ is positive integer set. $I$ is identity matrix; $*$ represents the elements below the main diagonal of a symmetric block matrix; Real matrix $P > 0 (< 0)$ denotes $P$ is a positive definite (negative definite) matrix; $\mathbb{N}[a, b] = \{a, a+1, \cdots, b\}$; $\lambda_{min}(\lambda_{max})$ denotes the minimum (maximum) eigenvalue of a real matrix.

## II. Preliminaries

Consider the following discrete-time uncertain system with time delay described by

$$\Sigma : \begin{cases} x(k+1) = A(k)x(k) + B(k)x(k-\tau) + C(k)u(k) \\ y(k) = D(k)x(k), k \in \mathbb{N}^+ \\ x(t) = \varphi(t), \ -\tau \leq t \leq 0. \end{cases} \quad (1)$$

Z. Liu, S. Lü and S. Zhong are with School of Applied Mathematics, University of Electronic Science and Technology of China, Chengdu, 610054, China. Z. Liu is also with School of Mathematics and Statistics, Guizhou College of Finance and Economics, Guiyang, 550004, China. E-mail:xinxin905@163.com

M. Ye is with School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu 610054, China.

where $x(k) = [x_1(k), x_2(k), \cdots, x_n(k)]^T \in \mathbb{R}^n$ denotes the state vector; $u(k) = [u_1(k), u_2(k), \cdots, u_n(k)]^T \in \mathbb{R}^n$ is the control input vector; $y(k) = [y_1(k), y_2(k), \cdots, y_n(k)]^T \in \mathbb{R}^n$ is the control output vector; Positive integer $\tau$ represents the transmission delay; $\varphi(\cdot)$ is vector-valued initial function and $\|\varphi\|_\tau$ is defined by $\|\varphi\|_\tau = \sup_{i\in\mathbb{N}[-\tau,0]} \|x(i)\|$; $A(k) = A + \triangle A(k)$, $B(k) = B + \triangle B(k)$, $C(k) = C + \triangle C(k)$, $D(k) = D + \triangle D(k)$; $A, B, C, D \in \mathbb{R}^{n\times n}$ represent the weighting matrices; $\Delta A(k), \Delta B(k), \Delta C(k), \Delta D(k)$ denote the time-varying structured uncertainties which are of the following form:

$$[\Delta A(k)\ \Delta B(k)\ \Delta C(k)\ \Delta D(k)] = GF(k)[E_a\ E_b\ E_c\ E_d],$$

where $G, E_a, E_b, E_c, E_d$ are known real constant matrices of appropriate dimensions; $F(k)$ is unknown time-varying matrix function satisfying $F^T(k)F(k) \le I, \forall k \in \mathbb{N}^+$.

Let $u(k)$ be linear gain local state feedback with the reference input $r(k)$ for system (1) as follows:

$$u(k) = Kx(k) + r(k) \tag{2}$$

so as to ensure stabilization of the closed-loop delayed system.

To obtain our main results, we need introduce the following definitions and lemmas.

*Definition 2.1:* A real discrete-time vector $r(k) \in L^n_\infty$ if $\|r(k)\|_\infty \triangleq \sup_{k\in\mathbb{N}[0,\infty]} \|r(k)\| < +\infty$.

*Definition 2.2:* The control system (1) is said to be BIBO stabilization by the local control law (2) if for every solution of the system (1), $y(k)$ satisfies

$$\|y(k)\| \le \theta_1\|r(k)\|_\infty + \theta_2, k \in \mathbb{N}^+.$$

where $\theta_1, \theta_2$ are known positive constants for every reference input $r(k) \in L^n_\infty$.

*Lemma 2.1:* [28](Tchebychev Inequality) For any given vectors $v_i \in \mathbb{R}^n, i = 1, 2, \cdots, n$, the following inequality holds:

$$[\sum_{i=1}^n v_i]^T[\sum_{i=1}^n v_i] \le n\sum_{i=1}^n v_i^T v_i.$$

*Lemma 2.2:* [29] Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$, where $\Sigma_1^T = \Sigma_1$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T\Sigma_2^{-1}\Sigma_3 < 0$ if and only if

$$\begin{pmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{pmatrix} < 0 \ or \ \begin{pmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{pmatrix} < 0.$$

*Lemma 2.3:* [8] Let $N$ and $E$ be real constant matrices with appropriate dimensions, matrix $F(k)$ satisfying $F^T(k)F(k) \le I$, then, for any $\epsilon > 0$, $EF(k)N + N^TF^T(k)E^T \le \epsilon^{-1}EE^T + \epsilon N^TN$.

