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A Probabilistic Optimization Approach for a Gas Processing Plant under Uncertain Feed Conditions and Product Requirements

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***Abstract*—**This paper proposes a new optimization techniques for the optimization a gas processing plant uncertain feed and product flows. The problem is first formulated using a continuous linear deterministic approach. Subsequently, the single and joint chance constraint models for steady state process with time-dependent uncertainties have been developed. The solution approach is based on converting the probabilistic problems into their equivalent deterministic form and solved at different confidence levels Case study for a real plant operation has been used to effectively implement the proposed model. The optimization results indicate that prior decision has to be made for in-operating plant under uncertain feed and product flows by satisfying all the constraints at 95% confidence level for single chance constrained and 85% confidence level for joint chance constrained optimizations cases.

***Keywords*—**Butane, Feed composition, LPG, Product specification, Propane.

I. Introduction

THE operation of a real gas plant is mainly challenged with uncertainties from the plant inlet [1]. However, on the plant outlet side, some of the composition and flow rates have also uncertain product requirement. Consequently, feed and product specifications are key parameters which must be taken into account during the plant operation [2]. Gas processors want to know long term specification requirements for the quality of the product to be delivered to their customers. In addition, they seek to maximize their revenue by boosting the production level based on the market conditions. Thus, due to the competitive nature of the market environment for some of the products, there exist certain restrictions on reliability of meeting the product requirements and quality specification. This effect is mainly pronounced for those products in which their product requirement cannot be easily determined in the market. For example, the C_3 content in propane product may vary depending on the customer's specification and also for butane (C_{4s} content). Sometimes, it may be also advantageous to produce LPG based on the market outlook. During such situations, the plant can be operated using the depropanizer column only and the debutanizer column may be shut down for energy saving. As a result, the amount of C_3 and C_{4s} in the LPG may also have different specification. Thus, treating these two uncertainties (inlet and outlet) at the same time has not been adopted before. Furthermore, the need to make an optimal decision under such uncertainty is a major issue which needs to be addressed well.

Most of the solution approaches which have been used previously are based on deterministic and "worst case" strategy. In deterministic approaches, the nominal value of the uncertain parameter is usually employed. However, solving the optimization problem by fixing the uncertain parameter to a nominal value may lead to a poor solution [3]. In some cases, even a small perturbation of the nominal value may result in a severe infeasibility. In the "worst case" strategy, the uncertain variable is replaced with a new value which has the maximum displacement from its nominal value. While the approach can ensure the reliability of the process, it may lead to a drastically reduced profit [4]. Thus, there should be a comprising measurement for the reliability and profitability of the plant in order to make an optimal decision under uncertainty.

The two competitive methods for optimization under uncertainties are two stage programming and probabilistic or chance constrained programming. In two-stage approach the decision variables are partitioned into two stages. The first stage variable has to be decided before the realization of the second stage uncertain variables [5, 6, 7]. This method has been applied in many planning problems under uncertainty [8]. However, as the number of scenario increases, it will need more computational efforts and the resulting problem remains to be unsolved. The chance constrained approach is also a competitive method for optimization of problems under uncertainty. The main advantage of this method is that the engineers are able to specify the reliability and profitability of the plant, which are very influential in decision making purpose [9]. The solution approach is to relax the probabilistic constraint to its corresponding equivalent deterministic form which can then be solved using the available commercial software routines like GAMS [10]. In this work, we have developed a model to deal uncertainties from both inlet and outlet for a gas processing plant. The model is used in such a way that it can incorporate the uncertainties of feed

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compositions and flow rates from the plant inlet as well as the uncertainty of product compositions and flow rates from the plant outlet.

II. Problem Formulation

A. Inflows and Outflows

The plant produces the material and energy products (outflows) by processing some raw materials and utilities (inflows). The general representation for the raw material, utility, product, and energy product flows is shown in Fig. 1. On the plant inlet side, the inflows consist of both raw materials as feed streams and utilities like electrical power, heating steam and cold water. These inflows may be brought from upstream suppliers (feed streams) and from the nearby cogeneration plant (utilities). Some of them can supply as much as we demand, such kind of inflows are certain and can be decided. However, the supply of some other inflows may have some degree of uncertainty. On the plant outlet side, the amount of some material and energy products can be treated as decision variables, if how-much-ever produced can be sold out to the market. The amount of other material and energy products, however, may depend on the random demands of the customers. As a result, the demands of those products will become uncertain in the future time period. Accordingly, the material and energy flows are the basic factors which determine the performance of the plant. Thus, such analysis helps to optimize the consumption of raw material and energy by pursuing systematically internal flows of mass and energy in the production process.

