Speed Regulation of a Small BLDC Motor using Genetic-Based Proportional Control

S. Poonsawat, and T. Kulworawanichpong

Abstract—This paper presents the speed regulation scheme of a small brushless dc motor (BLDC motor) with trapezoidal back-emf consideration. The proposed control strategy uses the proportional controller in which the proportional gain, k_p, is appropriately adjusted by using genetic algorithms. As a result, the proportional control can perform well in order to compensate the BLDC motor with load disturbance. This confirms that the proposed speed regulation scheme gives satisfactory results.

Keywords—BLDC motor, proportional controller, genetic algorithms.

I. INTRODUCTION

A brushless DC motor is one of a small-scale motor used in small electric devices such as CD players, hard disk drives, or even small electric cars. Its rotor is mounted with permanent magnet. There is no need for extra field excitation. This motor is well-known and popular for position and speed control drive applications [1,2]. The key advantage of this motor over other types in the same rating is higher ratio of produced torque per weight, faster response, accurate position control, lower moment of inertia, less maintenance, etc [3,4].

In this paper, mathematical modeling of a small BLDC motor and its electrical drives are the main purpose as described in Section II. Section III presents power loss and efficiency calculation for the BLDC motor. Section IV presents speed regulation technique. Section V and VI are simulation results and conclusions, respectively.

II. PRINCIPLES OF THE BLDC MOTOR

A. Mathematical Model of BLDC Motors

Modeling of a BLDC motor can be developed in the similar manner as a three-phase synchronous machine [1,2]. Since its rotor is mounted with a permanent magnet, some dynamic characteristics are different. Flux linkage from the rotor is dependent upon the magnet. Therefore, saturation of magnetic flux linkage is typical for this kind of motors. As any typical three-phase motors, one structure of the BLDC motor is fed by a three-phase voltage source as shown in Fig. 1. The source is

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not necessary to be sinusoidal. Square wave or other wave-shape can be applied as long as the peak voltage is not exceeded the maximum voltage limit of the motor. Similarly, the model of the armature winding for the BLDC motor is expressed as follows.

$$v_a = Ri_a + L\frac{di_a}{dt} + e_a \tag{1}$$

$$v_b = Ri_b + L\frac{di_b}{dt} + e_b \tag{2}$$

$$v_c = Ri_c + L\frac{di_c}{dt} + e_c \tag{3}$$

Or in the compact matrix form as follows.

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R+pL & 0 & 0 \\ 0 & R+pL & 0 \\ 0 & 0 & R+pL \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{4}$$

Where $L_a = L_b = L_c = L = L_S - M$ [H]

L_S is the armature self-inductance

M is the mutual inductance

$R_a = R_b = R_c = R$: armature resistance [Ω]

v_a, v_b, v_c : terminal phase voltage [V]

i_a, i_b, i_c : motor input current [A]

e_a, e_b, e_c : motor back emf [V]
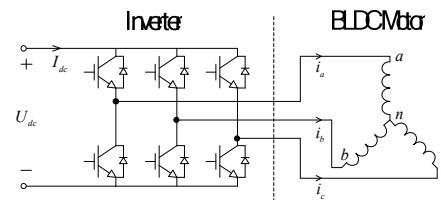
p in the matrix represents d/dt

Fig. 1 BLDC Motor Control System

Due to the permanent magnet mounted on the rotor, its back emf is trapezoidal as shown in Fig. 2. The expression of back emf must be modified as expressed in (5) – (7).

$$e_a(t) = K_E \cdot \phi(\theta) \cdot \omega(t) \tag{5}$$

$$e_b(t) = K_E \cdot \phi\left(\theta - \frac{2\pi}{3}\right) \cdot \omega(t) \qquad (6)$$

$$e_c(t) = K_E \cdot \phi\left(\theta + \frac{2\pi}{3}\right) \cdot \omega(t) \qquad (7)$$

Where K_E is the back emf constant and ω is the mechanical speed of the rotor.
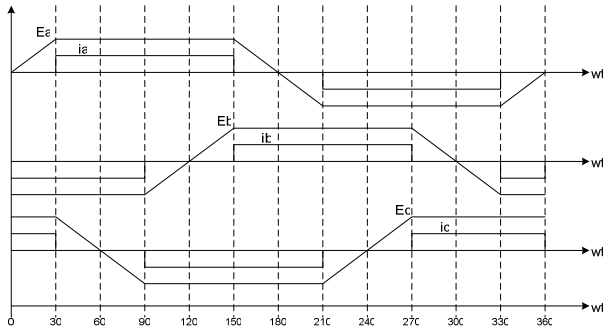