*Lemma 2.4:* For any real vector $X, Y$ and positive definite matrix $\Sigma > 0$ with appropriate dimensions, it follows that

$$2X^TY \le X^T\Sigma X + Y^T\Sigma^{-1}Y.$$

For designing the linear feedback control $u(k) = Kx(k) + r(k)$ such that the closed-loop system (1) is BIBO stabilization, we first consider the nominal $\Sigma_0$ of $\Sigma$ defined by

$$\Sigma_0 : \begin{cases} x(k+1) = Ax(k) + Bx(k-\tau) + Cu(k) \\ y(k) = Dx(k), k \in \mathbb{N}^+, \\ x(t) = \varphi(t), -\tau \le t \le 0. \end{cases} \tag{3}$$

Substituting (2) into system (3) yields a closed-loop systems as follows:

$$\Sigma_0 : \begin{cases} x(k+1) = (A+CK)x(k) + Bx(k-\tau) + Cr(k) \\ y(k) = Dx(k), k \in \mathbb{N}^+, \\ x(t) = \varphi(t), -\tau \le t \le 0. \end{cases} \tag{4}$$

Then, we can obtain the following BIBO stabilization results.

## III. Main results

*Theorem 3.1:* For given positive integer $\tau > 0$, the delayed system (4) with feedback gain matrix $K$ by the local control law (2) is BIBO stabilization, if there exist positive-definite matrices $Q, H, P_1, P_2$, positive diagonal matrix $Z$ with appropriate dimensions, such that the following LMI holds:

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix} < 0, \tag{5}$$

where $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}$,

$$\Xi_{11} = A^T(Q_{11} + Q_{22} + Q_{12} + Q_{12}^T)A$$
$$-Q_{11} + H + (1+\tau)Z + P_1 + CK + K^TC^T,$$
$$\Xi_{12} = -A^T(Q_{11} + Q_{22}) + B,$$
$$\Xi_{13} = A^T(Q_{11} + Q_{22} + Q_{12} + Q_{12}^T) - I + K^TC^T,$$
$$\Xi_{14} = A^T(Q_{12} + Q_{22}) - Q_{12}^T,$$
$$\Xi_{22} = Q_{22} - H,$$
$$\Xi_{23} = -(Q_{12}^T + Q_{22}) + B^T,$$
$$\Xi_{24} = -Q_{22},$$
$$\Xi_{33} = Q_{11} + Q_{22} + Q_{12} + Q_{12}^T - 2I + P_2,$$
$$\Xi_{34} = Q_{12} + Q_{22},$$
$$\Xi_{44} = -(1+\tau)^{-1}Z.$$

*Proof.* Constructing an augmented Lyapunov-Krasovskii function candidate as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k),$$

where

$$V_1(k) = \widetilde{X}^T(k)Q\widetilde{X}(k),\ \widetilde{X}^T(k) = [x^T(k), \sum_{i=k-\tau}^{k} x^T(i)],$$

$$V_2(k) = \sum_{i=k-\tau}^{k-1} x^T(i)Hx(i),$$

$$V_3(k) = \sum_{i=k-\tau}^{k}\sum_{i=j}^{k-1} x^T(i)Zx(i).$$

Set $X^T(k) = [x^T(k), x^T(k-\tau), \eta^T(k), \sum_{i=k-\tau}^k x^T(i)]$, $\eta(k) = x(k+1) - Ax(k)$. Define $\Delta V(k) = V(k+1) - V(k)$, then along the solution of system (4) we can obtain that

$$\begin{aligned}\Delta V_1(k) &= X^T(k+1)QX(k+1) - X^T(k)QX(k)\\ &= x^T(k)[A^T(Q_{11}+Q_{12}+Q_{12}^T+Q_{22})A - Q_{11}]x(k)\\ &\quad -2x^T(k)A^T(Q_{12}+Q_{22})x(k-\tau)\\ &\quad +2x^T(k)A^T(Q_{11}+Q_{22}+Q_{12}+Q_{12}^T)\eta(k)\\ &\quad +2x^T(k)[A^T(Q_{12}+Q_{22}) - Q_{12}^T](\sum_{i=k-\tau}^{k} x(i))\\ &\quad +x^T(k-\tau)Q_{22}x(k-\tau) - 2x^T(k-\tau)(Q_{12}^T\\ &\quad +Q_{22})\eta(k) - 2x^T(k-\tau)Q_{22}(\sum_{i=k-\tau}^{k} x(i))\\ &\quad +\eta^T(k)(Q_{11}+Q_{22}+Q_{12}+Q_{12}^T)\eta(k)\\ &\quad +2\eta^T(k)(Q_{12}+Q_{22})(\sum_{i=k-\tau}^{k} x(i)), \qquad (6)\end{aligned}$$