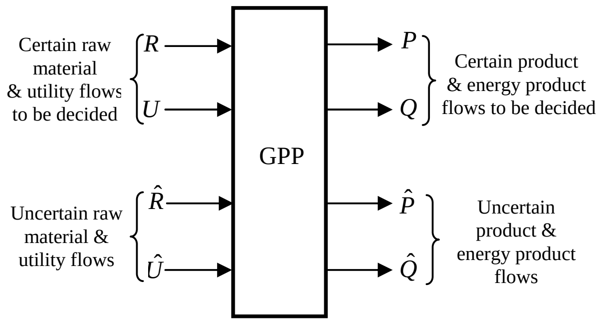

Fig. 1 A schematic representation for inflows and outflows of gas processing plant

where R, U, P, Q are for certain material, utility, product, and energy product flows, respectively. The corresponding uncertain flows are represented by $\hat{R}$, $\hat{U}$, $\hat{P}$, $\hat{Q}$, respectively.

B. Deterministic Formulation

Problem formulations based on deterministic approach emphasis on ensuring the feasibility of the solutions over a given domain of uncertain variables. The optimal solutions of a deterministic programming problem may become severely infeasible even if the nominal data is slightly perturbed [11]. Moreover, Sen & Higle (1999) reported that deterministic formulation in which uncertain variables are mathematically and statistically replaced by their expected values may not provide a solution that is feasible with respect to the uncertain variables. However, developing the deterministic formulation initially helps to easily transform to probabilistic problem. For the sake of clarity, in the next sections, we have considered only the uncertain feed and product flows.

1. Objective Function

The common optimization approach to an industrial process is to maximize the profitability of the plant, or minimize the overall costs. The former is adopted in this work (for simplicity, we considered only the material flow):

$$\max \text{Profit} = \sum_{j=1}^{J} \bar{C}_j^{P} P_j + \sum_{l=1}^{L} \bar{C}_l^{\hat{P}} \hat{P}_l - \sum_{I=1}^{I} \bar{C}_i^{R} R_i - \sum_{m=1}^{M} \bar{C}_m^{\hat{R}} \hat{R}_m \qquad (1)$$

where the parameters $\bar{C}^{R}$ and $\bar{C}^{P}$ are the expected price factors for the certain raw material and products flows, respectively, while $\bar{C}^{\hat{R}}$ and $\bar{C}^{\hat{P}}$ represent the expected price factor for the uncertain raw material and product flows. All the raw materials from the plant inlet and the products from the plant outlet are taken as decision variables as shown in equation (1).

2. Constraints

The aim of the constraints is to find an optimal solution to the optimization problem whose cost, evaluated as sum of the cost functions, is to be maximized. Thus, the constraints play a major role in satisfying the optimization problem with the defined decision variables. These constraints have been described below:

Inflet flow distribution to the plants:

$$R = \sum_{i=1}^{I} R_i, \quad \hat{R} = \sum_{m=1}^{M} \hat{R}_m \qquad (2)$$

Outlet flow distributions from the plant:

$$P = \sum_{j=1}^{J} P_j, \quad \hat{P} = \sum_{l=1}^{L} \hat{P}_l \qquad (3)$$

Availability constraint:

$$\sum_{j=1}^{J} a_{k,j} P_j + \sum_{l=1}^{L} \hat{a}_{k,l} \hat{P}_l \leq \sum_{i=1}^{I} b_{k,i} R_i + \sum_{m=1}^{M} \hat{b}_{k,m} \hat{R}_m \qquad (4)$$

Total material balance:

$$P + \hat{P} = R + \hat{R} \qquad (5)$$

Capacity restriction:

$$P_{j,\min} \leq P_j \leq P_{j,\max}, \quad \hat{P}_{l,\min} \leq \hat{P}_l \leq \hat{P}_{l,\max},$$

$$R_{i,\min} \le R_i \le R_{i,\max}\ ,\ \hat{R}_{m,\min} \le \hat{R}_m \le \hat{R}_{m,\max} \tag{6}$$

C. Probabilistic Formulation

During deterministic optimization, in which the expected values of the uncertain variables are employed, the implementation of the results will violate the inequality constraints in equation (4) with a probability of 50% [9]. The probabilistic formulation expresses the constraints in terms of a measurement unit called confidence level (α), which is assigned for each or the whole constraints.

