Least Square-SVM Detector for Wireless BPSK in Multi-Environmental Noise

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Abstract—Support Vector Machine (SVM) is a statistical learning tool developed to a more complex concept of structural risk minimization (SRM). In this paper, SVM is applied to signal detection in communication systems in the presence of channel noise in various environments in the form of Rayleigh fading, additive white Gaussian background noise (AWGN), and interference noise generalized as additive color Gaussian noise (ACGN). The structure and performance of SVM in terms of the bit error rate (BER) metric is derived and simulated for these advanced stochastic noise models and the computational complexity of the implementation, in terms of average computational time per bit, is also presented. The performance of SVM is then compared to conventional binary signaling optimal model-based detector driven by binary phase shift keying (BPSK) modulation. We show that the SVM performance is superior to that of conventional matched filter-, innovation filter-, and Wiener filter-driven detectors, even in the presence of random Doppler carrier deviation, especially for low SNR (signal-to-noise ratio) ranges. For large SNR, the performance of the SVM was similar to that of the classical detectors. However, the convergence between SVM and maximum likelihood detection occurred at a higher SNR as the noise environment became more hostile.

Keywords—Colour noise, Doppler shift, innovation filter, least square-support vector machine, matched filter, Rayleigh fading, Wiener filter.

I. INTRODUCTION

SUPPORT Vector Machine (SVM) is a recent class of statistical classification and regression techniques getting an increased attention on its application to classification problems in various engineering areas. SVM is based on the statistical learning theory initially developed by Vapnik [1] in 1979 and later developed to a more complex concept of structural risk minimization (SRM). SVM is formulated on the structural risk minimization (SRM) principle which minimizes an upper bound on the generalization error, as opposed to the classical empirical risk minimization (ERM) approach which minimizes the error on the training data and is embodied in statistical learning.

In a broad sense, two classes of classifiers are widely used in the literature: (1) *model-based* classifiers such as the maximum likelihood (ML) and maximum aposteriory (MAP) detectors and (2) *boundary-based* classifiers such as support vector machine, neural networks and fuzzy logic.

SVM claims to guarantee generalization, i.e., the decision rules reflect the regularities of the training data rather than the incapabilities of the learning machine.

SVM has been widely used in solving classification and function estimation problems due to its many attractive features and promising empirical performance with many successful applications in synthetic aperture radar image classification and pattern recognition [2]. Recently, SVM has been introduced to digital communication systems as a new method for *channel equalization* [3] − [5] and has proved to be very effective in overcoming intersymbol interference (ISI) and co-channel interference (CCI). SVM was also applied for the equalization of burst time division multiple access (TDMA) transmission [6]. To the best of our knowledge, SVM has not been implemented yet for *receiver detection* in digital communication systems in the presence of advanced additive colour interference noise and multiplicative channel fading noise in the presence of random Doppler shift. Notable exceptions are the diverse work of Dubois and Abdel-Latif [7] who first applied SVM to OOK-infrared channels in a local fading environment with partially developed multipath fading and additive white Gaussian interference noise (AWGN), then analysed MPSK detection in additive colour noise and fully developed Rayleigh fading [8], and then applied SVM detection to microwave radar and ultrasound images corrupted by partially developed speckle noise [9, 10]. Other notable exception is the initial work of Mokbel and Hashem [11] who applied SVM to a specific BNRZ detector (sampler and comparator) using multiple samples per binary period in the presence of AWGN in wire-line communication systems.

