

Air-Filled Circular Cross Sectional Cavity for Microwave Non-Destructive Testing

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Abstract—Dielectric sheet perturbation to the dominant TE₁₁₁ mode resonant frequency of a circular cavity is studied and presented in this paper. The dielectric sheet, placed at the middle of the air-filled cavity, introduces discontinuities and disturbs the configuration of electromagnetic fields in the cavity. For fixed dimensions of cavity and fixed thickness of the loading dielectric, the dominant resonant frequency varies quite linearly with the permittivity of the dielectric. This quasi-linear relationship is plotted using Maple software and verified using 3D electromagnetic simulations. Two probes are used in the simulation for wave excitation into and from the cavity. The best length of probe is found to be 3 mm, giving the closest resonant frequency to the one calculated using Maple. A total of fourteen different dielectrics of permittivity ranging from 1 to 12.9 are tested one by one in the simulation. The works show very close agreement between the results from Maple and the simulation. A constant difference of 0.04 GHz is found between the resonant frequencies collected during simulation and the ones from Maple. The success of this project may lead to the possibility of using the middle loaded cavity at TE₁₁₁ mode as a microwave non-destructive testing of solid materials.

Keywords—Middle-loaded cavity, dielectric sheet perturbation.

I. INTRODUCTION

UNIFORM CROSS - SECTIONAL CAVITIES are simply short circuited waveguides at both ends. Representing a very important group of microwave components, their applications are vast and range from the use as frequency meters and filters, to tools for material characteristics measurements [1 – 16, 18]. The use of cavity for measuring properties of dielectric materials has been reported in 1968 by Stinehelfer and it has been improved by Itoh in 1974 [1]. The simplicity of cavity design has been applied in microwave filtering even in a complex synthesis of elliptical filter [2]. The use of dielectric of high permittivity as a resonator is advised in reducing the loss and the dimensions of the filter. It is then interesting to analyze the dependency of the cavity on a dielectric perturbation as this can be used in the filtering application and the microwave non-destructive testing of materials.

Nowadays electromagnetic cavity has been one of the most popular tools in measuring materials' properties. Various techniques have been invented in using cavities for this purpose in the last 30 years and yet the more and more models and improved methods are presented. This is mainly due to the existence of infinity of resonance frequencies in a single cavity, representing every one of them its own mode. The mode with the lowest resonance frequency is called the fundamental mode and the rest are

called the higher modes. Every single mode of the cavity has in general its own distribution or form of fields in the cavity, giving birth to infinity of possibilities in introducing the material whose properties are to be measured. One of the most popular approaches is by using the bottom loaded cavity, where the material to be tested is inserted at one end of the cavity [3 – 15]. Open resonator/cavity has been used in the precision permittivity and loss-tangent measurement at very high frequencies [8-9]. An even more interesting technique is to use a coaxial resonator cavity where properties of liquid materials have been measured by Raveendranath et al.

As mentioned earlier, various ways have been presented in introducing dielectric to be measured into the cavity, such as the use of bottom-loaded cavity [3-15], open resonator/cavity [8-9] and Coaxial Resonator Cavity [10-12]. It is also important to highlight that the choice of mode will directly influence the exact place where the perturbation dielectric should be introduced in the cavity. The best place to insert the dielectric in this case is at the middle of half-part of the cavity (say at the first quarter or the third quarter of the cavity). This way, the presence of the dielectric will force the electric field at that point to fulfill the conditions at the materials boundaries, hence the change of the resonant frequency of the TE₁₁₂ mode. Another popular technique is to introduce the dielectric at the middle of the cavity. Gang Zhang et al. has reported the use of this technique where waveguides have been used to excite and collect the wave into and from the cavity [19]. The mode used was a higher mode TE₀₁₁, the same mode being used by Gordon in his experiments for permittivity measurements of low loss dielectrics [19].