1. Single Chance Constrained

The basic formulation for single chance constrained optimization can be derived from the deterministic formulation developed in the previous section. Accordingly, the objective function can be re-defined as:

$$\max \text{Profit} = \sum_{j=1}^{J} \overline{C}_j^P P_j + E\left[\sum_{l=1}^{L} C_l^{\hat{P}} \hat{D}_l\right] - \sum_{i=1}^{I} \overline{C}_i^R R_i - E\left[\sum_{m=1}^{M} C_m^{\hat{R}} \hat{S}_m\right] \tag{7}$$

where $\hat{S}_m$ is the supply for the uncertain raw material m, while $\hat{D}_l$ represent the demand for uncertain product l. The constraints for the single chance constrained formulation include equation (2), (3) and (5) as well as the probabilistic availability constraint and the new capacity restriction:

$$\Pr\left\{ r_k^{\xi} = \sum_{j=1}^{J} a_{k,j}^{(1)} P_j + \sum_{l=1}^{L} \hat{a}_{k,l}^{(1)} \hat{P}_l - \sum_{i=1}^{I} b_{k,i}^{(1)} R_i \le \xi_k \right\} \ge \alpha_k^{\xi},\quad k=1,\ldots,K$$
$$\Pr\left\{ p_k^{\zeta} = \sum_{j=1}^{J} a_{k,j}^{(2)} P_j - \sum_{i=1}^{I} b_{k,i}^{(2)} R_i - \sum_{m=1}^{M} \hat{b}_{k,m}^{(2)} \hat{R}_m \ge \zeta_k \right\} \ge \alpha_k^{\zeta},\quad k=1,\ldots,K \tag{8}$$

$$P_{j,\min} \le P_j \le P_{j,\max}\ ,\ R_{i,\min} \le R_i \le R_{i,\max} \tag{9}$$

where $\xi, \zeta \subseteq \Re^K$ are vectors of the uncertain feed and product *component* inflows and outflows, respectively. The small case letters r^{ξ} and p^{ζ} represent for the uncertain feed and product *component* flow rates, which actually consumed and produced from the plant, respectively.

2. Joint Chance Constrained

Joint probabilistic constraints express the condition that at minimum confidence level (α), certain trajectories satisfy the given constraints over the whole interval [13]. From the formal point of view, passing from individual probabilistic constraints to joint chance constraints involves a number of inequality constraints to be turned to a single inequality. The only changes exist on the probabilistic availability constraint stated in (8) to be replaced as:

$$\Pr\left\{\begin{array}{l} r_k^{\xi} = \sum_{j=1}^{J} a_{k,j}^{(1)} P_j + \sum_{l=1}^{L} \hat{a}_{k,l}^{(1)} \hat{P}_l - \sum_{i=1}^{I} b_{k,i}^{(1)} R_i \le \xi_k, k=1,\ldots,K \\ p_k^{\zeta} = \sum_{j=1}^{J} a_{k,j}^{(2)} P_j - \sum_{i=1}^{I} b_{k,i}^{(2)} R_i - \sum_{m=1}^{M} \hat{b}_{k,m}^{(2)} \hat{R}_m \ge \zeta_k, k=1,\ldots,K \end{array}\right\} \ge \alpha_k \tag{10}$$

