International Journal of Earth, Energy and Environmental Sciences
ISSN: 2517-942X
Vol:4, No:9, 2010

A Multi-objective Fuzzy Optimization Method of Resource Input Based on Genetic Algorithm

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***Abstract*—**With the increasing complexity of engineering problems, the traditional, single-objective and deterministic optimization method can not meet people's requirements. A multi-objective fuzzy optimization model of resource input is built for M chlor-alkali chemical eco-industrial park in this paper. First, the model is changed into the form that can be solved by genetic algorithm using fuzzy theory. And then, a fitness function is constructed for genetic algorithm. Finally, a numerical example is presented to show that the method compared with traditional single-objective optimization method is more practical and efficient.

***Keywords*—**Fitness function, genetic algorithm, multi-objective fuzzy optimization, satisfaction degree membership function.

I. Introduction

As the chlor-alkali chemical consumes a large number of resources and produces a lot of dangerous waste, which brings environmental and ecological problems, optimization method of resource input of chlor-alkali chemical is given more and more attention by designers. Good optimization method can be used to increase the overall optimization efficiency, and make the economic, environmental and energy objectives optimum.

In the optimization of resource input of chlor-alkali chemical, sometimes the objective functions or constraints are fuzzy. For example, the right hand side item of constraints often represents the limit of some resources. But sometimes may not absolutely can not exceed this limit. There is a allowable amount. When the amount of resources is between the limit and the allowable amount, the decision-maker's satisfaction degree decreases. Therefore, fuzzy theory is applied to the optimization of resource input of chlor-alkali chemical in this paper, and multi-objective fuzzy optimization method is used to give a feasible region, which can make the optimization staff have good choices.

Genetic algorithm (GA) as a global optimization algorithm, is robust, suitable for parallel processing, efficient, etc. It has been used to solve the optimization problem in many areas, such as power optimization, engineering design and some portfolio models[1].

In this paper, a multi-objective fuzzy optimization model of resource input is built for M chlor-alkali chemical eco-industrial park, which is solved by genetic algorithm. A numerical example is presented to show that the method compared with traditional single-objective optimization method is more practical and efficient.

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II. Mathematical Model of Multi-objective Fuzzy Optimization

In many practical problems, it's often expected that several indicators achieve optimal value at the same time, which is called multi-objective optimization problem. The general expression of the mathematical model is as follow:

$$\begin{cases} \min F(X) = [f_1(X), f_2(X), \cdots, f_m(X)]^T \\ \text{subject to:} \\ \quad g_j(X) \le 0 \quad j = 1, 2, \cdots, k \end{cases}$$

where X is decision variables, $F(X)$ the objective functions, and $g_j(X)$ the constraints.

When at least one of the decision variables, objective functions, and constraints exist ambiguity, the problem becomes multi-objective fuzzy optimization problem. When the constraints are fuzzy, multi-objective fuzzy optimization model is expressed as:

$$\begin{cases} \min F(X) = [f_1(X), f_2(X), \cdots, f_m(X)]^T \\ \text{subject to:} \\ \quad g_j(X) \subset G_j \quad j = 1, 2, \cdots, k \end{cases}$$

where fuzzy set G_j is the scope of $g_j(X)$ in the fuzzy sense[2].

III. The Multi-objective Fuzzy Optimization Model of Resource Input for M Chlor-alkali Chemical Eco-industrial Park

A. Overview of M Chlor-alkali Chemical Eco-industrial Park

There are chlor-alkali plant, cement plant, organic chlorine plant, sewage clarification plant, power plant, etc., in M chlor-alkali chemical eco-industrial park. In the park, the chlor-alkali plant is the core, and the industrial ecological communities of the park is composed of the core industry and its related ancillary industries. The resources that the chlor-alkali plant put into the production process include rock salt, calcium carbide and other raw materials, as well as coal, water, electricity and other energy. The main by-products and wastes include carbide slag, salt mud, liquid chlorine, CO_2 and wastewater. Carbide slag and salt mud are transferred to the cement plant for the production of cement. Liquid chlorine is transferred to the organic chlorine plant for the production of organic chlorine products. The plant's wastewater is transported to the sewage clarification plant. The park achieves energy saving, water saving, emission reduction, improving

International Journal of Earth, Energy and Environmental Sciences
ISSN: 2517-942X
Vol:4, No:9, 2010

resource utilization efficiency, and constructing cyclic economy development model through this kind of eco-industrial chain (see Fig. 1).

