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Use of Regression Analysis in Determining the Length of Plastic Hinge in Reinforced Concrete Columns

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***Abstract*—**Basic objective of this study is to create a regression analysis method that can estimate the length of a plastic hinge which is an important design parameter, by making use of the outcomes of (lateral load-lateral displacement hysteretic curves) the experimental studies conducted for the reinforced square concrete columns. For this aim, 170 different square reinforced concrete column tests results have been collected from the existing literature. The parameters which are thought affecting the plastic hinge length such as cross-section properties, features of material used, axial loading level, confinement of the column, longitudinal reinforcement bars in the columns etc. have been obtained from these 170 different square reinforced concrete column tests. In the study, when determining the length of plastic hinge, using the experimental test results, a regression analysis have been separately tested and compared with each other. In addition, the outcome of mentioned methods on determination of plastic hinge length of the reinforced concrete columns has been compared to other methods available in the literature.

***Keywords*—**Columns, plastic hinge length, regression analysis, reinforced concrete.

I. INTRODUCTION

WHEN identifying the responses of reinforced concrete buildings under the effect of earthquakes, the definition of the characteristic properties of plastic hinges in determining the non-linear response of the reinforced concrete building is quite significant. The accuracy of the results obtained from the analytical studies is directly related to the hinge definitions of the buildings. The determination of these plastic hinge zones, wherein the plastic deformations lumped, plays a crucial role in the global response of the building under seismic loading. These plastic hinges can be occurred in the load-carrier system elements, such as beams, columns and load – bearing walls. Nevertheless, since it is well-known that the columns from the structural elements are more effective in seismic response in comparison to the beams, the determination of the properties of plastic hinges for the columns is much more important for the load-carrier system.

The studies performed indicate that identifying the plastic hinge response in the reinforced concrete buildings and determining the parameters that affect such responses is a rather complex process. In the relevant literature, numerous studies have been conducted on the determination of the properties of plastic hinges in the reinforced concrete cross-sections and on the different empirical formulas produced particularly for the determination of the length of the plastic hinges [1]-[12].

A large number of experimental studies oriented towards identification of the response of reinforced concrete columns under the effect of cyclic lateral loading have been conducted [13]-[42]. In these studies, many parameters related to column cross-sections, material strength, longitudinal and transversal reinforcement ratios, loading history and support conditions of the column have been tested. However, none of these studies address the determination of plastic hinge length.

The basic objective of this study is to create a soft computing method-based algorithm that can estimate the length of a plastic hinge by making use of the outcomes of (lateral load- lateral displacement hysteretic curves) the experimental studies [13]-[42] conducted for the reinforced square concrete columns. For this aim, 170 different square reinforced concrete column tests results have been collected from the existing literature. The parameters which are thought affecting the plastic hinge length such as cross-section properties, features of material used, axial loading level, support condition, confinement of the column, shear force ratio, longitudinal reinforcement bars in the columns etc. have been obtained from these 170 different square reinforced concrete column tests. In the study, when determining the length of plastic hinge, using the experimental test results, Regression Analysis have been tested and compared with each other.

II. PLASTIC HINGE LENGTH

Today, the non-linear response of the load carrying system is calculated using the “lumped plasticity approach” based on the assumption that such behavior would concentrate at the ends of the carrier system elements. Pursuant to this hypothesis, plastic deformation in the beam and column-type of carrier system elements are assumed to occur by being properly diffused along “ℓ_p” zones with finite length, where they reach the capacities of internal forces. For the proper functioning of plastic hinges, the cross-sections within this zone must have the plastic curvature capacity. Moreover, to ensure the ability of this capacity, the plastic deformations that emerge should remain at an acceptable level. Lumped plasticity idealization of a cantilever column is a commonly used approach in models for deformation capacity estimates and given Fig. 1.