D. Relaxed Deterministic Formulation

The single and joint chance constrained formulation developed previously can not be solved unless they are relaxed to their equivalent deterministic form. The relaxation of the probabilistic constraints in (8) and (10) becomes:

$$\sum_{j=1}^{J} a_{k,j}^{(1)} P_j + \sum_{l=1}^{L} \hat{a}_{k,l}^{(1)} \hat{P}_l - \sum_{i=1}^{I} b_{k,i}^{(1)} R_i \le \Phi_k^{-1}\left(1-\alpha_k^{\xi}\right),\quad k=1,\ldots,K$$
$$\sum_{j=1}^{J} a_{k,j}^{(2)} P_j + \sum_{i=1}^{I} b_{k,i}^{(2)} R_i - \sum_{m=1}^{M} b_{k,m}^{(2)} \hat{R}_m \ge \Phi_k^{-1}\left(\alpha_k^{\zeta}\right),\quad k=1,\ldots,K \tag{11}$$

$$\prod_{k=1}^{K}\left[1-\Phi_k\left(\sum_{j=1}^{J} a_{k,j}^{(1)} P_j + \sum_{l=1}^{L} \hat{a}_{k,l}^{(1)} \hat{P}_l - \sum_{i=1}^{I} b_{k,i}^{(1)} R_i\right)\right]$$
$$\prod_{k=1}^{K} \Phi_k\left(\sum_{j=1}^{J} a_{k,j}^{(2)} P_j - \sum_{i=1}^{I} b_{k,i}^{(2)} R_i - \sum_{m=1}^{M} \hat{b}_{k,m}^{(2)} \hat{R}_m\right) \ge \alpha_k \tag{12}$$

E. Reliability vs Profitability

The relation between profitability and reliability of the process can be gained by solving the optimization problem for different confidence level. Fig. 2 shows the possible profit profiles with respect to confidence level [4,5]. Accordingly, if we have a slow decreasing profit profile, such as profile **A**, then point **a** should be chosen as the decision for the operating point. This is because increasing the confidence level from this point will lead to a considerable reduction in profit. For profit profile like **B**, it is difficult to determine the solution point. Thus the decision is based on the specific requirement or priority between the profit and reliability of the problem. For profile like **C,** the optimal value is determined in the higher confidence region at point **b** since profit is not much sensitive.

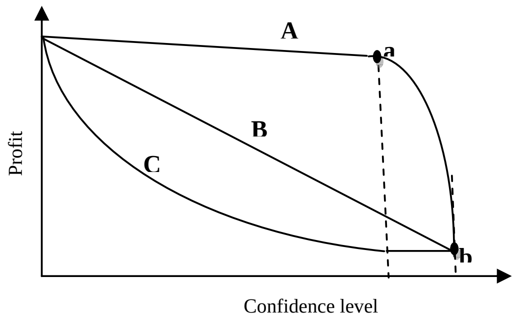

Fig. 2 Confidence level vs Possible profit profile

III. Case Study

A case study for a real gas plant operation has been taken to implement the developed model. Fig. 3 shows the schematic representation of the plant consisting of the main processes: pretreatment unit (PTU); acid gas removal unit (AGRU); low temperature separation unit (LTSU) and product recovery unit (PRU). The total feed which enters to the plant is represented as R_T and the products from the plant are: sales gas (P_1), ethane (P_2), propane (P_3), butane (P_4), condensate (P_5), and Carbon dioxide (P_6).The optimization is performed using GAMS version 2.0.20.0 (Module GAMS Rev 133). For single chance constrained optimization, the solver CPLEX 7.5 has been used. The joint chance constrained optimization was solved using CONOPT 3 solver. The probability density functions are generated using historical data from the plant on an hourly basis for a period of three months and all conform to the normal distribution. The feasibility analysis was carried out by allowing each uncertain feed component flow to vary according to their probability density function. The units for all raw material and product flow rates is in ton/h; the expected value for the total raw material price is $54/ton and for the products: sales gas ($101/ton), ethane ($21/ton), propane ($167/ton), butane ($212/ton) and LPG $189 /ton.