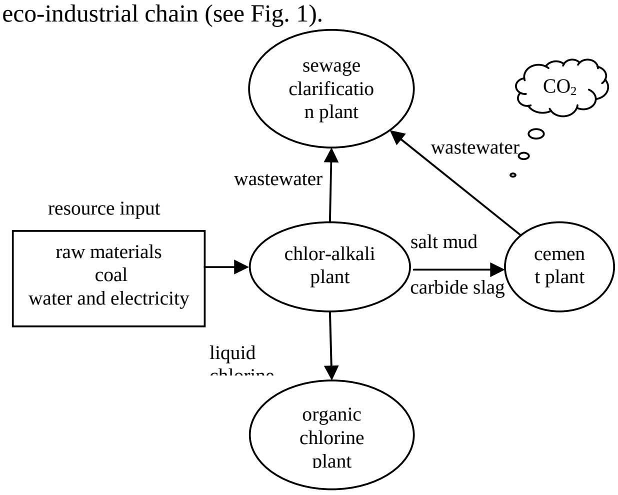

Fig. 1 M chlor-alkali chemical eco-industrial park

B. A Multi-objective Fuzzy Optimization Model of Resource Input

Economic objective was mainly paid attention to in the manufacturing process of M chlor-alkali chemical eco-industrial park in the past. With more and more pressing demands of sustainable development, in the optimization process the environmental objective and energy objective should also be considered to achieve the economic optimum, environmental impact minimum and energy consumption minimum. The economic objective mainly considers the production cost and minimizes it. Since M chlor-alkali chemical eco-industrial park builds a cyclic economy, solid and liquid wastes can be recycled. So the environment objective mainly considers CO_2 emissions and minimizes it. The energy objective mainly considers the consumption of raw materials, coal, water and electricity and minimizes it.

In the optimization of resource input, the single objective function is deterministic. So the ambiguity of the multi-objective fuzzy optimization problem is mainly reflected in the ambiguity of the multi-objective functions and constraints.

Let x_i ($i = 1, 2, \cdots, n$) be the proportion of type i resource input to the total resources input. Let $f_j(X)$ ($j = 1, 2, 3$) be the three objective functions, including economic objective, environment objective, and energy objective. Let p_i be the price coefficient of type i resource. Let c_i be the CO_2 emissions coefficient of type i resource. Let r_i be the conversion coefficient of type i resource. Let $g_k(X)$ be the fuzzy constraints that represent the permitted extent of objective functions[3]. Let R be the standard value of the upper limit of the total energy supply, S the standard value of the upper limit of the CO_2 emissions.

The multi-objective fuzzy optimization model is as follow:

$$\begin{cases} \min\ f_1(X) = \sum_{i=1}^{n} p_i x_i & \text{——economic objective} \\ \min\ f_2(X) = \sum_{i=1}^{n} c_i x_i & \text{——environment objective} \\ \min\ f_3(X) = \sum_{i=1}^{n} r_i x_i & \text{——energy objective} \\ \text{subject to:} & \\ \quad g_1(X) = \sum_{i=1}^{n} r_i x_i \underset{\sim}{\leq} R & \text{——energy constraint} \\ \quad g_2(X) = \sum_{i=1}^{n} c_i x_i \underset{\sim}{\leq} S & \text{——}CO_2 \text{ emissions constraint} \end{cases} \tag{1}$$

where the "~" indicates ambiguity.

IV. The Conversion of Multi-objective Fuzzy Optimization Model

Since the constraints are fuzzy, amplification coefficient method can be used to determine the tolerance of upper limit of the constraints. And satisfaction degree membership function is brought in the uncertain part of upper limit of the constraints. In order to obtain fuzzy objectives set, we can, firstly, find out the minimum and maximum of each objective function; then, draw the satisfaction degree membership function of each fuzzy objective; and at last, use level-cut method to transform the multi-objective fuzzy optimization model into the form that can be solved by genetic algorithm.

A. Amplification Coefficient Method

Amplification coefficient method is often used in engineering to determine the tolerance of upper limit of the constraints. It's a method that uses amplification coefficient (including upper amplification coefficient $\overline{\beta}$ and lower amplification coefficient $\underline{\beta}$) to determine the upper and lower bounds of the transition interval based on past general design specifications and design experience. Usually, let $\overline{\beta} = 1.05 \sim 1.30$, and $\underline{\beta} = 0.75 \sim 0.90$. In this paper, we use the upper amplification coefficient $\overline{\beta}$ to determine the tolerance of upper limit of the constraints.

B. Satisfaction Degree Membership Function of Fuzzy Constraints

According to experience, we can select the satisfaction degree membership distribution function as Fig. 2 for the uncertain part of upper limit of the energy constraint.

