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Further Thoughtson a Sequential Life Testing Approach using an Inverse Weibull Model

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Abstract**—**In this paper we will develop further the sequential life test approach presented in a previous article by [1] using an underlying two parameter Inverse Weibull sampling distribution. The location parameter or minimum life will be considered equal to zero. Once again we will provide rules for making one of the three possible decisions as each observation becomes available; that is: accept the null hypothesis H_0; reject the null hypothesis H_0; or obtain additional information by making another observation. The product being analyzed is a new electronic component. There is little information available about the possible values the parameters of the corresponding Inverse Weibull underlying sampling distribution could have.To estimate the shape and the scale parameters of the underlying Inverse Weibull model we will use a maximum likelihood approach for censored failure data. A new example will further develop the proposed sequential life testing approach.

Keywords**—**Sequential Life Testing, Inverse Weibull Model, Maximum Likelihood Approach, Hypothesis Testing.

I. Introduction

THE two-parameter Inverse Weibull distribution was derived by [2]. It has been used in Bayesian reliability estimation to represent the information available about the shape parameter of an underlying Weibull sampling distribution [2]; [3]; [4]. It has a location (or minimum life), a scale and a shape parameter. The location parameter will be considered to be equal to zero.Both parameters are positive.The Inverse Weibull density function $f(t)$is given by.

$$f(t)=\frac{\beta}{\theta}\left(\frac{\theta}{t}\right)^{\beta+1} exp\left[-\left(\frac{\theta}{t}\right)^{\beta}\right];t\geq 0 \qquad (1)$$

Here, $t$representsthe time to failure of a component or part. The scale parameter θ (the characteristic life) is positive and is the 63.21 percent point of the distribution of T. The shape parameter β, which is also positive, specifies the shape of the distribution. The hypothesis testing situations will be given by:

1. For the scale parameterθ: H_0: $\theta \geq \theta_0$; H_1: $\theta < \theta_0$

The probability of accepting the null hypothesis H_0 will be set at $(1-\alpha)$ if $\theta = \theta_0$. Now, if $\theta = \theta_1$ where $\theta_1 < \theta_0$, then the probability of accepting H_0 will be set at a low level γ. H_1 represents the alternative hypothesis.

2. For the shape parameter β: H_0: $\beta \geq \beta_0$; H_1: $\beta<\beta_0$

The probability of accepting H_0will be set again at $(1-\alpha)$in the case of $\beta = \beta_0$. Now, if $\beta = \beta_1$, where $\beta_1<\beta_0$, then the probability of accepting H_0 will also be set at a low level γ.

II. Sequential Testing

The development of a sequential test uses the likelihood ratio (LR) given by the following relationship proposed by [1] and [5]:

$$LR = L_{1;n}/L_{0;n}$$

The sequential probability ratio (SPR) will be given by:

$$SPR = L_{1;n}/L_{0;n}$$

Based on the paper from [1], for the Inverse Weibull case the (SPR) will be given by:

$$\text{SPR} = \left(\frac{\beta_1}{\theta_0^{\beta_0}}\times\frac{\theta_1^{\beta_1}}{\beta_0}\right)^n \prod_{i=1}^{n}\left[\frac{(t_i)^{\beta_0+1}}{(t_i)^{\beta_1+1}}\right]\times$$

$$\times exp\left[-\sum_{i=1}^{n}\left(\frac{\theta_1^{\beta_1}}{(t_i)^{\beta_1}}-\frac{\theta_0^{\beta_0}}{(t_i)^{\beta_0}}\right)\right] \qquad (1)$$

Therefore, the continue region becomes $A< SPR < B$, where $A = \gamma/(1-\alpha)$ and $B = (1-\gamma)/\alpha$. We will accept the null hypothesis H_0 if $SPR \geq B$ and we will reject H_0 if $SPR \leq A$. Now, if $A <SPR< B$, we will take one more observation. Then, we will have:

$$n\,ln\left(\frac{\beta_1}{\theta_0^{\beta_0}}\times\frac{\theta_1^{\beta_1}}{\beta_0}\right) - ln\left[\frac{(1-\gamma)}{\alpha}\right] < \text{W} <$$

$$< n\,ln\left(\frac{\beta_1}{\theta_0^{\beta_0}}\times\frac{\theta_1^{\beta_1}}{\beta_0}\right) + ln\left[\frac{(1-\alpha)}{\gamma}\right] \qquad (2)$$

$$\text{W} = \sum_{i=1}^{n}\left(\frac{\theta_1^{\beta_1}}{(t_i)^{\beta_1}}-\frac{\theta_0^{\beta_0}}{(t_i)^{\beta_0}}\right)+(\beta_1-\beta_0)\sum_{i=1}^{n}ln(t_i) \qquad (3)$$