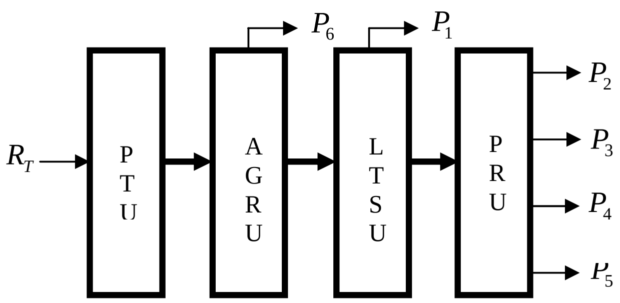

Fig. 3 Simplified block diagram of gas processing plant

A. Single Chance Constrained Optimization

The probability density functions are generated from historical data on an hourly basis for a period of three months. The normal distribution of each uncertain feed component flows ($\xi_{1,...,}\xi_7$) has been shown in appendix II with mean and standard deviation: N (201, 12), N (35, 3), N (22, 3), N (13, 2), N (9, 1.5), N (2.5, 0.5) and N (53, 4), respectively. The units are in ton/h; the expected value for the total raw material price is $54/ton and for the products: sales gas ($101/ton), ethane ($21/ton), propane ($167/ton), butane ($212/ton). The prices of condensate and carbon dioxide products have been considered nil. The optimal profit profile under single and joint chance constraints with confidence level starting from 50% is shown in Fig 4. Thus, the relation between the achievable profit and reliability to hold the constraints has been quantified for decision making purpose. The profit profiles in Fig. 6 resemble to profit profile '**A**' in Fig. 4, and decreases rapidly after α_C = 0.95. Accordingly, moving further from this point to the right direction guarantees the reliability of the process; however, the profit decreases dramatically. On the other hand, moving to the left direction from the critical point (α_C = 0.95) improves the profit, but at the expense of losing the reliability of the process. Hence, the 95% confidence level will be a suitable choice that can compromises between profit and reliability.

The corresponding product profiles for propane and butane are shown in Figs. 5 and 6, respectively. For example, for single chance constraint case, if the plant decided to produce P_2 = 195 ton/h, P_3 = 3.5 ton/h, P_4 = 17 ton/h and P_5 = 10 ton/h, then with this decision the production can be satisfied with a probability of 95%. Accordingly, there is only a 5% probability, risk of violation of constraints, that the amount of the uncertain feed component flow would not be enough to produce the desired amount of products.