Fig. 2 BLDC Motor back emf and the motor phase currents

The permanent magnet also influences produced torques due to the trapezoidal flux linkage. Given that K_T is the torque constant. The produced torques can be expressed as described in (8).

$$T_E = (e_a i_a + e_b i_b + e_c i_c)/\omega \qquad (8)$$

Substitute (5) – (7) into (8), the resultant torque, T_E, can be obtained by the following expressions.

$$T_a(t) = K_T \cdot \phi(\theta) \cdot i_a(t) \qquad (9)$$

$$T_b(t) = K_T \cdot \phi\left(\theta - \frac{2\pi}{3}\right) \cdot i_b(t) \qquad (10)$$

$$T_c(t) = K_T \cdot \phi\left(\theta + \frac{2\pi}{3}\right) \cdot i_c(t) \qquad (11)$$

$$T_E(t) = T_a(t) + T_b(t) + T_c(t) \qquad (12)$$

With the Newton's second law of motion [5], the angular motion of the rotor can be written as follows.

$$T_E(t) - T_L(t) = J\frac{d\omega(t)}{dt} + B \cdot \omega(t) \qquad (13)$$

Where T_L : load torque $[\text{N} \cdot \text{m}]$

J : rotor inertia $[\text{kgm}^2]$

B : damping constant

B. Supply Source

Power supply for BLDC motor drives can be in various forms. Sinusoidal supply is typical as the standard power supply of an electric utility, while square waves and PWM are widely used in small power applications. In this paper, only PWM shape is used for study. It is slightly complicated. To generate the PWM shape, it requires operation of high-frequency switching devices of electronic inverters, e.g. transistor, MOSFET, etc. The example of natural sampling or sinusoidal PWM wave shape is shown in Fig. 3.
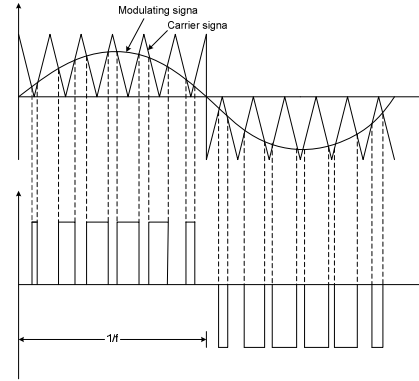

Fig. 3 Natural sampling PWM wave form

III. POWER LOSS AND EFFICIENCY CALCULATION

In the BLDC motor, the power losses consist of core losses in the magnetic core, copper losses in the winding and mechanical losses [6].

A. Copper Losses

The copper losses are $I^2 R$ loses. All three-phase windings must be taken into account. Thus, the total armature copper loss (P_{cu}) is equated as follows.

$$P_{cu} = \left(i_a^2 + i_b^2 + i_c^2\right)R \qquad (14)$$

B. Core Losses and Mechanical Losses

Mechanical losses are caused by friction (mostly in the bearings) and the dynamic drag to oppose the motion of the rotor or so-called the friction and windage losses (P_{fw}). Core losses (P_{core}) are the open circuit losses due to hysteresis property and induced eddy current in the core, which exist as long as the excitation winding energized. To be convenient and very useful for efficiency computation, the core losses, friction losses and windage losses are summed up to give rise to the no-load rotational loses (P_{rot}).

$$P_{rot} = P_{core} + P_{fw} \qquad (15)$$

C. Efficiency

The efficiency of a motor is an important performance characteristic. Although its formula is as very simple as (16), to compute the efficiency is problem-dependent due to the assumption of power conversion in the machine. In this paper, input and output powers are defined accordingly.

$$\eta = \frac{P_{out}}{P_{in}} \times 100 \qquad (16)$$

$$P_{in} = v_a i_a + v_b i_b + v_c i_c \qquad (17)$$

$$P_{out} = \frac{1}{2} J \omega^2 + T_L \omega \qquad (18)$$

IV. SPEED REGULATION TECHNIQUE

To regulate the motor speed at the desired level, a speed regulation scheme is proposed. Although disturbances can be caused by several events, e.g. supply change, sudden load change, etc, in this paper the speed regulation under mechanical load disturbances is only our particular study. To regulate the speed level, the proportional control scheme is employed and can be summarized as shown in the block diagram of Fig. 4.
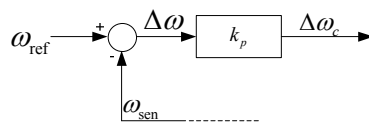