II. THE SYSTEM: MODULATION, NOISE, AND DETECTION

A. Modulation

When making a decision on the choice of model-based or boundary-based classifiers, we must take into consideration the driving modulation scheme. Binary phase shift keying (BPSK) is increasingly being adopted as a physical layer modulation technique for existing and future wireless technologies due to its simplicity, particularly when compared with its competitor quadrature amplitude modulation (QAM). Most notably, the most popular wireless LAN standard, IEEE 802.11b [12], uses a variety of different PSKs depending on

the data-rate required. At the basic-rate of 1 Mbit/s, it uses DBPSK (differential BPSK). The higher-speed wireless LAN standard, IEEE 802.11g, uses BPSK for the lowest of its eight data rates (6 and 9 Mbit/s modes). BPSK is also used in RFID standards such as ISO 14443 (which has been adopted for biometric passports, credit cards, and many other applications) because it is simple and appropriate for low-cost passive transmitters. In addition, ZigBee (a similar technology to bluetooth, also known as IEEE 802.15.4) employs BPSK in the frequency band 868–915MHz.

Since SVM is essentially a binary classifier, it is only logical to apply SVM to BPSK to improve its BER performance.

B. Stochastic Noise Modeling

Rayleigh fading: The Rayleigh fading channel is widely assumed in the literature for wireless systems, especially for mega- and marco-cells in the absence of dominant line of sight [13]. The fading envelope obeys the scattering stochastic model

$$\gamma_L = \left| \sum_{k=1}^{L} A_k e^{j\phi_k} \right|, \qquad (1)$$

with fading power being $v_L = \gamma_L^2$. A_k is the random amplitude of the kth scatterer, ϕ_k is the random phase of the kth scatterer assumed to be uniformly distributed between $[0, 2\pi)$, and L is the random number of scatterers in the channel. When L is sufficiently large so that the central limit theorem (CLT) holds and the scattered field is approximately a circular complex Gaussian random variable, we say that the fading *is fully developed*. Even for statistically dependent scatterers, the scattered field would still be asymptotically circularly Gaussian hinging on the fact that the sequence of scattering random variables satisfies the α-mixing property or the conditions of the Lindeberg-Feller CLT. A simple random variable transformation results in the fading envelope γ_L asympotically Rayleigh distributed (with parameter $P_{dif} = E(\gamma_L^2)$), and the fading power v_L exponentially distributed (central chi-square with 2 degrees of freedom).

Colour noise: In general, the received wireless BPSK signal is corrupted by two types of noises: (1) *multiplicative* channel fading noise γ_L and (2) *additive* background colour Gaussian noise (ACGN) which characterizes more complicated noise interference. This stochastic model is widely used in wireless optical communication systems [14] in the presence of interfering ambient or incandescent/fluorescent light and non-ideal photo-detectors. Technically, these noise models respectively correspond to shot noise and "random telegraph signal" or laser-phase noise. Most commonly, ambient light induces shot noise in the photo-detector of optical receivers.

By studying colour noise, we are assuming a more severe noise environment. Without loss of generality, we will consider the autocorrelation function of a random telegraph signal (or bi-phase laser noise)

$$R_N(\tau) = \varepsilon^2 \exp(-2\lambda |\tau|), \qquad (2)$$

where 2ε is the peak-to-peak noise amplitude and λ is the rate of the influencing underlying Poisson point process [14]. The power spectral density (PSD) of the noise is given by

$$P_N(f) = \frac{\varepsilon^2 \lambda}{1 + \pi^2 f^2}, \qquad (3)$$

which has a de-emphasis filter-like characteristics.

In addition to optical systems, receiver performance in time division multiple access (TDMA) wireless systems such as Global System for Mobile Communications (GSM), Enhanced Data GSM Evolution (EGDE), and Digital Advanced Mobile Service (DAMPS), is often interference-limited. Interference is statistically characterized as colour noise and occurs in the form of (1) co-channel interference (CCI) caused by other users operating on identical carrier frequencies in neighboring cells, and (2) adjacent-channel interference (ACI) caused by users operating on adjacent carrier frequencies. ACI is typically dominated by interference from first adjacent channels since interference from secondary adjacent channels is usually filtered out. A receiver demodulator based on matched filter-driven maximum likelihood detector is optimal only in the presence of white noise. Therefore, colored noise can significantly degrade the performance of the receiver if it is not compensated for.