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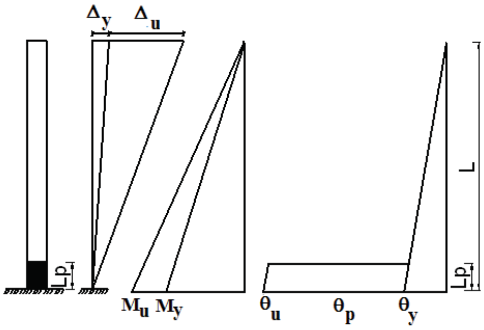


Fig. 1 Lumped plasticity idealization of a cantilever reinforced concrete column

TABLE I
FORMULAS FOR PLASTIC HINGE LENGTH

Formulation	Researchers	
$\ell_{\mathrm{p}} = 0{,}25d + 0{,}075L$	(Sawyer 1964)	[1]
$\ell_p = 0{,}5d + 0{,}2\sqrt{d}\left(\frac{L}{d}\right)$	(Corley 1966)	[2]
$\ell_p = 0{,}5d + 0{,}05L$	(Mattock 1967)	[3]
$\ell_p = 0{,}42h$	(Park et al. 1982)	[4]
$\ell_p = 0{,}08L + 6d_b$	(Priestley and Park 1987)	[5]
$\ell_p = 0{,}08L + 0{,}022d_b f_{sy}$	(Paulay and Priestley 1992)	[6]
$\ell_p = h$	(Sheikh and Khoury 1993)	[7]
$\ell_p = h$	(Sheikh et al. 1994)	[8]
$\ell_p = h$	(Bayrak and Sheikh 1998)	[9]
$\ell_p = 0{,}12L + 0{,}014\alpha d_b f_{sy}$	(Panagiotakos and Fardis 2001)	[10]
$\ell_p = 0{,}05L + \frac{0{,}1d_b f_{sy}}{\sqrt{f_c}}$	(Berry et al. 2008)	[11]
$\ell_p = \left[0{,}3\left(\frac{N}{N_0}\right) + 3\left(\frac{A_s}{A_g}\right) - 0{,}1\right](L) + 0{,}25 \geq 0{,}25$	(Bae and Bayrak 2008)	[12]

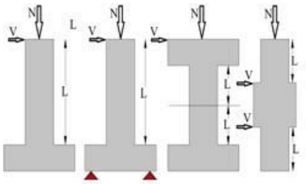
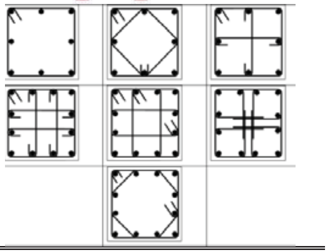
Several empirical equations have suggested estimating the plastic hinge length ℓ_p as summarized in Table I. In the table, formulation, researchers' name, and abbreviation are given. Due to the complexity of the problem, simplified expressions have been used so far.

The plastic hinge length ℓ_p of RC members depends on a number of parameters, including level of the axial load, longitudinal reinforcement ratio, transverse reinforcement ratio, concrete compressive strength, cross-sectional geometry, bond-slip characteristics between concrete and reinforcing steel, support condition, influence of shear etc. These parameters also affect the section ductility of the member and shape of moment-curvature curve. Therefore, the relationship between the cross-section ductility and the plastic joint length must be accurately determined. According to the proposed various formulas, some important parameters that are more effective on the plastic hinge length ℓ_p have been used. In general, the methods proposed by the researchers give constant ℓ_p regardless of the important parameters given above. For instance, Sawyer [1], Corley [2], Mattock [3] consider only length and height of the related section. Park et al. [4] and Sheikh [7] formulas the ℓ_p length depends on only cross section height. It is interesting that only Bae and Bayrak's [12] expression used the axial load level in ℓ_p calculation. In the formulations, L is column pier height, h is height of column, d_b is diameter of longitudinal reinforcement, f_{sy} is yield strength of longitudinal reinforcement, f_c is compressive strength of concrete, N/N_0 is normalized axial load and α represents the slippage of longitudinal reinforcement from the anchorage zone as 1 or 0.

