

# Precision Identification of Nonlinear Damping Parameter for a Miniature Moving-Coil Transducer

Yu-Ting Tsai, Yu-da Lee, and Jin H. Huang

**Abstract**—The nonlinear damping behavior is usually ignored in the design of a miniature moving-coil loudspeaker. But when the loudspeaker operated in air, the damping parameter varies with the voice-coil displacement corresponding due to viscous air flow. The present paper presents an identification model as inverse problem to identify the nonlinear damping parameter in the lumped parameter model for the loudspeaker. Theoretical results for the nonlinear damping are verified by using laser displacement measurement scanner. These results indicate that the damping parameter has the greatly different nonlinearity between in air and vacuum. It is believed that the results of the present work can be applied in diagnosis and sound quality improvement of a miniature loudspeaker.

**Keywords**—Miniature loudspeaker, non-linear damping, system identification, Lumped parameter model.

## I. INTRODUCTION

IN the past, the frequency response and the distortion characteristics of the moving-coil loudspeaker are often analyzed and simulated using the lumped parameter model (LPM) of a moving-coil transducer (MT) [1]. In practical, the nonlinear distortion of a MT is often illustrated by nonlinear parameters of electrical inductance  $L_e(x)$ , mechanical stiffness  $K_m(x)$ , and force factor  $Bl(x)$ , which have the nonlinearity variation [2], [3] depended on the voice-coil displacement  $x$  to describe nonlinear vibration characteristics of a MT. Earlier to references [4]-[8], the solution of the nonlinear LPM was tried by Volterra series expansion considering nonlinear behavior of  $K_m(x)$ ,  $L_e(x)$ ,  $Bl(x)$  at low frequencies in [9]. These parameters were modeled by a truncated power series to obtain 2<sup>th</sup> and 3<sup>rd</sup>-order harmonics and intermodulation distortions. However, Klippel [10] proposed that only the  $K_m(x)$ ,  $L_e(x)$ ,  $Bl(x)$  assumed to be nonlinearity cannot explain some particularities found in the miniature moving-coil transducer (MMT), such as mechanical and acoustic losses and highly distortion in 3<sup>rd</sup>-order harmonic components versus frequencies. For MMT, due to its small size, tiny structure, and subtle displacement of cone vibration, the viscous air flow in the gap and at leaks may

generate a nonlinear term of mechanical damping corresponding to the symmetrical of asymmetrical direction of the air flow. Thus the nonlinearity originating from the nonlinear damping  $R_m(x)$  may be the dominant factor responsible for highly distortion in MMT.

In order to identify the physical parameters in LPM, the system identification method [11]-[15] has been used as the main method for measuring the parameters of a MT. The system identification model requires the input signal, voice-coil current, and voice-coil displacement as input data. For measuring the voice-coil displacement is to use an optical linear displacement sensor [16], [17]. Along with the voice-coil displacement and the known values of physical parameters determined by system identification method, this paper develops an extended identification model to identify the coefficients of damping  $R_m(x)$ . The proposed method as an inverse problem of LPM which involves separate calculation procedures: (a) Nonlinear conjugate gradient method (CGM) [18], as the iterative optimization method for identifying the coefficients of damping  $R_m(x)$ . (b) Direct problem, as the numerical solutions of differential equations in model, which derived from LPM of MMT. (c) Adjoint problem, The Lagrange equations for getting gradient direction of CGM. (d) Sensitivity problem, for obtaining the search step of CGM.

This article is organized as follows. Section II presents the mathematical framework of MMT. Section III contains the description of the construction of the presented method and its calculation steps. In Section IV, numerical simulation and the experiment are presented to verify the accuracy of the proposed method. In addition, the result indicates that the  $R_m(x)$  has the different nonlinearity variation between in air and vacuum. Finally, related work is summarized in Section V.

## II. THE MATHEMATICAL FRAMEWORK OF MMT

A typical MMT, as depicted in Fig. 1, consists of a magnetic system (magnet under yoke and polar piece) and a vibration system (diaphragm and voice coil). The magnetic system of MMT transfers electrical-to-magnetic force to drive the voice coil, and that a diaphragm suspension system is used to generate vibration, causing the voice-coil to suspend and begins to vibrate. According to the LPM, the governing equations of MMT including the mechanical damping  $R_m(x)$  as a nonlinear function of voice-coil displacement  $x$  can be written as