Fig. 4 Closed-loop speed regulation scheme

To demonstrate the proposed speed regulation scheme, the proportional control (P-controller) is introduced. As obviously found in other controller design problems, the proportional gain (k_p) is the key to accomplish this regulation. From Fig. 4, the Proportional controller output is assigned by (19). To find an appropriate value of k_p, some efficient search algorithm, such as genetic algorithms, is employed.

$$\Delta\omega_c = k_p \Delta\omega \qquad (19)$$

Where $\Delta\omega_c$ is proportional controller output

k_p is gain of proportional controller

$\Delta\omega$ is the speed error

A. Genetic Algorithms: GAs

There exist many different approaches to tune controller parameters. The GAs is well-known [6] there exist a hundred of works employing the GAs technique to design the controller in various forms. The GAs is a stochastic search technique that leads a set of population in solution space evolved using the principles of genetic evolution and natural selection, called genetic operators e.g. crossover, mutation, etc. With successive updating new generation, a set of updated solutions gradually converges to the real solution. Because the GAs is very popular and widely used in most research areas [7-8] where an intelligent search technique is applied, it can be summarized briefly as shown in the flowchart of Fig. 5.

In this paper, the GAs is selected to build up an algorithm to tune k_p parameters. The procedure to perform the proposed parameter tuning is described as follows. First, time-domain results of the motor speed obtained by simulating the BLDC motor system in MATLAB™ [9] are collected. Second, the

Genetic Algorithms (GADS TOOLBOX in MATLAB™) [9] is employed to generate a set of initial random parameters. With the searching process, the parameters are adjusted to give response best fitting close to the desired response in the *abc* reference signals. To perform the searching properly, its objective function is the key. In this paper, the objective function is defined by using the power loss function in (14-15).
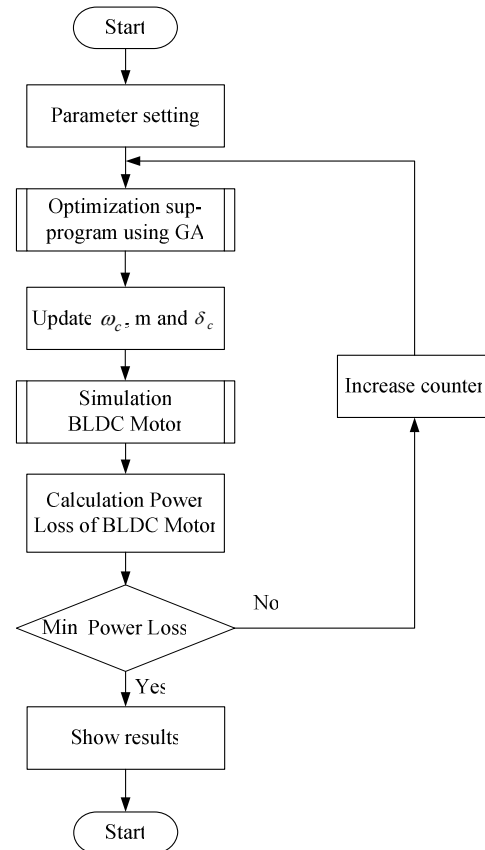

Fig. 5 Flow chart of the genetic algorithm

V. SIMULATION RESULTS

In this section, demonstration by simulation is illustrated. A small brushless DC motor is assumed to be operated with an adjustable voltage source inverter of the PWM type. The peak voltage magnitude is fixed at 12 V. For parameters of the test motor and GADS's MATLAB TOOLBOX are given below.

TABLE I
PARAMETERS OF THE TEST BLDC MOTOR

Parameter	Symbol	Data
Armature Inductance	L	1.13 mH
Armature Resistance	R	3.21 Ω
Rotor Inertia	J	0.932×10^{-11} kg.m^2
Damping Constant	B	1.09×10^{-12} N.m.s/rad
Back EMF Constant	K_e	0.593×10^{-4} V.sec
Torque Constant	K_t	0.0145×10^{-6} N.m/A

TABLE II
PARAMETERS OF GADS MATLAB TOOLBOX

Total Generation	500
Population Size	100
Stall Generation Limit	20
Mutation rate	0.05
Crossover rate	0.5
Population Range	$k_p \in [0,1], \delta^{abc} \in [0,360], m \in [1,12]$