III. DATABASE

In this study, various test configurations and different equivalent length of columns namely cantilever, flexible base, double curvature and double ended were selected from previous studies. Material properties, geometric characteristics, section properties, height of columns and all essential parameters affecting the column behavior under cycling loadings as shown in Table II.

TABLE II
RANGE OF PARAMETERS USED

Parameters	Definition	Range of parameters
b (mm)	Width of the cross-section	180-600
h (mm)	Depth of the cross-section	180-600
L (mm)	Length of the equivalent cantilever	323-2335
f_{sy} (MPa)	Yield stress of longitudinal reinforcement	331-511
f_{su} (MPa)	Tensile strength of longitudinal reinforcement	494-772
f_{yw} (MPa)	Yield stress of transverse reinforcement	249-616
f_c (MPa)	Mean compressive strength of concrete	16-46.5
ρ_ℓ (%)	Ratio of longitudinal reinforcement	0.00463-0.0412
ρ_s(%)	Ratio of transverse reinforcement	0.000678-0.02512
N/N_0	Normalized axial load	0-0.77
υ	Shear ratio (defined as L/h)	0-4.84
FT	Failure type	Flexure-Shear -Hybrid
CFT	Column fixing type	
CCST	Column Cross Section Type	

IV. CALCULATION OF PLASTIC HINGE LENGTH

In this study, the plastic hinge length ℓ_p was calculated according to the revealed moment and rotation capacity results obtained from the experiments. First, the plastic displacement amount occurring at the upper ends of the columns was calculated utilizing the experimental data of 170 individual columns contained in said data sets. The displacement corresponding to the start of plasticization in the column (σ_y) was assumed as 30% of the maximum load and the displacement values (σ_u) in the final state, 15% of the maximum load, and the graph was consulted to obtain the relevant values. Column plastic rotation was calculated by dividing the difference between the displacement values acquired from the net column length (L). In Fig. 2, an experimental load-displacement relationship is schematically shown, and the plastic rotational computation-oriented relationship is provided in (1).

$$\theta_p = \frac{\left(\Delta_u - \Delta_y\right)}{L} \tag{1}$$

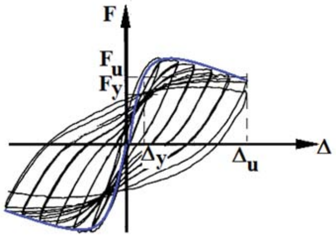


Fig. 2 Load-Displacement Hysteretic Curve of a Reinforced Concrete Column

Since the ratio between the amount of plastic rotation (θ_P) and plastic curvature (φ_p), as ascertained according to the experimental findings, would provide the plastic hinge length, the moment-curvature relationship was determined according to load, material and sectional properties of 170 individual reinforced concrete columns used testing processes, in the second-stage of the study. The moment-curvature relationship of the cross-section of reinforced concrete is as equally associated with the stress-strain (σ-ε) models of the steel and concrete selected as it is with the sectional characteristics.

The distribution and boundaries of sectional deformation at the cross-section of a column under the combined axial force and bending moment can be determined by force and strain compatibility equations. Behavior of the cross-section is determined whether or not the reinforcement at the tension zone would behaviorally exceed the yield point given for the reinforcement at the moment when the concrete has reached at the maximum deformation in the compression zone. Unlike the beams, the axial load level found in the deformations of the concrete and reinforcement is extremely dominant. The difference between cover and core concrete should definitely be taken into consideration when conducting an σ-ε correlation analysis. In Fig. 3, analytical modeling and sectional properties are shown, and in (2), the force correlation required for the modeling is given. F_{cc} and F_{cu} indicate the compressive force product for the cover and core concretes, respectively, while σ_{si} indicates tensile reinforcement and A_{si} the reinforcement area. N is the force that influences the column. The method was used in obtaining pressure forces for the confined and unconfined concretes contained in the force equilibrium equation.