In this paper, the cavity that will be used is an air-filled circular cross sectional cavity with metallic wall. The dimensions are set such as to have an unloaded fundamental resonant frequency at about 4 GHz. Two probes are used to excite the fundamental TE₁₁₁-mode wave into the cavity to give a resonant frequency at 4.206 GHz. These probes will be placed at the first quarter and the third quarter along the axis of the cavity. The dependency of the resonance frequency on the probes' lengths is studied to have the best choice of probes' lengths in achieving the most accurate results with less loss. The placement of the perturbation or insertion will be at the center of the cavity and the dielectric sheet is chosen among the materials available in the market. The work started with the equation relating the resonant frequency of the mode chosen to the characteristic of the dielectric sheet perturbation. The resonant frequency variation towards the dielectric sheet permittivity is visualized and plotted using mathematical software. The validity of the equation can then be verified using a 3D electromagnetic simulation software.

Understanding the variation of the resonant frequency towards the known perturbation allows one to tune the

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resonant frequency of the cavity only by choosing the correct dielectric as the perturbation. No change in the dimensions of the cavity will then be required. Another direct application of this is that the characteristics of the dielectric perturbation can be determined if the resonant frequency is known - hence the idea of using the cavity in the Microwave Non-Destructing Testing in determining the dielectric constant of a dielectric.

II. THE CAVITY AND THE DIELECTRIC SHEET PERTURBATION

A. The Unloaded Cavity

A circular cross sectional cavity is formed by closing both ends of a circular waveguide with plates, as shown in Fig. 1. Coupling in and out the cavity can be done using irises or probes. The analysis can be done by modifying the traveling waves of the z variations of the circular waveguide in order to obtain the standing waves of the cavity. A diagram of the cavity is as shown in Fig. 1 in where both TE and TM modes can exist.

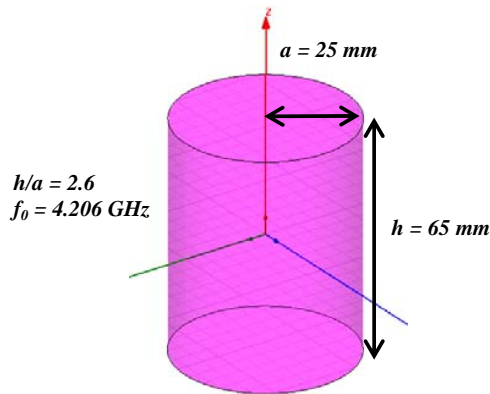


Fig. 1 The unloaded cavity with dimensions

The resonant frequencies of the TM_{mnp} and TE_{mnp} modes are functions of the h/a ratio. The TE_{mnp} mode with the smallest resonant frequency is the TE₁₁₁, and its cutoff frequency is given by:

$$(f_r)_{111}^{TEZ} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{1.8412}{a}\right)^2 + \left(\frac{\pi}{h}\right)^2} \quad (1)$$

Similarly, the TM_{mnp} mode with the smallest resonant frequency is the TM₀₁₀, and its cutoff frequency is given by:

$$(f_r)_{010}^{TMZ} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \left(\frac{2.4049}{a}\right) \quad (2)$$

Where f_r : resonant frequency

a : radius of the cavity

h : length of the cavity

μ : permeability of the dielectric filling the cavity

ϵ : permittivity of the dielectric filling the cavity

Equating both above equations, an identical frequency is found when

$$\frac{h}{a} = 2.03 \approx 2 \quad (3)$$

When $h/a < 2.03$, the dominant or fundamental mode is the TM₀₁₀ whereas for $h/a > 2.03$, the dominant mode is the TE₁₁₁ mode. The dimensions of the cavity to be studied in this project are fixed such that to have the dominant mode resonant frequency at 4.206 GHz, which is of TE₁₁₁ mode. In this case, the length h and the radius a of the cavity are fixed to have the ratio of h/a which is bigger than 2.03. The mode to be analyzed is than the TE₁₁ mode. As mentioned earlier, infinity of modes would coexist in the cavity, and Table I shows some lowest modes and their resonant frequencies.

TABLE I
THE RESONANT FREQUENCIES OF SOME LOWER MODES OF THE UNLOADED CAVITY

Mode	Resonant Freq (GHz)
TE ₁₁₁	4.206
TM ₀₁₀	4.593
TM ₀₁₁	5.140
TE ₁₁₂	5.802
TE ₂₁₁	6.273
TM ₀₁₂	6.511
TM ₁₁₀	7.318
TE ₂₁₂	7.438
TM ₁₁₁	7.673

B. The Middle-Loaded Cavity

The analysis of a middle-loaded cavity is done by dividing the loaded cavity into three main regions, I, II and III as shown in Fig. 2. The electromagnetic fields are then solved in the cavity, verifying the boundary conditions at the dielectric-air, dielectric-metal and air-metal boundaries.