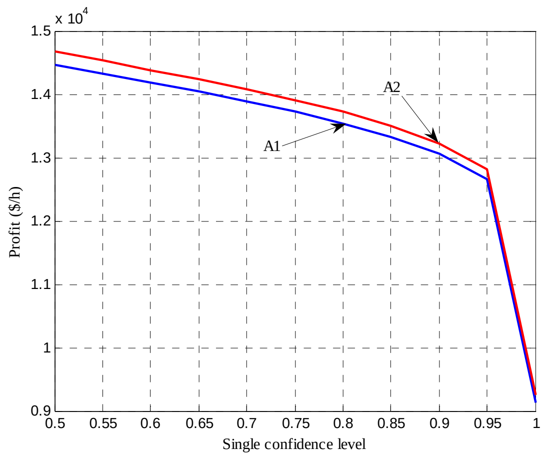

Fig. 4 Single confidence level vs profit profile

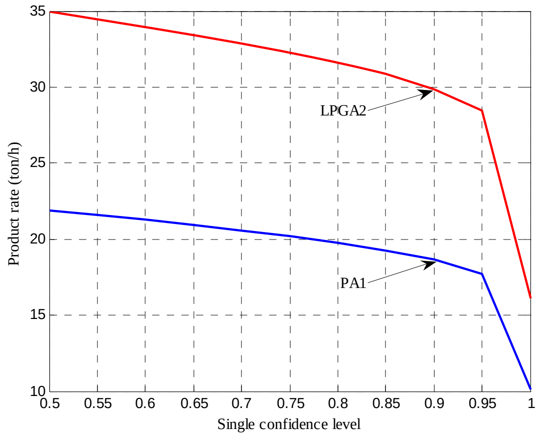

Fig. 5 Single confidence level vs propane production

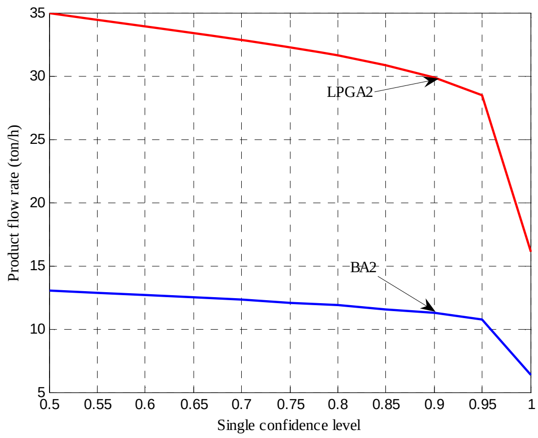

Fig. 6 Single confidence level vs butane production

B. Joint Chance Constrained Optimization

The achievable profit profile under joint chance constrained optimization for independent uncertain variables is shown in Fig. 7. The condition for profit profile A3 and A4 are same as A1 and A2, respectively, except that they are under joint chance constraints. The corresponding product profiles for propane (PA3 & LPGA4) and butane (BA3 &LPGA4) products are shown in Figs. 8 and 9, respectively. From Fig. 7, the optimal decision for A3 can be made at 85% confidence level, while for A4 at 70% confidence level. For joint chance constrained optimization, the reliability of the process has decreased compared to the single chance constrained cases. This is because the solution space for joint chance constraint case is limited compared to the single chance constraint and thus may not reach to 100% confidence level. For example, if the plant wants to produce P_1 = 186 ton/h (SA3), P_2 = 3 ton/h for a target of $11,340/ton, then the optimal decision will be satisfied at 85% confidence level and the LPG production will be set at 21 ton/h. In other words, there is only a 15% risk of violation in which the uncertain feed and LPG product component flows would not be enough to satisfy the required amounts of products to be produced. The same scenario is also applicable for A3.

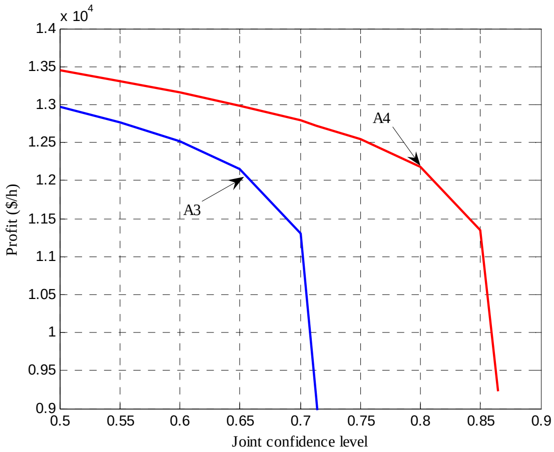

Fig. 7 Joint confidence level vs profit profile

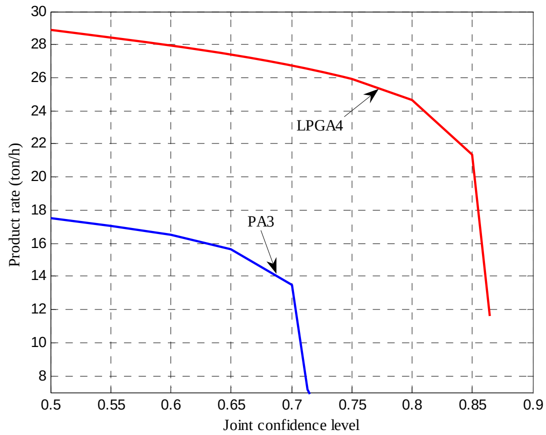

Fig. 8 Joint confidence level vs propane production

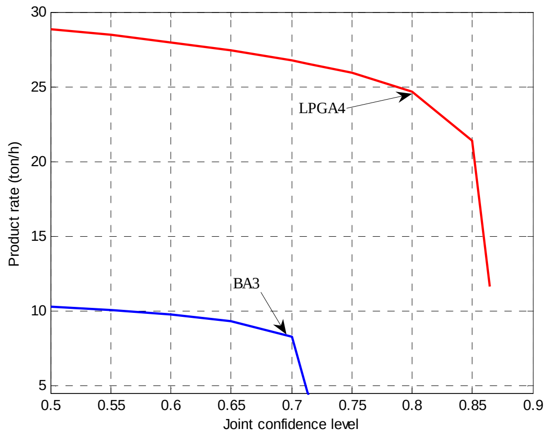

Fig. 9 Joint confidence level vs butane production

IV. Conclusion

A general optimization model for a gas processing plant with uncertain feed and product flows has been presented. The uncertain feed and product component flows have been explicitly introduced in the optimization to incorporate their effects during the plant performance. The computational results from the single and joint chance constrained optimization shows the need for making decision under uncertainty is very important to comprise the reliability and profitability of the plant by satisfying all the constraints at certain confidence level. In addition, a prior-decision can be made for the in-operating plant under uncertain feed and products flows.

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