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An AK-Chart for the Non-Normal Data

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***Abstract*—**Traditional multivariate control charts assume that measurement from manufacturing processes follows a multivariate normal distribution. However, this assumption may not hold or may be difficult to verify because not all the measurement from manufacturing processes are normal distributed in practice. This study develops a new multivariate control chart for monitoring the processes with non-normal data. We propose a mechanism based on integrating the one-class classification method and the adaptive technique. The adaptive technique is used to improve the sensitivity to small shift on one-class classification in statistical process control. In addition, this design provides an easy way to allocate the value of type I error so it is easier to be implemented. Finally, the simulation study and the real data from industry are used to demonstrate the effectiveness of the propose control charts.

***Keywords*—**Multivariate control chart, statistical process control, one-class classification method.

I. Introduction

THE function of SPC (Statistical Process Control) techniques is to detect the shift of production or service processes, so the decision makers can take corrective actions before quality is deteriorated. In reality, the data are often multivariate and are correlated. Thus, Multivariate Statistical Process Control (MSPC) techniques are applied to those processes. It is common to monitor several measurements of a process simultaneously. One important tool in MSPC is the multivariate control chart which is used to detect the shift in the process and to keep the process in an in-control state. Bersimiset al. [1] had reviewed the multivariate control charts thoroughly. Hotelling [2] develops T^2 chart which use T^2 statistic to monitor multivariate process. T^2 statistic is computed from (1):

$$T^2 = (\mathbf{x} - \bar{\mathbf{x}})^T \mathbf{S}^{-1} (\mathbf{x} - \bar{\mathbf{x}}) \qquad (1)$$

where $\bar{\mathbf{x}}$ and $\mathbf{S}$ is the sample mean and covariance matrix determined from the in-control state. When $\mathbf{x}$ follows a multivariate normal distribution, the T^2 statistic follows an F distribution. Other multivariate control charts include Multivariate Cumulative Sum (MCUSUM) chart [3], Multivariate Exponentially Weighted Moving Average (MEWMA) chart [4], and others. However, the above multivariate control charts usually assume that measurement follows a multivariate normal distribution. In many case this assumption may be difficult to verify. In order to monitor quality of the process with non-normal data, nonparametric multivariate control charts are developed.

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Liu [5] proposes a control scheme based on data depth can be applied on different non-normal processes. Then Liu and Tang [6] improve the estimation of control limits on Liu's control scheme. However, Stoumbos and Sullivan [7] argue that the control schemes developed by Liu cost large samples and are difficult to implement. This control scheme seems inappropriate to be applied to the skewed distributions. Thus,Qiu and Hawkins [8], [9] present Antirank-based Cumulative Sum (ACUSUM) chart which uses the CUSUM procedure based on antirank statistic. Zou and Tsung [10] comment that the computation of ACUSUM chart is too complicated to use. They proposes Multivariate Sign Exponentially Weighted Moving Average (MSEWMA) chart which use EWMA procedure based on sign statistic. Qiu [11] also suggests a multivariate CUSUM procedure based on log-linear modeling. But this log-linear modeling will be difficult to compute as the data dimension increases. Alternatively, Stoumbos and Sullivan [12] extend MEWMA chart to non-normal data. Zouet al. [13] further present Spatial Rank Exponentially Weighted Moving Average (SREWMA) chart which uses EWMA procedure based on spatial rank statistic. These control charts use less computation time and are more sensitive to shift. However, the above charts are difficult to construct because the settings of the weighting parameter depend on the deviation from the actual measurement distribution form to multi-normal distribution. And this deviation is far from measuring in practice. In addition, the rank-based nonparametric control chart is usually inefficient for large shifts.

Some studies have developed a control chart based on machine learning or data mining. Sun and Tsung [14] proposed the K charts based on a Support Vector Data Description (SVDD) algorithm. Kumar et al. [15], Camci et al. [16], Cheng and Cheng [17] have put efforts on improving the design of K chart. Gani et al. [18] use real data to demonstrate the application of K chart. Ning and Tsung[19] propose a systematic design of K chart and have a complete analysis of the design of K chart. In addition, Sukchotrat et al. [20] present K^2 chart based on k-Nearest Neighbors Data Description (kNNDD) algorithm. Ning and Tsung [21] use the method of outlier detection to develop control chart. However, most of these control charts are difficult to establish because the determination of the charting parameter are usually needed and the guideline to determine theses parameters are seldom found.