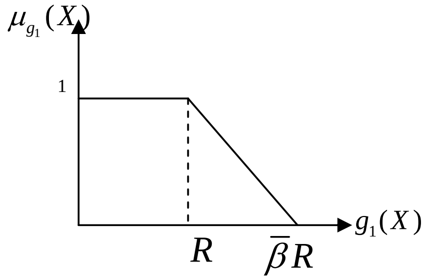

Fig. 2 Satisfaction degree membership distribution function of fuzzy constraints

The function is as follow:

$$\mu_{g_1}(X)=\begin{cases}1 & g_1(X)\le R\\ \dfrac{\bar{\beta}R-\sum_{i=1}^{n} r_i x_i}{\bar{\beta}R-R} & R<g_1(X)<\bar{\beta}R\\ 0 & g_1(X)\ge \bar{\beta}R\end{cases}.$$

Similarly, the membership function of the CO2 emissions constraint is as follow:

$$\mu_{g_2}(X)=\begin{cases}1 & g_2(X)\le S\\ \dfrac{\bar{\beta}S-\sum_{i=1}^{n} c_i x_i}{\bar{\beta}S-S} & S<g_2(X)<\bar{\beta}S\\ 0 & g_2(X)\ge \bar{\beta}S\end{cases}.$$

C. Satisfaction Degree Membership Function of Fuzzy Objectives

We can, firstly, use conventional optimization methods to find the possible optimal value $f_j(X_j^*)$ ($j=1,2,3$) and optimal solution X_j^* for each single objective function under the most relaxed constraints; then, substitute X_j^* into the rest of the single objective function and draw $f_l(X_l^*)$ ($l=1,2,3, l\neq j$); and at last, find out the possible minimum m_j and maximum M_j of each objective function.

The satisfaction degree membership distribution function of each objective is as Fig. 3.

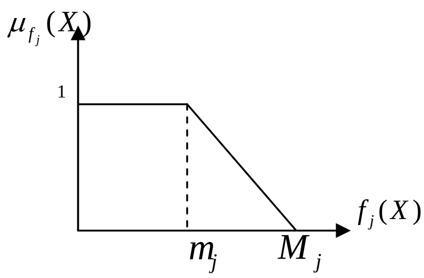

Fig. 3 Satisfaction degree membership distribution function of fuzzy objectives

The function is as follow:

$$\mu_{f_j}(X)=\begin{cases}1 & f_j(X)\le m_j\\ \dfrac{M_j-f_j(X)}{M_j-m_j} & m_j<f_j(X)<M_j\\ 0 & f_j(X)\ge M_j\end{cases}.$$

D. Level-cut Method

We can set a level $\lambda(0\le\lambda\le1)$ for the fuzzy objectives set to construct a level-cut. The level-cut is expressed as follow:

$$D_j(X)=\left\{X \mid \mu_{f_j}(X)\ge\lambda\right\} \quad (j=1,2,3).$$

According to fuzzy theory, the multi-objective fuzzy optimization model (1) can be transformed into:

$$\begin{cases} max\lambda\\ \text{subject to:}\\ \quad \dfrac{\bar{\beta}R-\sum_{i=1}^{n} r_i x_i}{\bar{\beta}R-R}\ge\lambda\\ \quad \dfrac{\bar{\beta}S-\sum_{i=1}^{n} c_i x_i}{\bar{\beta}S-S}\ge\lambda\\ \quad \dfrac{M_j-f_j(X)}{M_j-m_j}\ge\lambda\\ \quad i=1,2,\cdots n;\quad j=1,2,3\end{cases}$$

which can be solved by genetic algorithm.

V. DESIGN OF GENETIC ALGORITHM

A. The Basic Idea of Genetic Algorithm

Genetic algorithm is a random method which simulates some laws of nature to solve large-scale combinatorial optimization problems. It abandons the traditional search methods, simulates the natural process of biological evolution, uses artificial evolution methods to conduct random searches on the target space. With robust, global optimality, independent of the properties of the problem model, and parallel and high-efficiency, GA is more and more widely used in various fields with a unique appeal.