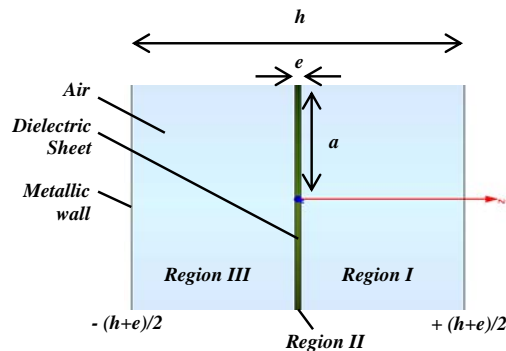


Fig. 2 Loaded cavity divided into three regions

The equation relating the properties of the dielectrics in the three regions to all the dimensions of the configuration is given by [2, 21]:

$$\beta_{II} \tan\left(\beta_{II} \cdot \frac{e}{2}\right) = \beta_I \cdot \cot\left(\beta_I \cdot \frac{h}{2}\right) \quad (4)$$

where $\beta_{II} = \sqrt{\epsilon_r \cdot \frac{\omega^2}{c^2} - k_c^2}$ and, $\beta_I = \sqrt{\frac{\omega^2}{c^2} - k_c^2}$ being the permittivity of the dielectric sheet inserted in the cavity. For the reason of simplicity of the numerical calculation, the above equation is written as follow (for TEMn modes) where χ'_{mn} is the m^{th} root of Bessel function of n^{th} order.

$$\sqrt{\epsilon_r \cdot \frac{\omega^2}{c^2} - \frac{\chi'_{mn}{}^2}{a^2}} \cdot \tan\left[\frac{e}{2} \sqrt{\epsilon_r \cdot \frac{\omega^2}{c^2} - \frac{\chi'_{mn}{}^2}{a^2}}\right] - \sqrt{\frac{\omega^2}{c^2} - \frac{\chi'_{mn}{}^2}{a^2}} \cdot \cot\left[\frac{h}{2} \sqrt{\frac{\omega^2}{c^2} - \frac{\chi'_{mn}{}^2}{a^2}}\right] = 0 \quad (5)$$

It is clearly seen from the above equation that for fixed value of frequency, the permittivity of the dielectric sheet can be determined, and vice versa. The equation now is keyed into Maple to solve the dominant resonant frequency for a given value of permittivity. Fig. 3 shows, as an example, the curve of the equation (5) versus the frequency, for a chosen ϵ_r equals 2.1. The frequency where the curve cuts the x-axis is the resonant frequency of the TE111 mode (the fundamental mode) of the loaded cavity.

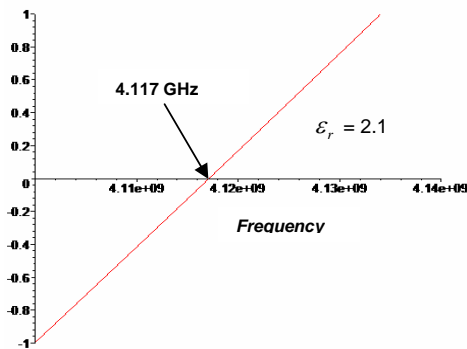


Fig. 3 Graphical solution of the equation (1)

It is sufficient to change the value of ϵ_r of the dielectric used to solve the corresponding resonant frequency. A number of dielectrics with different ϵ_r are then tested with this calculation to find for each of them the resulting resonant frequencies to be compared with the simulation. Fig. 4 shows the graph of ϵ_r versus the resonant frequency of the fundamental mode of the cavity. The graph shows that the permittivity of the dielectric inserted in the cavity varies quite linearly with the resonant frequency. In the next paragraph, a set of simulations will be held to verify the results found using Maple concerning the equation that has been presented earlier.