This study proposes a new multivariate SPC methodology for monitoring the processes with non-normal data. This methodology based on integrating the one-class classification method and the adaptive technique. The proposed method will be easier to be implemented because this design provides an intuitive way to allocate the value of type I error. We also analyze the effectiveness of the propose control charts by revisiting a simulation study and the real data from industry.

II. A Brief Review of SVDD-Based Chart and Adaptive Technique

SVDD is one of one-class classification algorithms which popularly used for SPC. It is a mixture of Support Vector Machine (SVM) and the data description method for solving one-class classification problems [22]. The SVDD is originally developed from SVM methods. We first give a brief description of the SVM method. Then, we will illustrate SVDD algorithms and explain the K chart which is derived from SVDD. Finally, we the introduction of the adaptive technique will be provided.

A. SVM

An SVM is usually used for both regression and classification problem. The basic concept of SVM is to find a hyperplane by solving convex optimization problem which can simultaneously minimize the generalization error and maximize the margin between the classes.

Let $\mathbf{X}_i = [x_1, x_2, \ldots, x_p]^T$, for i =1,2,…,N be a p-variate training data. Each $\mathbf{X}_i$ has a corresponding label (the class of data point in training set), $d_i \in \{-1,1\}$. The decision function for data point, $\mathbf{X}$,is given as (3):

$$f(x) = \text{sign}(\mathbf{w} \times \mathbf{X} + \mathbf{b}) \quad (3)$$

where$\mathbf{w} \times \mathbf{X} + \mathbf{b} = 0$is the separating hyperplane for the two classes. We can see from Fig. 1 that the margin is the distance in the direction perpendicular to the hyperplane between the closest points of two classes. So we can minimize the generalization error by maximizing the margin. To maximize the margin, one can solve the following optimization problem:

$$\min \tfrac{1}{2}\|\mathbf{w}\|^2 \quad (4)$$

$$\text{s.t.} d_i(\mathbf{w} \times \mathbf{X} + \mathbf{b}) \geq 1, \forall i \quad (5)$$

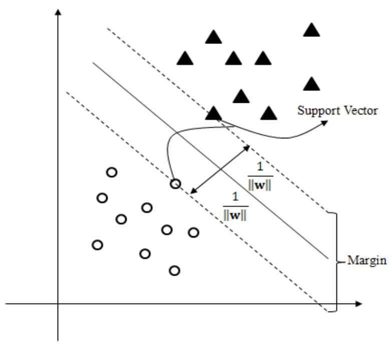

Fig. 1The basic concept of SVM

In some cases, the data set cannot be completely separated. In order to allow variability in non-linear program, Cortes and Vapnik [23] add slack variables into the model. That is,

$$\min \tfrac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{N} \xi_i \quad (6)$$

$$\text{s.t.} d_i(\mathbf{w} \times \mathbf{X} + \mathbf{b}) \geq 1 - \xi_i, \forall i \quad (7)$$

$$\xi_i \geq 0, \forall i \quad (8)$$

whereξ_i is the slack variable which is corresponding to the constraints. $C > 0$is the penalty coefficient. This model allows the misclassification of data points so that the SVM can solve the problem with non-linearly separated data.

B. SVDD

Usually the SVM is used to solve the binary classification problem. However, in process control, we do not have two-class data but the in-control data. So the SVM method cannot be directly applied to monitoring process. Tax and Duin [22] first propose the SVDD method derived from SVM and they use it to solve the one-class classification problem (i.e. in-control data) and was used to monitor processes later.

The basic concept of SVDD is to find a hypersphere (see Fig. 2) by solving optimization problem which is minimizing the volume of hypersphere and maximizing the training data points capture by hypersphere.