Doppler shift: Random Doppler shift is caused by the relative motion between the transmitter and receiver. Local scattering typically comes from many angles randomly distributed around the mobile causing a range of random Doppler shifts, known as the Doppler spectrum. Different arrival angles ψ_k from each k-th scattered wave will cause the transmitted signal to be received at different frequencies. The Doppler shift of each wave component is $\xi_{d_k} = (V/c)f_c \cos(\psi_k)$, V being the relative velocity between target and sensor and $c = 3(10^8)$ m/s is the speed of light. The maximum Doppler shift corresponds to the received local scattering component whose direction exactly opposes the mobile's trajectory, that is, $\psi_k = \pi$, and is given by the expression $f_D = V f_c / c$.

The expression of the received faded signal must also include the effects of motion induced frequency and is expressed as $\bar{s}(t) = \gamma \cos(2\pi v_c t + \phi), v_c = f_c + \xi_D, \gamma \perp \phi \perp v_c$. In a separate paper, we derived the statistical distribution of Doppler shift and proved it to be consistent with the classical Clark's model [15].

The Doppler shift may appear to be insignificant. For example, if $f_c = 1$ GHz, and $V = 60$ km/hr (16.7 m/s), then the Doppler shift will be 55.5 Hz. This shift of 55 Hz in the carrier will, in general, not affect the transmission for most modulation schemes. However, Doppler shift can cause significant problems in BPSK driven OFDM because this modulation technique is sensitive to carrier frequency offsets (a slight shift in frequency will cause carriers to become non orthogonal). Doppler shift can also be significantly large when

the relative speed is higher (for example in air force planes moving at the speed of sound and in low earth orbiting satellites).

C. Model-Based Detection Schemes

Classical filters: Matched filters (correlator structure) and wiener filters have received widespread attention in the literature and have been extensively explained [13]. In this section we will highlight a novel filter, termed innovation, and also discuss the process of whitening the colour noise.

Innovation filter: The generalized matched filter (GMF) has the same structure and behavior as the matched filter but is used when the signal is random. The general expression for the GMF is given by

$$H_o(f) = \frac{P_S^-(f)e^{-j2\pi fT_b}}{P_N(f)}, \qquad (4)$$

for signals with power spectral density (PSD) separable as $P_S(f) = P_S^+(f)P_S^-(f), P_S^-(f) = P_S^{+^*}(f)$, and is given by

$$H_o(f) = \frac{\sqrt{P_S(f)}e^{-j2\pi fT_b}}{P_N(f)}, \qquad (5)$$

for non separable PSD, in which case the GMF is sub-optimal. For optimal binary signal detection, $s(t) = s_d(t) = \bar{s}_1(t) - \bar{s}_0(t)$, where $\bar{s}_i(t)$ is the random scattered faded signal for bit i ($i = 0, 1$). T_b is the bit period and is equal to the reciprocal of the incoming bit rate ($T_b = 1/R_b$).

The resulting signal-to-noise ratio at the output of the GMF is given by

$$SNR_{0,\max} = \int_{-\infty}^{\infty} \frac{P_S(f)}{P_N(f)} df \; . \qquad (6)$$

When the noise is white, the GMF has a structure identical to an innovation filter:

$$\left|H_o(f)\right| = \frac{2}{N_0}\left|P_S^+(f)\right|, \qquad (7)$$

where $P_S^+(f)$ is an innovation filter for the signal $s(t)$, and hence the association of the nomenclature "*innovation*" with the generalized matched filter.

Whitening filter: The primary role of the whitening filter is to whiten the coloured noise (that is, transform its PSD to a constant) since the matched filter cannot process coloured noise. There are two primary approaches to whitening coloured noise: 1) implicit whitening, and 2) explicit whitening.