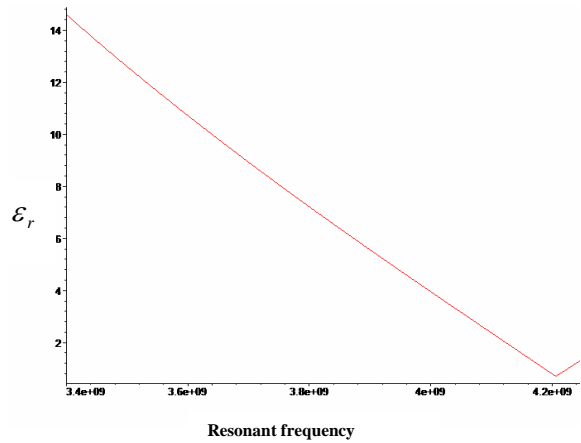


Fig. 4 Graph of permittivity versus resonant frequency from MAPLE

III. THE ELEGROMAGNETIC 3D SIMULATION

Fig. 5 shows the simulated middle-loaded cavity. Two probes are drawn and placed at the first and the third quarter of the cavity to introduce and to collect into and from the cavity. The properties of the probes (such as their dimensions) are taken from the existing probes which are the stub contact panel sockets.

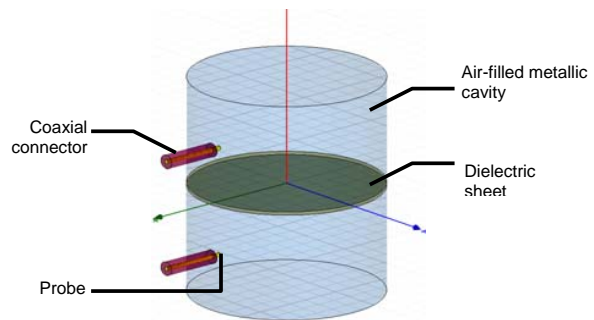


Fig. 5 Probes introduction into the cavity

It is known that for the resonant frequency calculated in the exact formula given earlier, the cavity has been considered as perfect, which means there is no parasite element such as the probe length, to disturb the resulting the resonant frequency. It has even been shown in [17] that the resonant frequency might vary with the length of the inserted probes. This leads to the importance of simulating the cavity to be tested using some different length of probes to help in choosing the correct probe length such that the simulation results will be as closed as the one had been calculated using Maple. Five probe lengths are tested in the simulation to give the $|S_{21}|_{\text{dB}}$ level and the resonant frequency for unloaded cavity. The results are given in Table II and plotted in Fig. 6.

TABLE II
PROBE LENGTHS AND THE CORRESPONDING $|S_{21}|$ dB AND f_0

Probe Length (mm)	$ S_{21} $ dB	f_0 (GHz)
1.5	13.60	4.240
2.0	11.83	4.230
2.5	3.70	4.226
3.0	3.47	4.222
4.0	0.75	4.230

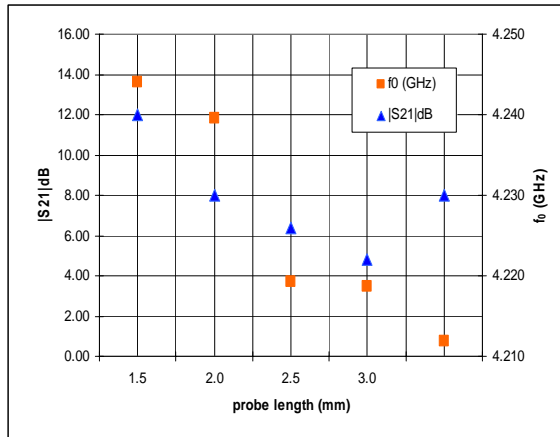
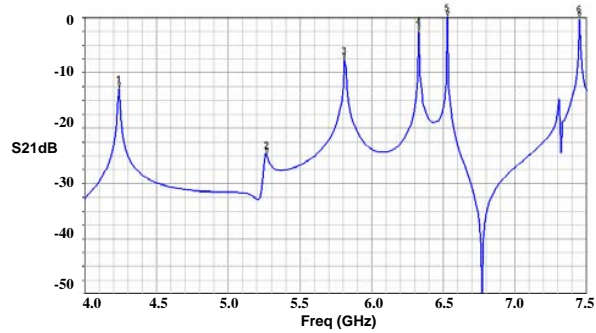


Fig. 6 Plot of $|S_{21}|$ dB and f_0 versus probe lengths

It is seen from the above results that the best probe length in giving the less loss in transmission is 4 mm, where only 0.75 dB has been detected for $|S_{21}|$. Meanwhile, the 3-mm probes have shown the best result in term of the resonant frequency where it gives the closest resonant frequency (4.222 GHz) to the one calculated (which is at 4.206 GHz). Considering the importance of the correct resonant frequency is more than of the loss (note that the quantity that helps in identifying the permittivity of the loading dielectric is the resonant frequency), the 3-mm probes are chosen to be used during all the rest of the simulations.