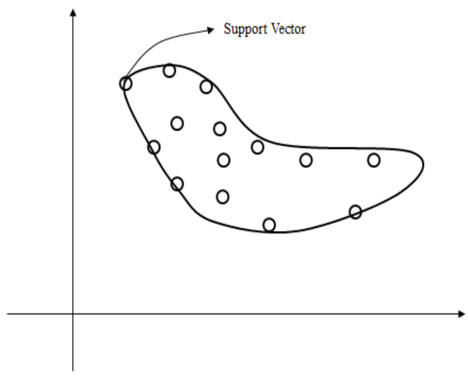

Fig. 2The basic concept of SVDD

Let$\mathbf{X}_i = [x_1, x_2, \ldots, x_p]^T$, for i =1,2,…,N be the training set. Note that there is only one class. We give a center of hypersphere, **O**, and a radius of hypersphere, R. This optimization model can be formulated as follows:

$$\min R^2 + C\sum_{i=1}^{N} \xi_i \quad (8)$$

$$\text{s.t.} (\mathbf{X}_i - \mathbf{O})^T(\mathbf{X}_i - \mathbf{O}) \leq R^2 + \xi_i \quad (9)$$

$$\xi_i \geq 0, \forall i \quad (10)$$

whereξ_i is the slack variable which is corresponding to the constraints. $C > 0$ is the trade-off between the volume of hypersphere and the training data points captured by hypersphere. If C is large, the corresponding boundary would be wide and more data points are captured by hypersphere. In contrast, if C is small, the corresponding boundary would be narrow and less data points are captured by hypersphere.

This model can be solved by using the Lagrangian function:

$$L(R, \mathbf{O}, \pi_i, \tau_i, \xi_i) = R^2 + C\sum_{i=1}^{N} \xi_i - \sum_{i=1}^{N} \pi_i \{R^2 + \xi_i - (\mathbf{X}_i - \mathbf{O})^T(\mathbf{X}_i - \mathbf{O})\} - \sum_{i=1}^{N} \tau_i \xi_i \tag{11}$$

where π_i and τ_i are the Lagrangian multipliers and are greater than or equal to zero. New constraints are:

$$2R - 2R\sum_{i=1}^{N} \pi_i = 0 \tag{12}$$

$$-2\sum_{i=1}^{N} \pi_i (\mathbf{X}_i - \mathbf{O}) = 0 \tag{13}$$

$$\mathrm{C} - \pi_i - \xi_i = 0 \tag{14}$$

The solving procedures can be found on Tax and Duin [22] and the simplified model is:

$$\max \sum_{i=1}^{N} \pi_i \mathbf{X}_i{}^T\mathbf{X}_i - \sum_{i,j=1}^{N} \pi_i\pi_j \mathbf{X}_i{}^T\mathbf{X}_j \tag{15}$$

$$\text{s.t.} \sum_{i=1}^{N} \pi_i = 1 \tag{16}$$

$$0 \le \pi_i \le C, \forall i \tag{17}$$

Then we can determine whether the test point, $\mathbf{Z}$, is belonged to the class (i.e. in-control data) by calculating the distance between test point, $\mathbf{Z}$, and center, $\mathbf{O}$. If the distance, $\|\mathbf{Z} - \mathbf{O}\|^2$, is larger than radius, *R*, it is be identified as an abnormal point and can be formulated as:

$$\|\mathbf{Z} - \mathbf{O}\|^2 = \left(\mathbf{Z} - \sum_{i=1}^{N} \pi_i\mathbf{X}_i\right)^T \left(\mathbf{Z} - \sum_{i=1}^{N} \pi_i\mathbf{X}_i\right) = \mathbf{Z}^T\mathbf{Z} - 2\sum_{i=1}^{N} \pi_i \mathbf{X}_i{}^T\mathbf{Z} + \sum_{i,j=1}^{N} \pi_i\pi_j \mathbf{X}_i{}^T\mathbf{X}_j \tag{18}$$