The implicit whitening approach incorporates a whitening function in a so-called pre-filter with length that is much longer than a span of a corresponding channel when a

decision feedback equalizer (DFE) or a decision feedback sequence estimator (DFSE) are employed. The drawback of implicit whitening is that an ideal pre-filter (i.e., a whitened matched filter) is anti-causal, unlimited in time, and can only be approximated. In addition, it is computationally complex and expensive to setup and process the pre-filter because it involves spectrum factorization of the propagation channel and inversing the maximum/phase factor of the channel.

On the other hand, explicit whitening involves employing a whitening filter with the structure

$$H_w(f) = \frac{1}{P_N^+(f)}, \qquad (8)$$

with $P_N(f) = P_N^+(f)P_N^-(f)$, $P_N^-(f) = P_N^{+^*}(f)$, or $P_N(s) = M_N(s)M_N(-s)$, $M_N(s) = P_N^+(s = j2\pi f)$.

For the colour noise whose PSD is given in (3), the whitening filter is given by

$$H_w(f) = \frac{\lambda + j\pi f}{\varepsilon\sqrt{\lambda}}, \qquad (9)$$

or, in time domain,

$$h_w(t) = \frac{1}{\varepsilon\sqrt{\lambda}}\left(\lambda\delta(t) + \frac{1}{2}\delta'(t)\right). \qquad (10)$$

III. SUPPORT VECTOR MACHINE

In this section, we provide a succinct introduction to the SVM approach. The reader is referred to the initial work of Vapnik [1] and the book of Christianini [16] for more in-depth treatment of the SVM theory.

The relation between the capacity of a learning machine and its performance is ruled by a set of boundaries, which is referred to as the bound on the generalization performance. Statistical pattern recognition techniques face two problems: the identification problem and the parameters estimation problem. The identification problem is the problem of determination of the degree of freedom or complexity of the model and is generally the more complex problem [17]. The estimation problem is how to get an optimal estimate of the model parameters regarding the training data set.

Let us consider a mapping $\Phi : \Box^d \mapsto H$, which maps the training data from $\Box^d$ to a higher Euclidean space H, that may have an infinite dimension. In this high dimension space, the data is linearly separable, hence linear SVM formulation above can be applied for any type of data [2]. In the SVM formulations, the training data only appear in the form of dot products $\mathbf{x}.\mathbf{x}$. These can be replaced by dot products in the Euclidean space H, i.e., $\varphi(.).\varphi(.)$.

The dot product in the high dimension space can also be replaced by a kernel function. By computing the dot product directly using a kernel function, one avoids the mapping $\Phi(x)$. This is desirable because H has possibly infinite dimensions and $\Phi(x)$ can be tricky or impossible to compute. Using a

kernel function, a SVM that operates in infinite dimensional space can be constructed [16].

Given a training set of N data points $\{y_k, x_k\}^N$, where x_k denotes the kth input pattern and y_k the kth output pattern, the SVM aims at constructing a decision function or classifier

$$f(x) = sign\left[\mathbf{w}^T \boldsymbol{\varphi}(x) + b\right]$$
$$= sign\left[\sum_{k=1}^{N} \alpha_k y_k K(x, x_k) + b\right], \quad (11)$$

where $\mathbf{w}$ is the weight vector in the reproducing kernel Hilbert space (RKHS), α_k are support values (Lagrangian multipliers), b is the bias term, and the kernel function

$$K(x, x_k) = \varphi(x)\varphi(x_k). \quad (12)$$

For every new test data, the kernel functions for each SV (support vector) need to be recomputed.

For any kernel function suitable for SVM, there must exist at least one pair of $\{H, \Phi\}$, such that (12) is satisfied. The kernel that has these properties is said to obey the Mercer's condition, i.e., for any $g(x)$ with finite L_2 norm,

$$\int g^2(x)\,dx < \infty, \quad (13)$$

$$\iint K(x, y)g(x)g(y)dxdy \geq 0 \quad (14)$$

By choosing different kernel functions, the SVM can emulate some well known classifiers [18], as shown in Table I.