The test case employs a PWM voltage source inverter to energize the BLDC motor as commonly found for small BLDC motors in real-world applications. To verify its responses, voltage and current waveforms together with mechanical rotor speed in rpm are characterized. Due to the test of mechanical load disturbance, controller parameter tuning can be performed by using the GADS MATLAB TOOLBOX to find an optimal k_p parameter. Fig. 6 shows solution convergence for the k_p tuning process. To avoid too many voltage and current waveform presentations, only the results of the uncompensated system are selected as shown in Figs 7 – 10 for the motor's input voltages, the motor's current, the motor's back emfs and the mechanical speed of the rotor, respectively. Similarly, Figs 11 – 14 give responses for the case of load disturbance event.
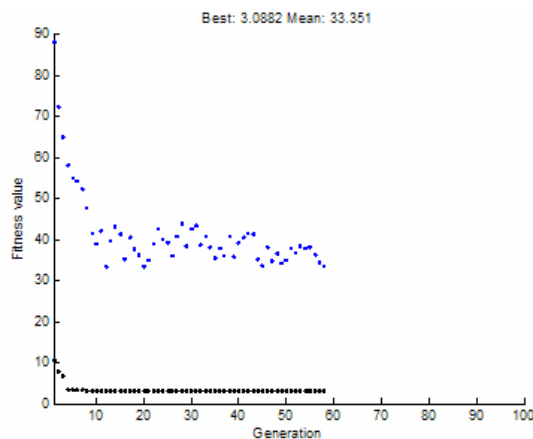

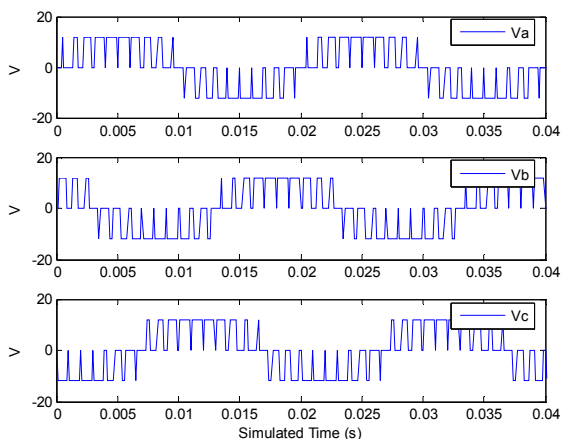Fig. 6 Convergence of GA solution of the k_p parameter
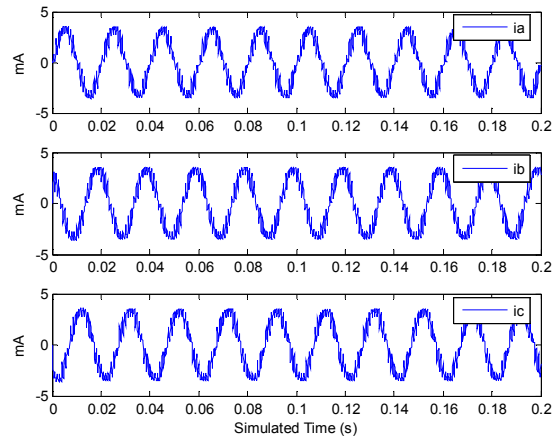