The first simulation is done to observe the resonant frequency of the TE₁₁₁-mode when the cavity is unloaded. In the same time, other resonant frequencies of other modes are also visualized and analyzed. All resonant frequencies are collected and the wave form of each existing mode (from 4.00 to 7.50 GHz) is studied and compared with what have been presented earlier. Fig. 7 shows $|S_{21}|$ dB versus frequency, indicating the resonant frequencies of the first lowest modes ranging from 4.00 to 7.50 GHz.



TE ₁₁₁	TM ₀₁₁	TE ₁₁₂
X1 = 4.24 GHz	X2 = 5.27 GHz	X3 = 5.81 GHz
Y1 = -25.81 dB	Y2 = -49.76 dB	Y3 = -15.53 dB

TE ₂₁₁	TM ₀₁₂	TE ₂₁₂
X4 = 6.33 GHz	X5 = 6.53 GHz	X6 = 7.45 GHz
Y1 = -5.12 dB	Y1 = -0.13 dB	Y1 = -0.72 dB

Fig. 7 Resonant frequencies of some lower modes in the unloaded cavity

This first run of simulation gives the fundamental TE₁₁₁-mode reasonable resonant frequency at 4.24 GHz, but with enormous transmission loss of 25 dB. Other modes that can be observed are TM₀₁₁, TE₁₁₂, TM₀₁₂ and TE₂₁₂. The following diagrams (Fig. 8, Fig. 9) show the field forms or configurations in the cavity for TE₁₁₁-mode detected during this first simulation.

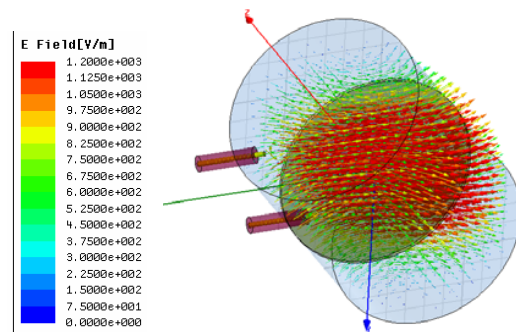


Fig. 8 E-field of TE₁₁₁ of unloaded cavity

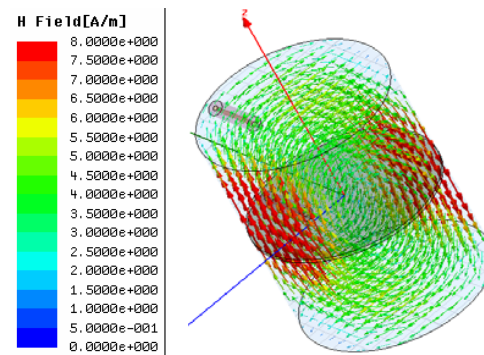


Fig. 9 H-field of TE₁₁₁ of unloaded cavity

TABLE III

TABLE OF RESULTS FROM SIMULATION AND CALCULATION USING MAPLE

No.	Dielectric	Er	loss tan	f_0 (Sim) GHz	f_0 (Maple) GHz	delta f_0 GHz	S21 dB
m1	Air	1.00	0.0000	4.222	4.187	0.035	3.474
m2	Teflon (tm)	2.10	0.0010	4.161	4.118	0.043	8.980
m3	Arlon AD 250 (tm)	2.50	0.0030	4.135	4.092	0.043	12.294
m4	Neltec NH9300 (tm)	3.00	0.0023	4.104	4.060	0.044	14.798
m5	Neltec NH9350 (tm)	3.50	0.0030	4.069	4.028	0.041	17.126
m6	Silicon_dioxide	4.00	0.0000	4.042	3.998	0.044	8.873
m7	Rogers TMM 4 (tm)	4.50	0.0020	4.011	3.966	0.045	15.156
m8	Glass	5.50	0.0000	3.945	3.904	0.041	7.338
m9	Rogers TMM 6 (tm)	6.00	0.0023	3.916	3.874	0.042	19.947
m10	Silicon_nitrite	7.00	0.0000	3.853	3.813	0.040	5.932
m11	Marble	8.30	0.0000	3.775	3.736	0.039	7.349
m12	Al2_O3_ceramic	9.80	0.0000	3.687	3.651	0.036	12.299
m13	Silicon	11.90	0.0000	3.571	3.537	0.034	15.563
m14	gallium_arsenide	12.90	0.0000	3.518	3.485	0.033	5.282