TABLE I
KERNEL FUNCTIONS' CLASSIFIERS

Kernel Function	Type of Classifier
$K(x,y) = xy$	Linear
$K(x,y) = \exp\left(\|x-y\|_2^2 / \sigma^2\right)$	Gaussian radial bias function (RBF)
$K(x,y) = (xy + \tau)^d$	Polynomial of degree d
$K(x,y) = \tanh(\kappa xy + \theta)$	Multi layer perceptron

While standard SVM solutions involve solving quadratic or linear programming problems, the least square version of SVM (LS-SVM), which has been adopted for this research, corresponds to solving a set of linear equations. In LS-SVM, the Mercer's condition is still applicable. Hence several types of kernels can be used, yet the RBF is the adopted one since it gives a Gaussian distribution for the errors in the feature space yielding an optimal estimate of the support values [19]. Many reasons could be stated for preferring LS-SVM over other models of SVM, yet the most important one is that LS-SVM is an iterative method that could be used to solve large scale problems with robustness in the sense of the choice of the regularization and smoothing parameters. Moreover, it offers a fast method for obtaining classifiers with good generalization performance in many real life applications [20].

So far, the formulation of SVM was based on a two-class problem (SVM is essentially a binary classifier). Various schemes can be applied to the basic SVM algorithm to handle the M-class pattern classification problem. Such schemes are useful when applying SVM to M-ary signaling (such as QPSK), a study we have already published in [8].

IV. SVM-BASED BPSK DETECTOR

The BPSK SVM-based detector is illustrated in Fig. 1. The received noisy signal is processed through a simple correlator, where it is mixed with locally generated reference signal and then integrated over each bit period. This has the effect of enhancing the received SNR.
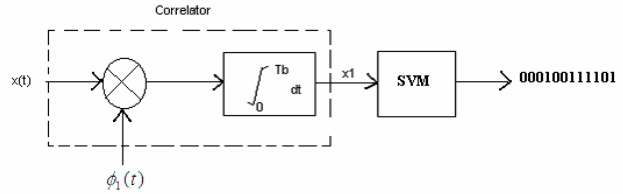

Fig. 1 BPSK SVM-based receiver

The output of the correlator becomes input to the support vector machine, which classifies the received data and produces a stream of bits representing the received message.

V. SIMULATION, RESULTS, AND DISCUSSIONS

For simulation purposes, Matlab is used due to its enhanced mathematical capabilities and engineering based structure. The LS-SVM model was simulated using Matlab code on a 1.7 GHz Pentium IV computer with 256 MB RAM.

Without loss of generality (wlg) and for the purpose of simulation, we assumed $P_{dif} = 1$ (average diffuse power-Rayleigh driving parameter), $\lambda = \varepsilon = 1$ (color noise parameters), $f_D = 240$ Hz for $f_c = 2.4$ GHZ (maximum Doppler shift). To take full advantage of the SVM scheme, we consider several samples of the BPSK signal in the bit period. This offers a generalization since SVM is applied in a wider space.

The results of the simulated LS-SVM-based BPSK system in the presence of Rayleigh fading and AWGN are shown in Fig. 2. We observe that the SVM-based detector outperforms (in terms of bit error rates) all the ML-based detector for low SNR. For high SNR, the SVM and the best of the ML-based detectors, namely the Wiener-ML scheme, produce close results and converge at SNR = 16.18 dB.

Yet this superior performance occurs at the cost of processing time as shown in the Table II. This drawback is expected because SVM is a block-data based method.