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$$M_m \frac{d^2x(t)}{dt^2} + R_m(x) \frac{dx(t)}{dt} + K_m(x)x = Bl(x)i(t) - \frac{dL_e(x)}{dx} \frac{i(t)^2}{2} \quad (1)$$

$$L_e(x) \frac{di(t)}{dt} + (R_e + \frac{L_e(x)}{dx} \frac{dx}{dt})i(t) + Bl(x) \frac{dx(t)}{dt} = e(t) \quad (2)$$

where  $e(t)$  is the input voltage. The nonlinear parameters  $R_m(x)$ ,  $Bl(x)$ ,  $K_m(x)$ , and  $L_e(x)$  can be represented by Nth- order polynomials as:

$$R_m(x) = \sum_{j=0}^N r_j x^j \quad (3)$$

$$Bl(x) = \sum_{j=0}^N b_j x^j \quad (4)$$

$$K_m(x) = \sum_{j=0}^N k_{m,j} x^j \quad (5)$$

$$L_e(x) = \sum_{j=0}^N a_j x^j \quad (6)$$

where  $r_j$ ,  $k_j$ ,  $b_j$ , and  $a_j$  are coefficients.

If the initial conditions of the input signal, voice-coil displacement, current are known, and the parameters  $M_m$ ,  $K_m(x)$ ,  $Bl(x)$ ,  $L_e(x)$ ,  $R_e$  are given, then one can solve (1) and (2) for the voice-coil displacement  $x(t)$ , velocity  $dx(t)/dt$ , acceleration  $d^2x(t)/dt^2$ , and current  $i(t)$ . This problem is called a well-posed problem, also known as the direct problem, and its solution is called a direct solution. Conversely, at any time interval  $t \in (0, t_f)$ , if the input voltage  $e(t)$ , the voice-coil displacement  $x(t)$ , and the voice-coil current  $i(t)$  are known, then (1) and (2) can be used to find the specific electroacoustic parameters, such as the non-linear damping  $R_m(x)$ , which is the unknown parameter in this study. This problem may be labeled an ill-posed problem, and its solution is known as an inverse solution. This is also the inverse problem explored in proposed method.

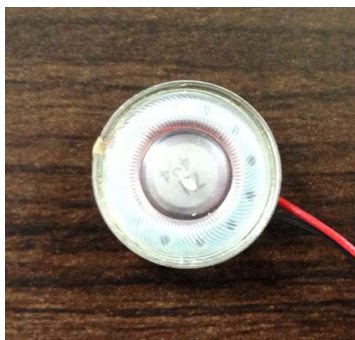


Fig. 1 A miniature moving-coil transducer

### III. INVERSE PROBLEM

Through the measured value  $x_{mea}(t)$  and estimated value  $x(t)$ , the objective function  $J$  can be defined as

$$J(\mathbf{w}) = \int_0^{t_f} [x(t; \mathbf{w}) - x_{mea}(t)]^2 dt \quad (7)$$

where  $\mathbf{w} = [r_0 r_1 \dots r_N]^T$  is a unknown vector to be determined. The above equation reveals that when the objective function  $J$  is at minimum value, the estimated value  $x(t)$  will approach the measured value  $x_{mea}$ . Solving the unknown  $R_m(x)$  in (1) as it gradually moves toward the minimum value will obtain the solution for an optimal set of  $R_m(x)$  through iterations. Therefore, the nonlinear conjugate gradient method is designed to optimize by repeated iteration and which leads to objective function minimization. The iterative equation is

$$\mathbf{w}^{(k+1)} = \mathbf{w}^{(k)} - \beta^{(k)} \mathbf{P}^{(k+1)} \quad (8)$$

Here the superscript  $k$  represents the  $k^{th}$  iteration,  $\beta^{(k)}$  denotes the  $k^{th}$  search step length, and  $\mathbf{P}^{(k+1)}$  is the  $(k+1)^{th}$  search direction with decreased value:

$$\mathbf{P}^{(k+1)} = \nabla J^{(k)} + \gamma^{(k)} \mathbf{P}^{(k)} \quad (9)$$

where  $\nabla J^{(k)}$  represents the gradient of the objective function at the  $k^{th}$  iteration. The conjugate gradients update parameter  $\gamma^{(k)}$  is given by

$$\gamma^{(k)} = \frac{\|\nabla J^{(k)}\|^2}{\|\nabla J^{(k-1)}\|^2} = \frac{\sum_{j=0}^{N_k} \left( \frac{\partial J^{(k)}}{\partial r_j} \right)^2}{\sum_{j=0}^{N_k} \left( \frac{\partial J^{(k-1)}}{\partial r_j} \right)^2} \quad (10)$$

Note that, when descending direction does not take into account  $\gamma^{(k)} \mathbf{P}^{(k)}$ , then  $\mathbf{P}^{(k+1)} = \nabla J^{(k)}$  in (9). At this point, CGM will degenerate into steepest decent method.