$$\Delta V_2(k) = x^T(k)Hx(k) - x^T(k-\tau)Hx(k-\tau). \quad (7)$$

From lemma 2.1 we have

$$\begin{aligned}\Delta V_3(k) &= \sum_{j=k+1-\tau}^{k+1}\sum_{i=j}^{k} x^T(i)Zx(i) - \sum_{j=k-\tau}^{k}\sum_{i=j}^{k-1} x^T(i)Zx(i)\\ &= \sum_{j=k-\tau}^{k}\sum_{i=j+1}^{k} x^T(i)Zx(i) - \sum_{j=k-\tau}^{k}\sum_{i=j}^{k-1} x^T(i)Zx(i)\\ &= \sum_{j=k-\tau}^{k} [x^T(k)Zx(k) - x^T(j)Zx(j)]\\ &= (1+\tau)x^T(k)Zx(k) - \sum_{i=k-\tau}^{k} x^T(i)Zx(i)\\ &= (1+\tau)x^T(k)Zx(k) - \sum_{i=k-\tau}^{k} (\sqrt{Z}x(i))^T\sqrt{Z}x(i)\\ &\le (1+\tau)x^T(k)Zx(k) - \frac{1}{1+\tau}[\sum_{i=k-\tau}^{k} x(i)]^T Z[\sum_{i=k-\tau}^{k} x(i)] \quad (8)\end{aligned}$$

On the other hand, by lemma 2.4, for any positive matrices $P_1, P_2$ with appropriate dimensions, we have

$$\begin{aligned}0 &= 2x^T(k)[CKx(k) + Bx(k-\tau) + Cr(k) - \eta(k)]\\ &= 2x^T(k)CKx(k) + 2x^T(k)Bx(k-\tau)\\ &\quad +2x^T(k)Cr(k) - 2x^T(k)\eta(k)\\ &\le x^T(k)(CK + K^TC^T + P_1)x(k) + 2x^T(k)Bx(k-\tau)\\ &\quad -2x^T(k)\eta(k) + r^T(k)C^TP_1^{-1}Cr(k)\\ &\le x^T(k)(CK + K^TC^T + P_1)x(k) + 2x^T(k)Bx(k-\tau)\\ &\quad -2x^T(k)\eta(k) + \|r(k)\|^2\|C^TP_1^{-1}C\|, \qquad (9)\end{aligned}$$

$$\begin{aligned}0 &= 2\eta^T(k)[CKx(k) + Bx(k-\tau) + Cr(k) - \eta(k)]\\ &= 2\eta^T(k)CKx(k) + 2\eta^T(k)Bx(k-\tau)\\ &\quad +2\eta^T(k)Cr(k) - 2\eta^T(k)\eta(k)\\ &\le 2\eta^T(k)CKx(k) + 2\eta^T(k)Bx(k-\tau)\\ &\quad +\eta^T(k)(P_2 - 2I)\eta(k) + r^T(k)C^TP_2^{-1}Cr(k)\\ &\le 2\eta^T(k)CKx(k) + 2\eta^T(k)Bx(k-\tau)\\ &\quad +\eta^T(k)(P_2 - 2I)\eta(k) + \|r(k)\|^2\|C^TP_2^{-1}C\|. \quad (10)\end{aligned}$$

Combining (6)-(10), we get

$$\Delta V(k) \le X^T(k)\Xi X(k) + \sigma\|r(k)\|^2, \qquad (11)$$

where $\sigma = \|C^TP_1^{-1}C\| + \|C^TP_2^{-1}C\|$. If the LMI (5) holds, it follows that there exists a sufficient small positive scalar $\varepsilon > 0$ such that

$$\Delta V(k) \le -\varepsilon\|x(k)\|^2 + \sigma\|r(k)\|^2. \qquad (12)$$

On the other hand, it is easy to get that

$$V(k) \le \alpha_1\|x(k)\|^2 + \alpha_2\sum_{i=k-\tau}^{k}\|x(i)\|^2, \qquad (13)$$

where

$$\alpha_1 = \lambda_{max}(Q),\ \alpha_2 = (1+\tau)[\lambda_{max}(Q)+\lambda_{max}(Z)]+\lambda_{max}(H).$$