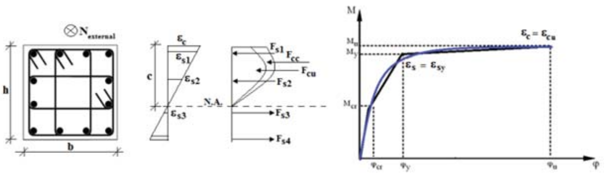


Fig. 3 Analytical modeling and sectional properties

$$N = F_{cc} + F_{cu} + \sum_{i=1}^{n} (\sigma_{si} \times A_{si}) = \left[\int_{0}^{\varepsilon_c} \sigma_c \times d\varepsilon_c \right] \times b + \sum_{i=1}^{n} (\sigma_{si} \times A_{si}) \tag{2}$$

To determine the moment-curvature relationship, the Kent-Park model [42] developed to explain the confinement concrete stress-strain relationship deformation, and the Mander hardening model [43], developed to explain the steel stress-strain deformation relationship were employed.

To create the (M-φ) graphics, two equilibrium equations (ΣF=0, ΣM=0) and strain compatibility equations were used. By failing to include the tension strength of the concrete in the analyses, it was assumed that all tension strain within the tension zone was met by the reinforcement bars. As in classical mechanics, it was assumed that pre-bending cross-sections of the plane remain as a plane after the bending. In the analyses, maximum deformation of the unconfined concrete was assumed to be 0,004.

In modeling and tests, the materials, cross-sections and load values given by the researchers were used. The column cross-sections given in the relevant references for the positions of longitudinal reinforcement inside the column cross- section were kept the same.

With said assumption and according to the modeling technique, M-φ graphics were obtained for 170 individual columns. According to the moment-curvature relationship obtained, the yielding curvature during the yielding pertaining to each of the column cross-section samples (M_y-φ_y) and the final moment-curvature (M_u-φ_u) values were determined using the Equivalent Energy Elastic-Plastic Curve (EEEP) method. While M-φ graphics were obtained in the manner shown in Fig. 2 due to dominance of the bending moment at the ductile cross-sections where the axial force is lower, a dramatic drop was observed at the cross-sections where normal force was higher following the maximum moment at the M-φ graphics. Final curvature moment for this situation was assumed to have been decreased by 20%.

Employing a simple approach, the plastic hinge length ℓ_p was calculated according to (3) by using the plastic rotation amount (θ_P) fixed upon utilization of the experimental data from (1) yielding curvature (φ_y), and the ultimate curvature (φ_u) values were analytically calculated, as explained in (2) and Fig. 3.

$$\ell_p = \frac{\theta_p}{(\varphi_u - \varphi_y)} \tag{3}$$

V. REGRESSION ANALYSIS

Regression analysis is a kind of mathematical modeling. In this study, column area, Shear reinforcement ratio, Shear reinforcement yielding, Shear reinforcement spacing, Concrete compressive strength, Column height, longitudinal reinforcement area, longitudinal reinforcement yielding and Normalized axial load were observed 171 times and measurement results using the Linear Regression Analysis with Plastic Hinge Length estimation were studied. Variables Correlation Matrix is given in Table III.

TABLE III
VARIABLES CORRELATION MATRIX

	Plastic Hinge Length	**Column Cross section**	**Shear reinforcement**	**Shear reinforcement yielding**	**Shear reinforcement spacing**	**Compressive strength**
Plastic Hinge Length	1.000	0.063	-0.227	0.623	0.241	0.434
Column Cross section	0.063	1.000	0.570	-0.069	0.322	-0.296
Shear reinforcement	-0.227	0.570	1.000	-0.084	-0.180	-0.063
Shear reinforcement yielding value	0.623	-0.069	-0.084	1.000	0.005	0.591
Shear reinforcement spacing	0.241	0.322	-0.180	0.005	1.000	-0.144
Compressive strength	0.434	-0.296	-0.063	0.591	-0.144	1.000

Analyzing the values in Table III “shear reinforcing yielding value” was found the variable with the largest correlation. This value is said to be positively correlated with Plastic Hinge Length dependent (response-result) of our variable.