Fig. 7 PWM voltage supply source
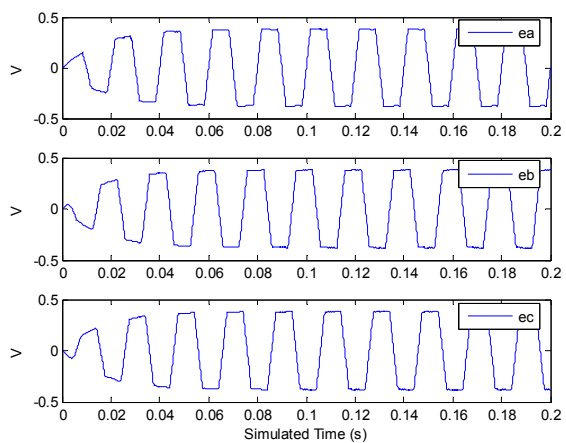

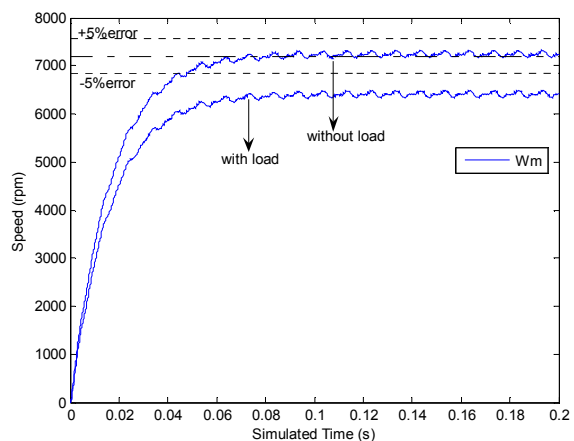Fig. 8 Current of the BLDC motor

Fig. 9 Trapezoidal motor back emf
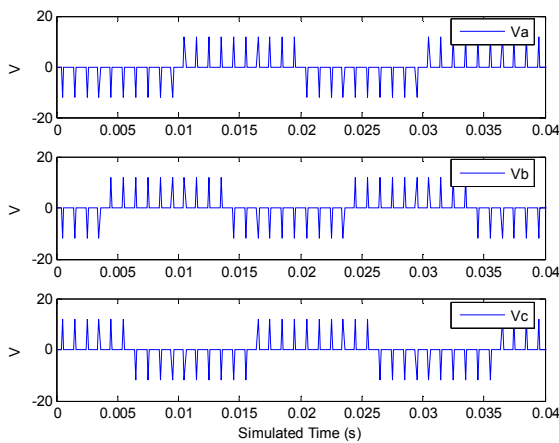

Fig. 10 Speed of the BLDC motor with and without load

Fig. 11 PWM voltage supply source

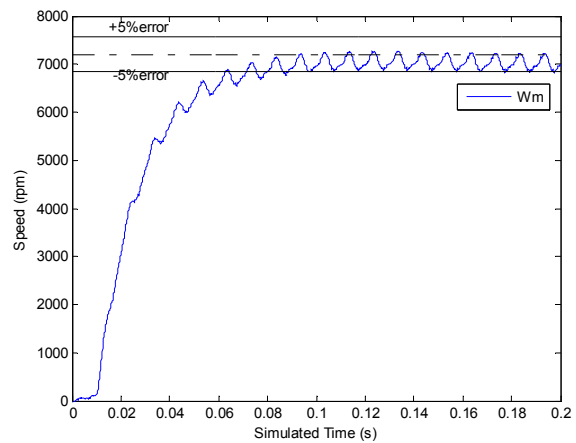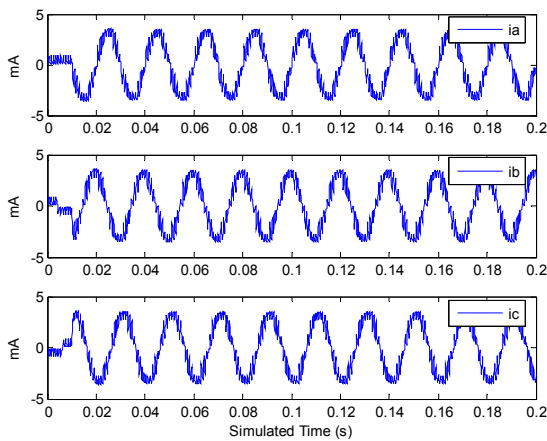Fig. 12 Input current of the BLDC motor

Fig. 13 Trapezoidal motor back emf

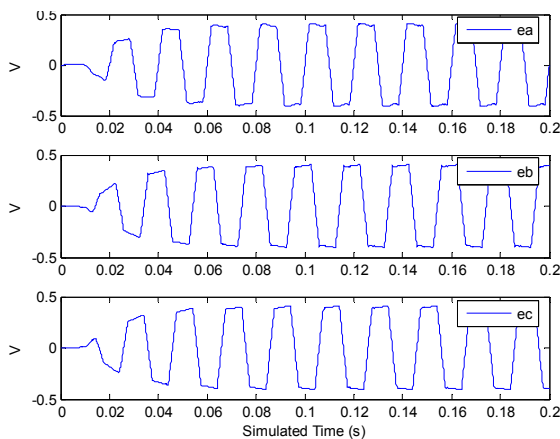Fig. 14 Speed of the BLDC motor with load

The results of the uncompensated system as shown in Figs. 7 – 10 show that the rotor speed drops considerably to just over 6000 rpm when the mechanical load torque disturbance is applied. To resume the rotor speed to its nominal value, the proportional gain needs to be tuned. With appropriately tuned proportional gain, the rotor speed can be regulated to operate within ±5% speed error band.

VI. CONCLUSION

In this paper, speed regulation scheme of a small brushless DC motor (BLDC motor) with trapezoidal back-emf consideration has been presented. The proposed control strategy uses the proportional controller in which the proportional gain, k_p, is appropriately adjusted by using genetic algorithms. As a result, with the load torque disturbance applied to the BLDC motor operation, the rotor speed can be regulated to operate within ±5% speed error band. To extend this study, thermal effect of the winding resistances resulting from the motor current should be taken into account as well as the use of different motor load models.

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