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III. THE MAXIMUM LIKELIHOOD APPROACH

According to [1], the maximum likelihood estimator for the shape and scale parameters of a two parameter Inverse Weibull sampling distribution is given by:

$$\frac{dL}{d\theta} = \frac{r\beta}{\theta} - \beta\theta^{\beta-1}\sum_{i=1}^{r}\left(\frac{1}{t_i}\right)^{\beta} - (n-r)\beta\theta^{\beta-1}\left(\frac{1}{t_r}\right)^{\beta} = 0 \quad (4)$$

$$\frac{dL}{d\beta} = \frac{r}{\beta} + r\ln(\theta) - \sum_{i=1}^{r}\ln(t_i) - \sum_{i=1}^{r}\left(\frac{\theta}{t_i}\right)^{\beta} ln\left(\frac{\theta}{t_i}\right) - (n-r)\left(\frac{\theta}{t_r}\right)^{\beta} ln\left(\frac{\theta}{t_r}\right) = 0 \quad (5)$$

From (4) we obtain:

$$\theta = \left(r \Big/ \sum_{i=1}^{r}\left(\frac{1}{t_i}\right)^{\beta} + (n-r)\left(\frac{1}{t_r}\right)^{\beta} \right)^{1/\beta} \quad (6)$$

Using (6) for θ in (5) and after some mathematical manipulation, (5) reduces to:

$$\frac{r}{\beta} - \sum_{i=1}^{r}\ln(t_i) - \frac{r\times\left[\sum_{i=1}^{r}\left(\frac{1}{t_i}\right)^{\beta} ln(t_i) + (n-r)\left(\frac{1}{t_r}\right)^{\beta} ln(t_r)\right]}{\sum_{i=1}^{r}\left(\frac{1}{t_i}\right)^{\beta} + (n-r)\left(\frac{1}{t_r}\right)^{\beta}} = 0 \quad (7)$$

Equation (7) must be solved iteratively.

In a previous article [1] an example was presented to illustrate the proposed approach. We will now analyze four new different situations for the hypothesis testing considered in this paper.

IV. EXAMPLE

A new electronic component will be life tested. Since this is a new product, there is little information available about the possible values that the parameters of the corresponding Inverse Weibull underlying sampling distribution could have. To estimate the shape and the scale parameters of this sampling model we will use a maximum likelihood approach for censored failure data. Some preliminarily life testing was performed in order to determine an estimated value for the two Inverse Weibull parameters. Using the maximum likelihood estimator approach we obtained the following values for these parameters:

$$\theta = 581.22 \text{ hours}; \quad \beta = 9.14$$

It was decided that $\alpha = 0.05$ and $\gamma = 0.10$. Initially, we elect the null hypothesis parameters to be $\theta_0 = 581.22$ hours; with $\beta_0 = 9.14$; $\alpha = 0.05$ and $\gamma = 0.10$ and choose some possible values for the alternative parameters θ_1 and β_1, and see how this choice will alter the results of the test. After that, we will change the values of the null hypothesis parameters and verify how the test results will behave. So, we choose $\theta_1 = 540$ hours and $\beta_1 = 8.5$. Then, using (2) and (3), we have:

$$nln\left(\frac{8.5}{540^{8.5}}\times\frac{581.22^{9.14}}{9.14}\right) - ln\left[\frac{(1-0.10)}{0.05}\right] =$$

$$n\times 102.141 - 2.890$$

$$nln\left(\frac{8.5}{540^{8.5}}\times\frac{581.22^{9.14}}{9.14}\right) + ln\left[\frac{(1-0.05)}{0.10}\right] =$$

$$n\times 102.141 + 2.251$$

Then, we get:

$$n\times 102.141 - 2.890 < W < n\times 102.141 + 2.251$$

$$W = \sum_{i=1}^{n}\left(\frac{t_i^{8.5}}{540^{8.5}} - \frac{t_i^{9.14}}{581.22^{9.14}}\right) - 0.64\times\sum_{i=1}^{n} ln(t_i)$$

After a sequential test graph has been developed for this life-testing situation, a random sample is taken.