One of the important technical terms in GA is *chromosome*, which is usually a string of symbols or numbers. A chromosome is a coding of a solution of an optimization problem, not necessarily the solution itself. GA starts with an initial set of randomly generated chromosomes called a *population*. The number of individuals in the population is a predetermined integer and is called *population size*. All chromosomes are evaluated by the so-called *evaluation function*, which is some measure of *fitness*. A new population will be formed by a *selection process* using some *sampling mechanism* based on the fitness values. The cycle from one population to the next one is called a *generation*. In each new generation, all chromosomes will be updated by the *crossover* and *mutation* operations. The revised chromosomes are also called *offspring*. The selection process selects chromosomes to form a new population and the genetic system enters a new generation. After performing the genetic system a given number of cycles, we decode the best chromosome into a solution which is regarded as the optimal solution of the optimization problem [4].

B. Construction Of fitness Function

GA uses the probability theory to search based on the fitness function. The fitness function determines the direction of search. Whether the fitness function is appropriate directly determines whether the final optimal solution evolved by GA is consistent with the expectations of decision makers. Thus the selection of fitness function becomes the key point of GA to solve multi-objective optimization problem[5].

There are three main methods to determine the fitness of chromosome using satisfaction degree membership function, including maximizing the minimum satisfaction degree method, maximizing the weighted sum of satisfaction degree method, and maximizing the decision objectives method. The maximizing the minimum satisfaction degree method concerns that all the objectives have to achieve the membership degree as high as possible. The maximizing the weighted sum of satisfaction degree method is to fit when the importance of each membership degree is accurately known. The maximizing the decision objectives method pursues a maximum single decision objective. According to the structural characteristics of the model after transformation, we choose the maximizing the minimum satisfaction degree method. Let fit be the fitness function of chromosome, and it's expressed as follow:

$$fit = min\left\{\mu_g(X), \mu_{f_j}(X) \mid j = 1,2,3\right\}.$$

After constructing the fitness function of chromosome, we can use the traditional GA, generate initial feasible chromosomes randomly, use roulette wheel spinning selection method, and conduct the crossover and mutation operations to solve the model.

VI. A NUMERICAL EXAMPLE

To illustrate the superiority of the model and the algorithm, a numerical example is given and performed in personal computer. It's supposed that there are four types of resources for input, including rock salt, calcium carbide, coal, and water and electricity. The resources coefficients are shown in Table I. Let $R = 0.37$, $S = 0.78$, and $\overline{\beta} = 1.20$.

TABLE I
COEFFICIENT SETTING

Types of Resources	Price Coefficient (p_i)	CO_2 Emissions Coefficient (c_i)	conversion coefficient (r_i)
rock salt (x_1)	1.5	0.8	1.43
calcium carbide (x_2)	2.8	0.6	12.14
coal (x_3)	0.4	1	0.71
water and electricity (x_4)	1.7	0	3.27

The GA described in the paper is used to solve the problem, and programmed with VC++6.0. The parameters in GA are set as follows: the population size is 100, the probability of crossover is 0.80, the probability of mutation is 0.005, and the evolutionary generation is 50.

Ignoring the ambiguity of constraints, and only considering the economic objective, we can obtain the results of traditional single-objective optimization. Two kinds of optimization methods were compared in Table II.

TABLE II
COMPARISON OF TWO OPTIMIZATION METHODS RESULTS

Types of Resources	Traditional Single-objective Optimization Method (%)	Multi-objective Fuzzy Optimization Method Based on GA (%)
rock salt (x_1)	33.9	30.6
calcium carbide (x_2)	15.7	21.0
coal (x_3)	25.8	23.2
water and electricity (x_4)	24.6	25.3

From Table II, we can draw:

1) In practice, the traditional single-objective optimization method and the multi-objective fuzzy optimization method both can achieve a satisfactory solution, and the results were consistent. But the multi-objective fuzzy optimization method gives a feasible region, which can make the optimization staff have good choices; and it also takes into consideration the fuzzy factors, which is more in line with engineering practice.

2) In the algorithm, it's increasingly difficult for the conventional simplex algorithm to find feasible solutions with variable size larger. But the convergence speed of GA and the scale of the problem are the approximate linear relationship. And GA can give a set of optimal or more sets of sub-optimal results for decision makers. So when the variables are within a certain range, GA has more advantages.

VII. CONCLUSION

In practice, due to the optimization of resource input is affected by many factors, multi-objective fuzzy optimization method is used in this paper. The general optimization of resource input is changed into the solution of multi-objective fuzzy optimization model. And GA is used to solve the model to achieve global optimization purposes. The multi-objective fuzzy optimization method fully considers many factors in the optimization process of resource input, while GA overcomes the limitations of the traditional algorithm. The numerical example shows that the method compared with traditional single-objective optimization method is more practical and efficient.

ACKNOWLEDGMENT

The authors would like to thank the editor and the anonymous referees for their valuable comments and suggestion.

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