For any $\theta > 1$, it follows from (13) that

$$\begin{aligned}\theta^{j+1}V(j+1) - \theta^jV(j) &= \theta^{j+1}\Delta V(j) + \theta^j(\theta-1)V(j)\\ &\le \theta^j[\sigma\theta\|r(j)\|^2 - \varepsilon\theta\|x(j)\|^2\\ &\quad +(\theta-1)\alpha_1\|x(j)\|^2\\ &\quad +(\theta-1)\alpha_2\sum_{i=j-\tau}^{j}\|x(j)\|^2]. \quad (14)\end{aligned}$$

Summing up both sides of (14) from 0 to $k-1$, we can obtain

$$\begin{aligned}\theta^kV(k) - V(0) &\le [\alpha_1(\theta-1) - \varepsilon\theta]\sum_{j=0}^{k-1}\theta^j\|x(j)\|^2\\ &\quad +\alpha_2(\theta-1)\sum_{j=0}^{k-1}\sum_{i=j-\tau}^{j}\theta^j\|x(i)\|^2\\ &\quad +\sum_{j=0}^{k-1}\sigma\theta^{j+1}\|r(j)\|^2\\ &\le \mu_1(\theta)\sup_{j\in\mathbb{N}[-\tau,0]}\|x(j)\|^2\\ &\quad +\mu_2(\theta)\sum_{j=0}^{k}\theta^k\|x(k)\|^2\\ &\quad +\sum_{j=0}^{k-1}\sigma\theta^{j+1}\|r(j)\|^2, \qquad (15)\end{aligned}$$

where $\mu_1(\theta) = \alpha_2(\theta-1)\tau^2\theta^\tau$, $\mu_2(\theta) = \alpha_2(\theta-1)\tau\theta^\tau + \alpha_1(\theta-1) - \varepsilon\theta$. Since $\mu_2(1) = -\varepsilon < 0$, there must exist a positive $\theta_0 > 1$ such that $\mu_2(\theta_0) < 0$. Then, we have

$$\begin{aligned}V(k) &\le \mu_1(\theta_0)(\frac{1}{\theta_0})^k\sup_{j\in\mathbb{N}[-\tau,0]}\|x(j)\|^2\\ &\quad +(\frac{1}{\theta_0})^kV(0) + \sigma\sum_{j=0}^{k-1}\frac{1}{\theta_0^{k-j-1}}\|r(j)\|^2\\ &\le \mu_1(\theta_0)(\frac{1}{\theta_0})^k\sup_{j\in\mathbb{N}[-\tau,0]}\|x(j)\|^2\\ &\quad +(\frac{1}{\theta_0})^kV(0) + \sigma\|r(k)\|_\infty^2\sum_{j=0}^{k-1}\frac{1}{\theta_0^{k-j-1}}\\ &\le \mu_1(\theta_0)\|\varphi\|_\tau^2 + V(0) + \frac{\sigma}{\theta_0-1}\|r(k)\|_\infty^2. \quad (16)\end{aligned}$$

On the other hand, set $\varpi = \alpha_1 + (1+\tau)\alpha_2$, we can obtain

$$V(0) \le \varpi \sup_{j\in\mathbb{N}[-\tau,0]} \|x(j)\|^2 \, and \; V(k) \ge \lambda_{min}(Q)\|x(k)\|^2. \quad (17)$$

It follows that $\|y(k)\| \le \theta_1 \|r(k)\|_\infty + \theta_2, k \in \mathbb{N}^+$, where

$$\theta_1 = \|D\|\sqrt{\sigma(\theta_0 - 1)^{-1}\lambda_{min}^{-1}(Q)},$$

$$\theta_2 = \|D\|\sqrt{[\mu_1(\theta_0) + \varpi]\lambda_{min}^{-1}(Q)}.$$

By Definition 2.2, system (4) is BIBO stabilization, which complete the proof of Theorem 3.1.