Then we can replace the inner product with kernel functions so one can obtain a more adaptive boundary of SVDD compared to the SVDD without kernel function. One popular form of kernel functions is Gaussian Radical Basis Function (Gaussian RBF). That is:

$$K(\mathbf{X}_i, \mathbf{X}_j) = \exp\left(-\frac{\|\mathbf{X}_i - \mathbf{X}_j\|^2}{g^2}\right) \tag{19}$$

where $g > 0$ is the window width. We replace the program with Gaussian RBF. The resulted model becomes:

$$\max \sum_{i=1}^{N} \pi_i K(\mathbf{X}_i, \mathbf{X}_i) - \sum_{i,j=1}^{N} \pi_i\pi_j K(\mathbf{X}_i, \mathbf{X}_j) \tag{20}$$

$$\text{s.t.} \sum_{i=1}^{N} \pi_i = 1 \tag{21}$$

$$0 \le \pi_i \le C, \forall i \tag{22}$$

And the kernel distance ($kd(\mathbf{Z})$) between test point and center is shown as:

$$kd(\mathbf{Z}) = K(\mathbf{Z}, \mathbf{Z}) - 2\sum_{i=1}^{N} \pi_i K(\mathbf{Z}, \mathbf{X}_i) + \sum_{i,j=1}^{N} \pi_i\pi_j K(\mathbf{X}_i, \mathbf{X}_j) \tag{23}$$

Finally, the $kd(\mathbf{Z})$ can be used to justify if the test data is in a specific category.

C. K Chart

Sun and Tsung [14] propose the K charts based on a SVDD algorithm. In their research, the Upper Control limit, *UCL*, is constructed by the SVDD boundary which is found from equations (20)-(22). And one can determine that the process is out-of-control if $kd(\mathbf{Z}) > UCL$.

Note that one must determine two parameters of K chart, the window width g and penalty coefficient *C*. Both of themhave a great effect on SVDD boundary. Ning and Tsung [19] have studied on how to set up these two parameters. Their idea is to find two parameters which have minimal total error, γ. Hereγ equals to the weighted sum of Type-I error and Type-II error. That is:

$$\gamma = (1 - v) \times \frac{\#SV}{N} + v \times f_{o+} \tag{24}$$

where *v* is the weight which weights the importance between Type-I errors and Type-II errors. Type-I errors is calculated by the proportion of the number of Supper Vector (SV), $\#SV$, and the number of training data. Type-II errors are calculated by the proportion of the number of artificial outlier that is identified as in-control data. And Ning and Tsung [19] also proposed the method to determine the weight, *v*. However, the users are redundant to determine the weight in (24) because the users are difficult to weight importance between Type-I errors and Type-II errors. Subsequently users are difficult to implement the K charts.

D. Adaptive Technique

The adaptive technique is widely used to improve the sensitivity on the shift of control charts. This technique uses previous information from process to determine the future sampling policy such as the sample size and sample interval between samples. Many studies have put efforts on using the adaptive technique to strengthen the multivariate control chart [24]-[28]. These studies show that Variable Sample Interval (VSI) control chart and Variable Sample Size and Sample Interval (VSSI) control chart are more sensitive to detect the shift of a process. And VSI technique is the easiest one to be implemented. In our study, we will use the VSI technique to construct theproposed multivariate control chart.

III. An Adaptive Kernel-based (AK) Control Chart

The construction of AK chart requires two parts: constructing the SVDD and determining of the adaptive parameters.

A. Construct SVDD

We first gather the training data set of size *N* in stable state. An assumption of one-class classification problem is that the data is independent and identically distributed. So we transform the data set by using Principal Component Analysis (PCA). In this way, we can eliminate the correlation of variables. The Principal Component (PC) resulted from PCA is then used to determine the two parameter of SVDD. However, because we only have one class data and we cannot calculate the classification error of setting of SVDD parameter. Tax and Duin [28] proposed a method on how to generate uniformly artificial outliers in hypersphere (see Fig. 3.). So one can get Type-I error and Type-II error. Ning and Tsung [19] use equation (24) to choose the parameters of SVDD, but it is too difficult to comprehend as far as user is concerned. So we rewrite it into following model:

$$\min_{g,C} \beta \tag{25}$$

$$\text{s.t.} \alpha \leq \alpha_0 \tag{26}$$

$$g > 0 \tag{27}$$

$$0 \leq C \leq 1 \tag{28}$$

where α and β is Type-I error and Type-II error. Objective function (25) is to minimize the Type-II error. $\alpha_0$stands for the targeted Type-I error which is specified by user. Equation (26) makes Type-I error smaller or equal to the targeted Type-I error. Equations (27) and (28) specify the range of the parameters. In this model, the users are more intuitively to understand the relationship between Type-I error and Type-II error.