The next simulations concern the observation of the TE₁₁₁-mode resonant frequency variation as a function of the permittivity of the dielectric sheet insertion. Fourteen dielectrics with different permittivity ranging from 1 (air) to 12.9 are simulated one by one for their dominant mode resonant frequencies and |S₁₁|dB level. Table III shows all the resonant frequencies for the different dielectrics collected from calculation using Maple and simulations, with the corresponding level of |S₂₁|dB. The variation of the resonant frequency in terms of the dielectric permittivity is plotted in Fig. 10.

From Fig. 10, both curves from simulation and Maple are found to be similar in terms of their shape. The same shape has had even been drawn during the demonstration using Maple (Fig. 4). In average, a translation of 0.04 GHz along the y-axis of the Maple curve will superpose it on the simulation curve. This error of 0.04 GHz made during the simulation might be suggested to be due to the parasitic elements presented by the presence of the probes for wave excitation. The comparison shows then very close results obtained from the simulations and calculation using Maple.

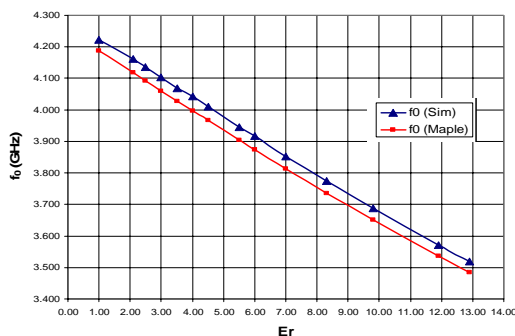


Fig. 10 Variation of f_0 against the permittivity – simulation and Maple

IV. CONCLUSION

Electromagnetic cavity has been one of the most popular tools in measuring materials' properties. Various techniques have been invented in using cavities for this purpose, encouraged by the simplicity of its use. Middle loaded cavity in the dielectric measurement has the advantage of having a relatively simple and direct calculation and equation in determining the relationship between the resonant frequency and the dielectric properties. Furthermore, as shown in the calculation and simulation, the frequency-permittivity relation is quasi-linear as long as all the dimensions of the cavity and the dielectric are fixed. This leads to the easiness in the analysis and experiment. But fixing the dimensions of the dielectric such as its thickness gives us less liberty in the preparation of the samples. Another drawback is that determining the way to hold the dielectric sheet in the middle of the cavity could become quite delicate. In this project, the dielectric sheets being tested are all in the form of plane having surface area bigger than the cross sectional circular surface of the cavity. The dielectric is then put in sandwiched by the halves of the cavity. But this technique may present quite a noticeable level of loss at the cavity-dielectric-cavity discontinuity.

Starting from the equation relating the resonant frequency of the loaded cavity to the permittivity of the loading dielectric, a curve has been plotted using Maple in this project showing their linear relationship. In the electromagnetic simulations, the fixed-dimension metallic circular cavity in question resonates its dominant TE₁₁₁ mode at 4.222 GHz without load, which is slightly different than what has been found in calculation, 4.026 GHz. The 3 mm probe length has been chosen for the wave excitation due to the close results it gave to the one calculated. Some resonant frequencies of the lowest modes have been observed and their field configurations have been verified and compared with theory. Thirteen different dielectric samples have been inserted one by one at the middle of the cavity for the shifted resonant frequency. The results from the simulations are compared to the ones from the calculation using Maple. It is found that an amount of 0.04 GHz in average separates the frequencies coming from simulations to the frequencies calculated using Maple. This small and constant error allows this technique to give quite accurate measurements of permittivity of solid dielectrics.

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