The procedure is then defined by the following rules:

1. If $W \geq n \times 102.141 + 2.251$, we will accept H_0.
2. If $W \leq n \times 102.141 - 2.890$, we will reject H_0.
3. If $n \times 102.141 - 2.890 < W < n \times 102.141 + 2.251$, we will take one more observation.

In this first case, 5 units were tested to allow the decision of accepting the null hypothesis H_0. The values for the corresponding number of cycles (time to failure) of these 5 units were the following: *598.94; 624.87; 729.65; 675.28; 748.34 hour*s. Fig.1 shows the results of this test.

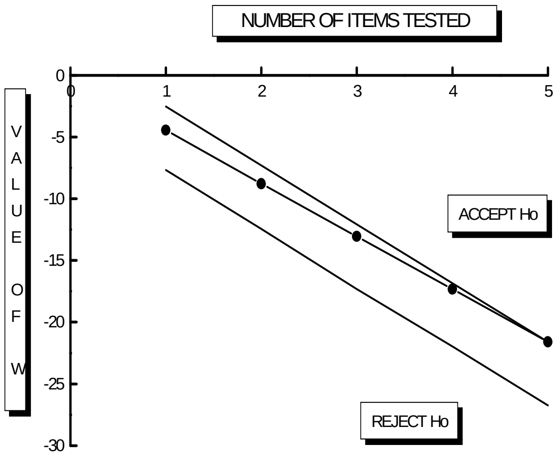

Fig. 1 Sequential test graph for the two-parameter Inverse Weibull model

Next, we modify the value of θ_1, the scale parameter of the alternative hypothesis, making it closer to the value of θ_0, the scale parameter of the null hypothesis. So, we choose the value of 555 hours for θ_1. Fig. 2 shows the results of this test.

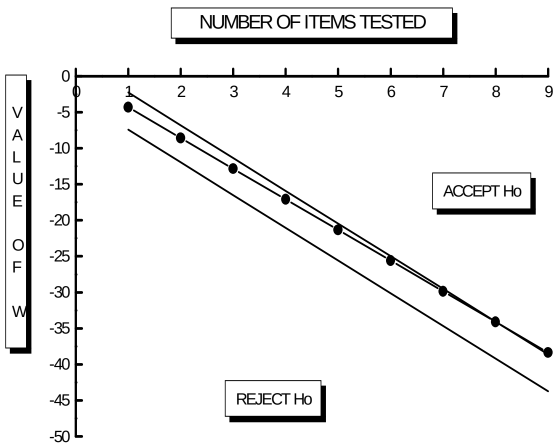

Fig. 2 Sequential test graph for the two-parameter Inverse Weibull model

The choice for the value of the alternative scale parameter (θ_1 = 555 hours) being closer to the value of the null hypothesis shape parameter (θ_0 = 581.22 hours) made it necessary to continue the test through 9 units, so a decision could be made to accept the null hypothesis.

Now, we decided to verify if a null scale parameter value relatively wrong will cause the null hypothesis to be rejected by this sequential life testing procedure. We choose the following values for the alternative and null scale parameters θ_1 and θ_0 (θ_1 = 540 hours; θ_0 = 520 hours). Fig. 3 shows the results of this test.

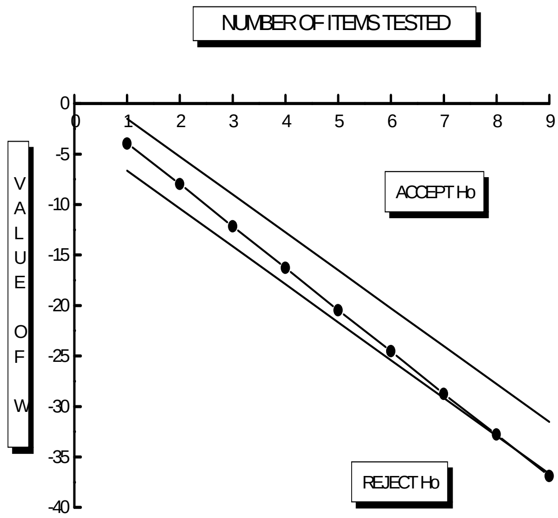

Fig. 3 Sequential test graph for the two-parameter Inverse Weibull model

In this third case, 9 units had to be life-tested to allow the decision of rejecting the null hypothesis H_0. A relatively poor choice of the value for the null scale parameter (θ_0 = 520 hours), caused this rejection. So, we can verify that this sequential life testing procedure is shown to be sensitive to "wrong" choices for the null scale parameter values.