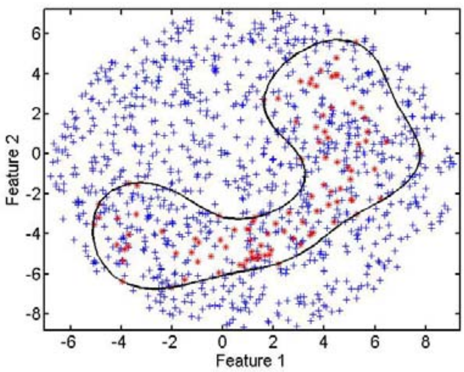

Fig. 3Uniformly artificial outliers in hypersphere

Grid Search (GS) algorithms are well suitable for solving such a problem because they have complete search in a solution space. Since the range of parameter C is from 0 to 1 and window width, g, changed from 2^{-3} to 2^{10} according to Ning and Tsung [19] 's research. We first identify the number of segments which is separated from range of two parameters and assign a value to each segment. We then pair the each segment of window width with the each segment of penalty coefficientas a sub solution, and use (20)-(22) with each sub solution to construct the candidate of optimal SVDD. Subsequently, one can solve (25)-(28) with all the candidates of SVDD to find the optima SVDD. Finally, the result will be used to construct the SVDD of AK chart.

B. Determination of Adaptive Parameter

An AK chart allows the sampling interval of chart to vary depend on the previous informations. In this method, we consider two different sampling intervals h_0 and h_1 and the placement of samples will fall on three regions divided by control limit, *UCL*, and warning limit, *WCL*. The region between zero and *WCL* is safe region which means process is stable (see Fig.4). The region between *WCL* and *UCL* is warning region indicating the process is close to out-of-control. The sample region over *UCL* is action region that implies the process is out-of-control. The decision to switch between two different sampling intervals h_0 and h_1 depends on the placement of prior sample. If the prior sample falls on safe region, the longer sampling interval h_0 will be used on current sample. If the prior sample falls on warning region, the shorter interval h_1 will used current sample. If the prior sample is on action region, the process is considered as out-of-control and a signal is triggered. However, the performance of AK chart is related to the setting of adaptive parameters including *UCL*, *WCL*, $h_0$and h_1. The performance measure of AK chart is based on Average Time to Signal (ATS) because the sampling interval is varied. When the process is in-control, the average time to signal (ATS_0) is larger and the process is better and the number of false alarms will be small. In addition, when the process is out-of-control, the average time to signal (ATS_1) is smaller and the process is better.That is, the time of detecting shift will be short. And the design of these parameters is formulated as following optimization problem:

$$\min_{UCL,WCL,h_0,h_1} \text{ATS}_1 \tag{29}$$

$$\text{s.t.} h_1 \geq h' \tag{30}$$

$$\text{ATS}_0 \geq \text{ATS}_0' \tag{31}$$

The objective function (29) is to minimize ATS_1. The equation (30) means that the limit h_1 must be more than the specify interval, h' (the shortest sampling interval that process can have). And equation (31) restricts ATS_0must be greater than the targeted average time to signal, ATS_0', which is determined by users.