*Remark 1.* It is worth pointing out that, since the existence like item $A^T(Q_{11}+Q_{22}+Q_{12}+Q_{12}^T)A$, this new criterion is difficult to be extended to robust BIBO stabilization by lemma 2.3. Thus, it is necessary to give another sufficient condition as follows:

*Theorem 3.2:* For given positive integer $\tau > 0$, the delayed system (4) with feedback gain matrix $K$ by the local control law (2) is BIBO stabilization, if there exist positive-definite matrices $Q, H, P_1, P_2$, positive diagonal matrices $Z$ with appropriate dimensions, such that the following LMI holds:

$$\Xi' = \begin{bmatrix} \Xi'_{11} & \Xi'_{12} & \Xi'_{13} & \Xi'_{14} \\ * & \Xi'_{22} & \Xi'_{23} & \Xi'_{24} \\ * & * & \Xi'_{33} & \Xi'_{34} \\ * & * & * & \Xi'_{44} \end{bmatrix} < 0, \quad (18)$$

where $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}$,

$$\Xi'_{11} = Q_{22} + Q_{12} + Q_{12}^T + H + (1+\tau)Z$$
$$+P_1 + CK + K^TC^T + A^T + A - 2I,$$
$$\Xi'_{12} = -Q_{11} - Q_{22} + B,$$
$$\Xi'_{13} = Q_{11} + Q_{22} + Q_{12} + Q_{12}^T + K^TC^T + A^T - 2I,$$
$$\Xi'_{14} = Q_{22} + Q_{12} - Q_{12}^T,$$
$$\Xi'_{22} = Q_{22} - H,$$
$$\Xi'_{23} = -(Q_{12}^T + Q_{22}) + B^T,$$
$$\Xi'_{24} = -Q_{22},$$
$$\Xi'_{33} = Q_{11} + Q_{22} + Q_{12} + Q_{12}^T - 2I + P_2,$$
$$\Xi'_{34} = Q_{12} + Q_{22},$$
$$\Xi'_{44} = -(1+\tau)^{-1}Z.$$

*Proof.* Similar to the proofs in Theorem 3.1, set $\eta(k) = x(k+1) - x(k)$, one can easily obtain this result.

*Theorem 3.3:* For given positive integer $\tau > 0$, the delayed system (1) with feedback gain matrix $K$ by the local control law (2) is robust BIBO stabilization, if there exist positive-definite matrices $Q, H, P_1, P_2$, positive diagonal matrix $Z$ with appropriate dimensions, and positive scalar $\epsilon > 0$, such that the following LMI holds:

$$\widetilde{\Xi} = \begin{bmatrix} \Xi' & \xi_1 & \epsilon\xi_2^T \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0, \quad (19)$$

where $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}$, $\xi_1^T = [G^T, 0, G^T, 0, 0, 0]$, $\xi_2 = [E_a + E_cK, E_b, 0, 0, 0, 0]$.

*Proof.* Replacing $A, B, C$ in inequality (18) with $A + GF(t)E_a$, $B + GF(t)E_b$ and $C + GF(t)E_c$, respectively. Inequality (18) for system (1) is equivalent to $\Xi' + \xi_1 F(t)\xi_2 + \xi_2^T F^T(t)\xi_1^T < 0$. From lemma 2.2 and lemma 2.3, we can easily obtain this result, which complete the proof.

*Remark 2.* The above criteria are all expressed in the form of LMIs. This make the design of the controller (2) become more easy, which not considered in previous literature for discrete-time BIBO stabilization.

## IV. Numerical examples

In this section, two numerical examples will be presented to show the validity of the main results derived above.

*Example 1.* Consider the delayed discrete-time system in (3) with parameters given by

$$C = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.7 \end{bmatrix}, A = \begin{bmatrix} 0.7 & 1.3 \\ 0.3 & 1.6 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}, \tau = 7.$$

One can check that LMI (5) in Theorem 3.1 and LMI (18) in Theorem 3.2 are feasible. By the Matlab LMI Toolbox, we find a feasible solution to the LMI (5) as follows:

$$Q_{11} = \begin{bmatrix} 45.4372 & -7.1208 \\ -7.1208 & 110.0019 \end{bmatrix}, Q_{12} = \begin{bmatrix} -0.3076 & 0.1655 \\ 0.2500 & -0.7777 \end{bmatrix},$$

$$Q_{22} = \begin{bmatrix} 0.8911 & -0.3784 \\ -0.3784 & 1.2830 \end{bmatrix}, H = \begin{bmatrix} 53.5644 & -0.0734 \\ -0.0734 & 20.8058 \end{bmatrix},$$

$$Z = \begin{bmatrix} 1.0519 & 0 \\ 0 & 4.1680 \end{bmatrix}, P_1 = \begin{bmatrix} 4.3607 & -2.7739 \\ -2.7739 & 10.9409 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 2.7923 & -4.3695 \\ -4.3695 & 20.4034 \end{bmatrix}, K = \begin{bmatrix} -30.6990 & -65.7751 \\ -70.0633 & -158.8050 \end{bmatrix}.$$