Finally, we decided to verify if a null shape parameter value relatively wrong will cause the null hypothesis to be rejected by this sequential life testing procedure. We choose the following values for the alternative and null shape parameters (β_1 = 8.5; β_0 = 11). Fig. 4 shows the results of this test.

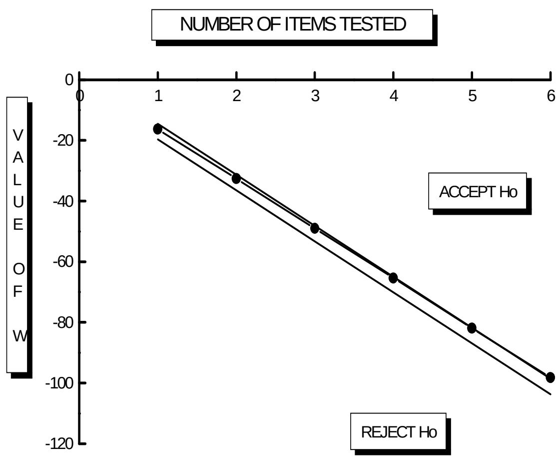

Fig. 4 Sequential test graph for the two-parameter Inverse Weibull model

In this last case with a null shape parameter value relatively wrong ($\beta_0 = 11$), after 6 units have been life testedwe still were able to make the decision of accepting the null hypothesis H_0. So, it seems that this sequential life testing procedure is shown not to be sensitive to "wrong" choices for the null shape parameter values when the underlying sampling distribution is the Inverse Weibull model.

V. Conclusions

The sequential life testing approach developed in this work provides rules for working with the null hypothesis H_0 in situations where the underlying sampling distribution is the Inverse Weibull model. After each observation one of three possible decisions is made:

1. Accept the null hypothesis H_0.
2. Reject the null hypothesis H_0.
3. Take one more observation.

In the example presented, we analyzed 4 different situations for the hypothesis testing considered in this paper.

Fig. 1 shows the sequential test results for the Inverse Weibull distribution, where $\beta_1 = 8.5$; $\theta_1 = 540$ hours; $\beta_0 = 9.14$; $\theta_0 = 581.22$ hours.In this first case it was necessary to use only 5 units of the product under analysis to reach the decision to accept the null hypothesis $H_{0.}$

In the second case, the test had to be continued through 9 units before a decision could be made to accept the null hypothesis. Fig. 2 shows the results of the test when $\beta_1 = 8.5$; $\theta_1 = 555$ hours;$\beta_0 = 9.14$; $\theta_0 = 581.22$ hours. We used the value of the alternative scale parameter ($\theta_1 = 555$ hours) because it is closer to the value of the null hypothesis scale parameter $\theta_0 = 581.22$ hours (the value we believe to be "true" for this parameter).

Fig. 3 shows the results of the test when we have $\beta_1 = 8.5$; $\theta_1 = 540$ hours; $\beta_0 = 9.14$; $\theta_0 = 520$ hours. In this third case, 9 units had to be life-tested to allow the decision of rejecting the null hypothesis H_0. A relatively poor choice of the value for the null scale parameter ($\theta_0 = 520$ hours) caused this rejection. So, we can verify that this sequential life testing procedure is shown to be sensitive to "wrong" choices for the null scale parameter values.

Finally, Fig. 4shows the results of the test when $\beta_1 = 8.5$;$\theta_1 = 540$ hours;$\beta_0 = 11$; $\theta_0 = 581.22$ hours. In this last casewith a null shape parameter value relatively wrong ($\beta_0 = 11$), after 6 units have been life tested we still were able to make the decision of accepting the null hypothesis H_0. So, it seems that this sequential life testing procedure is shown not to be sensitive to "wrong" choices for the null shape parameter values when the underlying sampling distribution is the Inverse Weibull model.

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