TABLE IV
REGRESSION ANALYSIS RESULTS TABLE

	b_j	$S(b_j)$	$BETA$	t	p
constant	-218.656	36.947	-	-5.918	0.000
Shear reinforcement yielding value	0.642	0.093	0.471	6.891	0.000
Shear reinforcement spacing	0.201	0.120	0.108	1.680	0.095
Shear reinforcement Area	-0.070	0.016	-0.339	-4.493	0.000
Column area	0.001	0.000	0.328	4.019	0.000
Compressive strength	2.544	0.742	0.247	3.429	0.001
	$n = 171$	$S = 109.51024$	$R^2 = 0.525$	$F = 36.192, p = 0.000$	

In Table IV, the results are given in the regression analysis. According to these results, we have established our regression model is significant ($F = 36.192, p = 0.000$). Accordingly, at least one variable in the model is said to contribute significantly. As a result of the values given in Table III, the regression equation to be derived;

$$\hat{Y}_i = -218.656 + 0.642x_1 + 0.201x_2 - 0.070x_3 + 0.001x_4 + 2.544x_5 + e_i$$

The values in the equation,
$\hat{Y}_i$: Plastic Hinge Length
x_1: Shear reinforcement yielding
x_2: Shear reinforcement spacing
x_3: Shear reinforcement area
x_4: Column area
x_5: Compressive strength
is abbreviated.

Also within the model e_i term is the error term in the regression analysis so, the model is:

$$\hat{Y}_i = -218.656 + 0.642x_1 + 0.201x_2 - 0.070x_3 + 0.001x_4 + 2.544x_5$$

VI. RESULTS AND CONCLUSION

In end of the study, the plastic hinge lengths ℓ_p for 170 square cross-section reinforced concrete columns whose cross-sections and material properties were completely different from each other and the load-displacement behaviors were experimentally acquired before were obtained using analytical methods. The plastic hinge lengths calculated using experimental and analytical methods were also checked with some of the approaches contained in the literature. Furthermore, using Regression analysis in this research, the plastic hinge lengths were estimated. In the study, the following findings were observed;

- The literature-proposed methods yield very different results from each other. The reason for such discrepancy is that the parameter selected for each formula is different. In other words, there are no common parameters in the formulas. These formulas are able to estimate the plastic hinge lengths between 4.69% - 10.65%.
- All of the formulas, including Bae and Bayrak [12], obtained the plastic hinge length by assuming that only the cross-sections were under the effect of bending. Nevertheless, in some of the experiments considered in this study, it was reported that the damage has resulted from shearing and bar-slip deformation.
- The fact that the length of plastic hinges is cross section dependent, particularly in the columns, is a situation open for further discussion. Each parameter exerting an effect on the moment-curvature relationship of the cross- section affects the plastic hinge length. It can be anticipated that the longitudinal reinforcement ratio would also have an increasing effect on ℓ_p length.
- Here, it might be mentioned that the most important element limiting the success ratio of the methods is the structure of the data set. In cases where the axial force level of the data set is separated into subgroups such as type of hinge (flexure, shear etc.), etc., a small incremental change might also be anticipated in the success of the methods employed.
- Given that both traditional methods and Regression analysis systems yielded results that were not high in the estimation of plastic hinge lengths demonstrate that there might be other parameters in the column behavior which could help define the plastic hinge length or the plastic hinge hypothesis should be discussed.

According to the authors, the greatest uncertainty surrounding the supportive system elements in the understanding of non-linear behavior of the reinforced concrete structures that have become common today is the preferred length of the plastic hinge. Since a change in the hinge length would lead to a significant differentiation in structural behavior, it would be appropriate to describe a hinge length according to the parameters of columns, such as normal force, cross-section, diameter of longitudinal reinforcement, etc.

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