TABLE II
PROCESSING TIME FOR THE VARIOUS BPSK SCHEMES IN AWGN

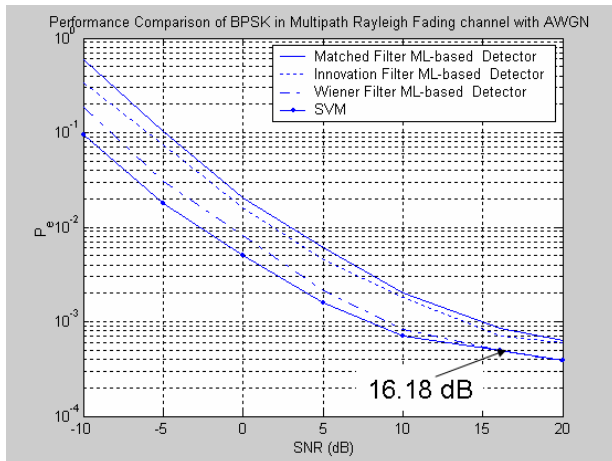| Adopted Scheme	Processing Time (micro secs/bit)
Matched filter	0.0255
Innovation filter	0.0312
Wiener filter	0.1025
SVM	0.6023

Fig. 2 Comparison of BPSK performance for different detection schemes in multipath Rayleigh fading channel with AWGN

Fig. 3 illustrates the results of the simulated LS-SVM-based BPSK system in the presence of Rayleigh fading and ACGN. We make the same observation that the SVM-based detector outperforms (in terms of bit error rates) all the ML-based detector for low SNR. For high SNR, the SVM and the Wiener-ML scheme produce close results and converge at slightly higher SNR (17.72 dB) than communication in an AWGN environment.
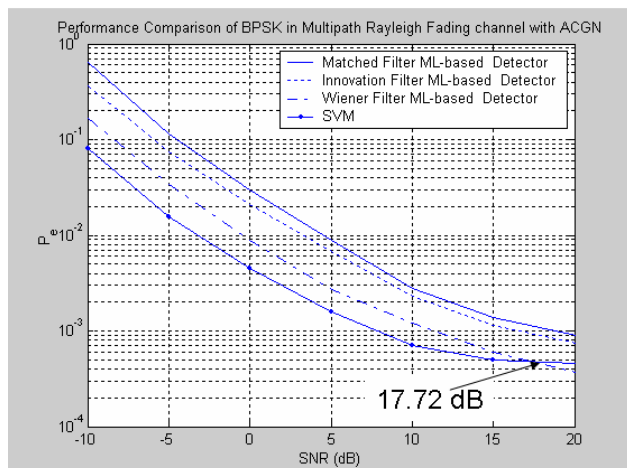

Fig. 3 Comparison of BPSK performance for different detection schemes in multipath Rayleigh fading channel with ACGN

This superior performance also occurs at the cost of processing time as shown in the Table III.

TABLE III
PROCESSING TIME FOR THE VARIOUS BPSK SCHEMES IN ACGN

Adopted Scheme	Processing Time (micro secs/bit)
Matched filter	0.0253
Innovation filter	0.0314
Wiener filter	0.1025
SVM	0.6023

Fig. 4 illustrates the results of the simulated LS-SVM-based BPSK system in a more severe *mobile* noisy environment

consisting of Rayleigh fading, AWGN, and the presence of random Doppler shift in the carrier caused by the relative motion between transmitter and receiver. We notice that the SVM-based detector outperforms (in terms of bit error rates) the wiener filter, the best of the ML-based detectors, for low SNR. For high SNR, the SVM and the Wiener-ML scheme produce close results and converge at a higher SNR (19.64 dB) than communication in non mobile environment. We also observe from Fig. 4 that the performance deteriorates when the communication environment is mobile with a Doppler shift.
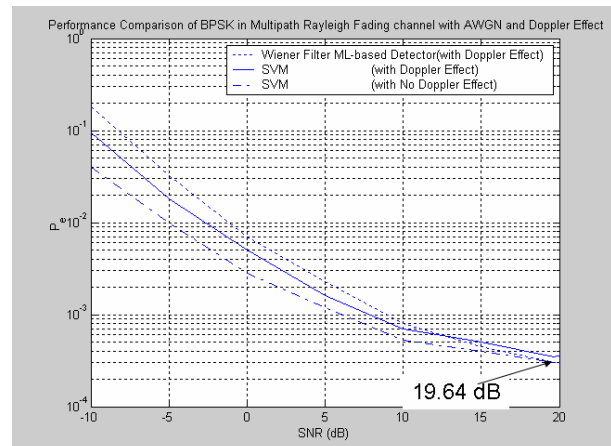