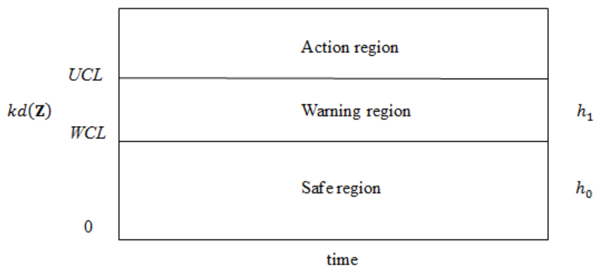

Fig. 4An AK chart

In this paper, Genetic Algorithms (GAs) are used for solving this design optimization problem because they have a less chance of converging to local optima in a multimodal space than the typical search algorithms. And many researches [27], [29]-[34] apply GAs to solve design optimization problem of control chart and found that GAs are well fitted for solving such a problem. GAs are search algorithms that were developed based on an analogy with natural selection and population genetics in biological system [35]. According to the research of [27], [29]-[34], the setting of parameters in GAs are chosen as: Population Size (PS) = 50, Crossover probability (CP) = 0.5, and Mutation Rate (MR) = 0.25.

IV. Performance of the AK chart

In this section, we evaluate the performances of the proposed AK chart and compare it toT^2 chart and K chart on simulation base. T^2chart can be conducted by (1) and K chart can be conducted by solving model (20)-(22). The comparison among the proposed AK chart, T^2 chart, and K chart are conducted for the situations that the multi-normal distribution in low dimensional case and high dimensional case. At low dimensional case where $p = 3$ and high dimensional case with $p = 5$ are considered. The covariance matrix $\mathbf{\Sigma}_0 = (\sigma_{ij})$ of multinormal distribution is chosen to let $\sigma_{ii} = 1$ and $\sigma_{ij} = 0.5^{|i-j|}$, for i,j = 1, 2, …, p. We fix α_0 at 0.01 and consider mean shift δ of the first one measurement of data point. Also, we present different degrees of process mean shift δ : 0.25, 0.5, 0.75, 1.00, 1.25, 1.50. In this paper, the out-of-control ATS is chosen as measurement of control chart performance because the sampling interval is allowed to vary. And all the ATS results in this section are obtained from 10,000 replications.

TABLE I
Low Dimensional Case

		ATS_1	
δ	T^2 chart	K-chart	AK-chart
0.25	36.14	22.74	14.52
0.50	7.52	1.28	1.13
1.00	1.41	1.00	1.00
1.25	1.03	1.00	1.00
1.50	1.00	1.00	1.00

Table I indicates that K chart and the proposed AK chart have satisfactory performance and the difference between the K chart and the proposed AK chart is small in low dimensional case. However, Table II shows, in a high dimensional case, that the K chart is worse than T^2 chart and it is poor on detecting small shift ($\delta < 1.00$). But the difference among the proposed AK chart, K chart and T^2 chart is small in large shift ($\delta > 1.25$). Note that, the AK chart usually has better performance compared to T^2 chart and K chart because the proposedAK chart use the previous informations from process to strengthen the power of detecting shift of process.

TABLEII
High Dimensional Case

		ATS_1	
δ	T^2 chart	K-chart	AK-chart
0.25	74.7770	100.00	46.7679
0.50	16.8960	45.9560	11.6749
1.00	1.8650	6.1170	1.6279
1.25	1.00	3.21	1.01
1.50	1.00	1.00	1.00

The AK chart improves the weakness of K chart which is not sensitive to small shift of process. In additional, the AK chart is easier than K chart to be implemented because AK chart provides an intuitive way to allocate the value of type I error. However, the construction of AK chart requires the distribution of measurement because the determination of adaptive parameter is based on simulation method. In real, this assumption sometime may not or difficult to be validated.

V. Conclusions

In this paper, an AK chart is proposed for monitoring process with non-normal data. The design of SVDD parameters of AK chart is formulated as an optimization problem which minimizes the Type-II error and limits the Type-I error to be smaller than the target level. It is easier to be allocated for the value of Type-I error and Type-II error compared to the original design because AK chart provides an intuitive way to decision maker. The performance of the proposed AK chart measured by out-of-control ATS is compared toT^2 chart and K chart. The results of simulation show that the performance of the proposed AK chart is better than T^2 chart and K chart. However, because the determination of adaptive parameter is based on simulation method, the proposed AK chart need the information on distribution of measurements which may not easy to identify in real.Finally, this article uses VSI technique to improve the sensitive of control chart to detect the shift of process. Future researches may use other adaptive techniques to improve the performance of the proposed AK chart.

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