During the convergence process of CGM, the voice-coil displacement  $x(t)$ , the gradient of objective function  $\nabla J$ , and the step length  $\beta$  must be solved. They are respectively the solutions of the direct problem, adjoint problem, and sensitivity problem, which will be explained in following subsections.

#### A. Solving Direct Problem for $x(t)$

Before solving the voice-coil displacement  $x(t)$  from differential equations (1) and (2), arbitrary entries in the unknown vector  $\mathbf{w}$  are given first and then apply the hybrid spline difference method (HSDM) [19], [20] to solve the differential equations. In view of HSDM, which has the characteristics of the ability to achieve second-order derivative function within the accuracy of up to  $O(\Delta t^2)$ , and its calculations are as simple and easy to implement as finite difference method. Therefore, the study uses the following discrete equations to discretize the differential equation of (1):

$$Bl(x)i - \frac{dL_e(x)}{dx} \frac{i(t)^2}{2} = M_m \frac{p^{n-1} - 2p^n + p^{n+1}}{\Delta t^2} + R_m(x) \left( \frac{p^{n+1} - p^{n-1}}{2\Delta t} - \Delta \dot{x}^n \right) + K_m(x) \frac{p^{n-1} + 10p^n + p^{n+1}}{12} \quad (11)$$

where  $p$  and  $n$  are respectively the calculation index of the spline parameters and calculation grid index on the time axis. The above equation can be further rearranged into of the compact form:

$$T_A p^{n-1} + T_B p^n + T_C p^{n+1} = T_D \quad (12)$$

where the coefficients  $T_A$ ,  $T_B$ ,  $T_C$  and  $T_D$  are,

$$T_A = \frac{K_m(x)}{12} + \frac{M_m}{\Delta t^2} - \frac{R_m(x)}{2\Delta t} \quad (13)$$

$$T_B = \frac{K_m(x)}{6} - \frac{2M_m}{\Delta t^2} \quad (14)$$

$$T_C = \frac{K_m(x)}{12} + \frac{M_m}{\Delta t^2} + \frac{R_m(x)}{2\Delta t} \quad (15)$$

$$T_D = Bl(x)i(t) - \frac{dL_e(x)}{dx} \frac{i(t)^2}{2} + R_m(x)\Delta \dot{x}^n \quad (16)$$

Since  $p^{n-1}$  and  $p^n$  are known values, the above equation can be used for rapid iteration to obtain the size of the new  $p^{n+1}$  for calculating the discrete function  $x(t^n)$  and its first- and second-order derivatives.

#### B. Solving Adjoint Problem for $\nabla J$

Using Lagrange multiplier method to multiply (1) by a Lagrange multiplier  $\lambda(t)$ , and substituting it into the objective function in (7) to obtain a new objective function. The adjoint problem is established to solve the Lagrange multipliers  $\lambda(t)$  as

$$M_m \frac{d^2 \lambda(t)}{dt^2} - \left[ \sum_{j=0}^N (j r_j x(t)^{j-1}) x(t) + R_m(x) \right] \frac{d \lambda(t)}{dt} + \lambda(t) K_m(x) = -2 [x(t) - x_{mea}(t)] \quad (17)$$

$, t \in (t_f, 0)$

subjected to

$$\lambda(t_f) = 0 \quad \text{and} \quad \left. \frac{d \lambda(t_f)}{dt} \right|_{=0} \quad (18)$$

Then gradient of the objective function can be readily obtained as,

$$\nabla J = \left[ \frac{\partial J}{\partial r_0} \dots \frac{\partial J}{\partial r_N} \right]^T \quad (19)$$

$$= \left[ \int_0^{t_f} \lambda(t) \dot{x}(t) x(t)^0 dt, \dots, \int_0^{t_f} \lambda(t) \dot{x}(t) x(t)^N dt \right]^T$$

Once  $\nabla J$  is solved, the search direction  $\mathbf{P}^{(k+1)}$  is readily solved.