Fig. 4 Comparison of BPSK performance for Wiener and SVM schemes in a wireless (mobile) environment with Rayleigh fading and AWGN with and without Doppler shift

Expectedly, this superior performance occurs at the cost of processing time as shown in the Table IV.

TABLE IV
PROCESSING TIME FOR BPSK SCHEMES IN AWGN WITH DOPPLER SHIFT

Adopted Scheme	Processing Time (micro secs/bit)
SVM	0.721
Wiener filter	0.1045

When we consider the results of Figs. 2 – 4 cumulatively, we make the important observation that for large SNR, there are diminishing differences in the BER curves between all detection schemes. For high SNR, the BER is very weak and cannot be measured with sufficient precision for model based- and SVM-based methods, so a much larger training data block must be used. The converging dB levels of the SNR for both SVM and Wiener-ML detectors are displayed in Table V. We notice that the convergence between SVM and Wiener-ML occurred at a higher SNR as the noise environment became more hostile.

TABLE V
WORKING REGION OF SVM-BASED DETECTOR FOR BPSK SYSTEM

Noise Environment	SNR Value (dB)
Rayleigh fading and AWGN	16.18
Rayleigh fading and ACN	17.72
Rayleigh fading, AWGN, and Doppler	19.64

VI. CONCLUSION

In this paper, we applied SVM to BPSK detection in wireless systems with severe noise conditions modeled by Rayleigh fading channel noise, additive white Gaussian noise, additive colour noise, and random Doppler shift. SVM was found to be a learning machine suitable for wireless communication with the ability to handle data coming out from a relatively hostile wireless channel at a low SNR with a considered superiority to the classical ML-based detector schemes at the cost of relatively longer processing time (this is expected because SVM is a block-data based method). For large SNR, the performance of the SVM detector was similar to that of the best of the ML-based detectors, namely the Wiener-ML scheme. However, the convergence between SVM and Wiener-ML occurred at a higher SNR as the noise environment became more hostile. The fact that SVM outperformed the Wiener filter can be justified by noting that the Wiener filter, being the theoretical optimal filter in the mean square sense, is a linear filter whereas SVM is non linear. We also note that the BER can be significantly reduced if the signaling order is increased (as shown in [8]) and if channel coding is employed. Detection of channel coded signals is a possible immediate extension of our work.

One major weakness of SVM is that it needs excessive training before implementation. Furthermore, the training of SVM is not as straightforward as it seems; numerical problems will cause the training to give non-optimal decision boundaries due to the formulation of the SVM training. In other words, the selection of the user-defined parameters, which has a significant effect on the generalization performance of the classifier, should be related to the Vapnik Chervonenkis (VC) dimension of SVM.

Since BPSK is widely used in existing wireless systems (e.g. IEEE 802.11 WLAN and RFID ISO 14443) and proposed for future technologies, we expect this research to give a clear insight of the performance of the new SVM-based system, thus triggering a newer generation of SVM-based wireless systems.

As future work, we propose to adopt one of the many pre-designed SVM chips [21] and implement a real-time system to compare results with the simulation outputs. As processors technology becomes faster SVM will be able to meet real-time computational requirement of high speed data communication.

We also propose to apply relevance vector machine (RVM) [22] to BPSK detection. RVM has an identical functional form to SVM and has been demonstrated to have a comparable generalization performance to SVM while requiring dramatically fewer kernel functions, thus enjoying faster computation.

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