*Example 2.* Consider a delayed discrete-time system in (1) with parameters given by

$$E_a = \begin{bmatrix} 0.1 & 0.13 \\ 0.1 & 0.12 \end{bmatrix}, E_b = \begin{bmatrix} 0.03 & 0.1 \\ 0.0 & 0.1 \end{bmatrix},$$

$$E_c = \begin{bmatrix} 0.02 & 0.00 \\ 0.00 & 0.02 \end{bmatrix}, G = \begin{bmatrix} 0.2 & 0.0 \\ 0.0 & 0.3 \end{bmatrix},$$

$A, B, C, \tau$ are the same as given in Example 1. One can check that LMI (19) in Theorem 3.3 is feasible. By the Matlab LMI Toolbox, we find a feasible solution to the LMI (19) as follows:

$$Q_{11} = \begin{bmatrix} 21.1459 & -1.0993 \\ -1.0993 & 20.3565 \end{bmatrix}, Q_{12} = \begin{bmatrix} -0.1607 & -0.0404 \\ 0.0621 & -0.0447 \end{bmatrix},$$

$$Q_{22} = \begin{bmatrix} 1.0376 & 0.0622 \\ 0.0622 & 0.5440 \end{bmatrix}, H = \begin{bmatrix} 28.4911 & 0.2924 \\ 0.2924 & 7.5679 \end{bmatrix},$$

$$Z = \begin{bmatrix} 0.8107 & 0 \\ 0 & 1.8655 \end{bmatrix}, P_1 = \begin{bmatrix} 3.2501 & -1.0388 \\ -1.0388 & 4.3021 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 1.7861 & -1.5901 \\ -1.5901 & 5.6875 \end{bmatrix}, K = \begin{bmatrix} -15.2670 & -12.5090 \\ -35.6235 & -31.0160 \end{bmatrix},$$

$$\epsilon = 15.4831.$$

## V. Conclusion

The main contribution of this paper is the results that ensure BIBO stabilization for discrete-time delayed control system. Combined with linear matrix inequality (LMI) technique, an augmented Lyapunov-Krasovskii function is constructed, and some new delay-dependent conditions ensuring BIBO stabilization or robust BIBO stabilization are obtained. Numerical examples show that the new results are valid.

## Acknowledgment

This work was supported by the program for New Century Excellent Talents in University (NCET-06-0811) and the Research Fund for the Doctoral Program of Guizhou College of Finance and Economics (200702).

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**ZiXin Liu** was born in Sichuan Province, China, in 1977. He received the B.S. degree from China West Normal University, Sichuan, in 1999 and the M.S. degree from the University of Electronic Science and Technology of China (UESTC), Sichuan, in 2006, both in applied mathematics. He is currently pursuing the Ph.D. degree with UESTC. His research interests include neural networks, chaos synchronization and stochastic delayed dynamic systems.

**Shu Lü** was born in 1963 in Jilin, China. She received B.S. degree in Mathematics Department of Northeast Normal University, Changchun, China, in 1984 and the M.S. degree from the School of mathematical statistics, China University of Technology. She is now a professor with UESTC. Her research interests include the theory and application of economics mathematics.

**Shouming Zhong** was born in 1955 in Sichuan, China. He received B.S. degree in applied mathematics from UESTC, Chengdu, China, in 1982. From 1984 to 1986, he studied at the Department of Mathematics in Sun Yat-sen University, Guangzhou, China. From 2005 to 2006, he was a visiting research associate with the Department of Mathematics in University of Waterloo, Waterloo, Canada. He is currently as a full professor with School of Applied Mathematics, UESTC. His current research interests include differential equations, neural networks, biomathematics and robust control. He has authored more than 80 papers in reputed journals such as the In International Journal of Systems Science, Applied Mathematics and Computation, Chaos, Solitons and Fractals, Dynamics of Continuous, Discrete and Impulsive Systems, Acta Automatica Sinica, Journal of Control Theory and Applications, Acta Electronica Sinica, Control and Decision, and Journal of Engineering Mathematics.

**Mao Ye** was born in September1973,obtained his doctoral degree from the Chinese University of Hong Kong. He is serving the University of Electronic Science and Technology Computer Science and Engineering College as professor and has to his credit 30 papers, of which more than 15 have SCI, EI above 12.