#### C. Solving Sensitivity Problem for Step Length $\beta$

After the search direction  $\mathbf{P}$  is obtained, the step length  $\beta$  needs to be determined in order to find the next better search result. The step length  $\beta$  is hence obtained as,

$$\beta = \frac{\int_0^{t_f} [x(t) - x_{mea}(t)] \delta x(t) dt}{\int_0^{t_f} \delta x^2(t) dt} \quad (20)$$

As for the subtle change  $\delta x$  in  $x$ , perturbation can be applied to (1) with a subtle change. A sensitivity problem can be obtained by substituting these subtle changes into (1) as

$$M_m \frac{d^2 \delta x(t)}{dt^2} + \left[ R_m(x) + x(t) \sum_{j=0}^N (j r_j x(t)^{j-1}) \right] \frac{d \delta x(t)}{dt} + K_m(x) \delta x(t) = - \frac{d \delta x(t)}{dt} \sum_{j=0}^N x^j \delta r_j \quad (21)$$

subjected to

$$\delta x(t) = 0 \quad \text{and} \quad \left. \frac{d \delta x(t)}{dt} \right|_{=0} \quad \text{for } t = 0 \quad (22)$$

then  $\delta x(t)$  can be solved from the above sensitivity problem by letting  $\delta r_j = P^{(k)}$ .

## IV. RESULTS AND DISCUSSION

#### A. Numerical Experiment

Consider an input transient excitation signal  $e(t)$  with an amplitude of 2 V into the system. Using HSDM to set  $t_f = 2$ s and the time step  $\Delta t = 1/8000$ , the voice-coil displacement  $x_{mea}(t)$  can be obtained through the computational steps. As shown in Fig. 2, the identified result from the proposed model is exactly the same with the exact solution.

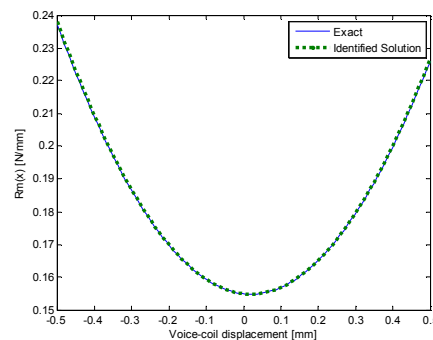


Fig. 2 Comparison of exact and identification solution of  $R_m(x)$  curves depended on voice-coil displacement

### B. Validation with Experiment

The loudspeaker is excited by a multi-tone test signal with 2 Vrms generated from the KLIPPEL analyzer system [10]. Since the voltage and displacement are measured simultaneously, the damping  $R_m(x)$  can be obtained instantaneously by the proposed method. A voltage sensor is required to measure the voltage of  $e(t)$  at loudspeaker terminals. The center point of diaphragm displacement  $x_{mea}(t)$  was measured using a laser displacement sensor in the KLIPPEL analyzer system.

Fig. 3 is the setup for the loudspeaker that is placed to measure the  $R_m(x)$  in air and vacuum, respectively. The measured results depict in Fig. 4 that the behavior of  $R_m(x)$  curves is much different between in air and vacuum. In air, the damping  $R_m(x)$  of the loudspeaker is strongly nonlinear.

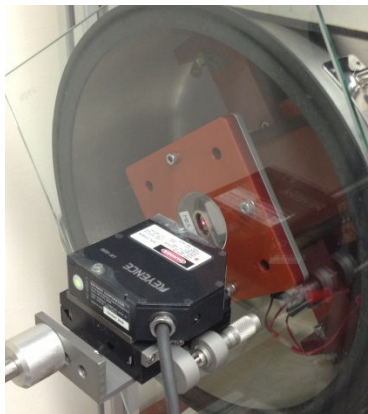


Fig. 3 The setup for laser displacement measurement in vacuum and in air (where the glass is removed)

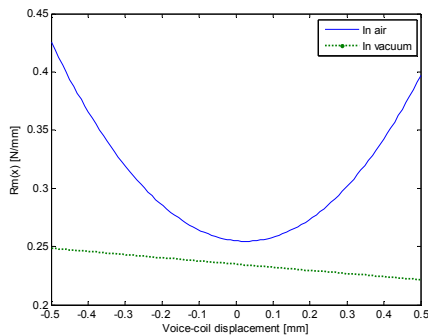


Fig. 4 The behaviors of  $R_m(x)$  in air and vacuum

### V. CONCLUSION

The identification method of nonlinear damping parameter  $R_m(x)$  of MMT is presented in this paper. The theoretical results are verified by comparing the nonlinear curves of  $R_m(x)$ . The accuracy of the numerical model was ascertained by comparing the modeled  $R_m(x)$  with theoretical solution. Through the measurement, the results show that the  $R_m(x)$  has the different nonlinearity variation between in air and vacuum. In the future, the identification method may be extended to account for the

coupling with nonlinear force factor  $Bl(x)$  or the other nonlinear factor of MMT in acoustical field.

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