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Calculation of Voided Slabs Rigidities

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***Abstract*—**A theoretical study of the rigidities of slabs with circular voids oriented in the longitudinal and in the transverse direction is discussed. Equations are presented for predicting the bending and torsional rigidities of the voided slabs. This paper summarizes the results of an extensive literature search and initial review of the current methods of analyzing voided slab. The various methods of calculating the equivalent plate parameters, which are necessary for two-dimensional analysis, are also reviewed. Static deflections on voided slabs are shown to be in good agreement with proposed equation.

***Keywords*—**voided slab, bending rigidity, torsional rigidity, orthotropic plate.

I. Introduction

An orthotropic plate is defined as one which has different rigidities in two orthogonal directions. In general two forms of orthotropic plate are identified, namely, material orthotropic plate and shape orthotropic plate. Most of the actual orthotropic plates are of the latter type, as like ribbed slabs and voided slabs. As voided slabs, voids running in the longitudinal direction are frequently in traduced into concrete slabs in order to reduce their self weight. Such voids are often of circular shape because they are simple then to fabricate, and it is relatively easy to ensure that compaction of the concrete under the void takes place during casting. Circular voided slabs of this nature are used both for floor slabs and for medium span slab bridge.

An approach commonly used is to assume the concrete is uncracked and linearly elastic, and thus ignore the reinforcement. This approach has the advantage of simplicity and of closely modeling the behavior of a slab. And the concept of converting an actual slab into an equivalent orthotropic plate for the purpose of determining the distributions of stresses is well established. Numerous investigators have suggested expressions of the determination of these bending rigidities but few have compared them. In this paper, the bending rigidities are derived from finite element analyses of cross sections of voided slabs, with the voids symmetrical with respect to the slab middle surface. And this paper summarizes the results of literature research and the review process, giving details of the various methods of analysis and calculating the properties of simplified mathematical models.

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II. Governing Equation of Plate

The governing equation of the plate ignoring the extensibility of the middle surface is given by

$$D_x \frac{\partial^4 w}{\partial x^4} + (D_{xy} + D_{yx} + D_1 + D_2)\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p(x, y) \quad (1)$$

In which D_x and D_y denote the bending rigidities, D_{xy} and D_{yx} are the torsional rigidities and D_1 and D_2 are the coupling rigidities. The values of solid slab are defined as follows;

Bending rigidity is

$$D = \frac{Et^3}{12(1-\nu^2)} \quad (2)$$

Torsional rigidity is

$$D_{xy} = \frac{Gt^3}{12} \quad (3)$$

The elasticity modulus, shear elasticity modulus and Poisson’s ratio are denoted by E, G and ν respectively.

It becomes necessary to calculate the equivalent parameters for a two-dimensional analysis. The various parameters may be required for all two-dimensional analyses of voided slab. The orthotropic plate rigidities are required for the orthotropic plate and two dimensional finite element analyses.

The cross section and the notation of voided slab are as shown in Fig. 1.

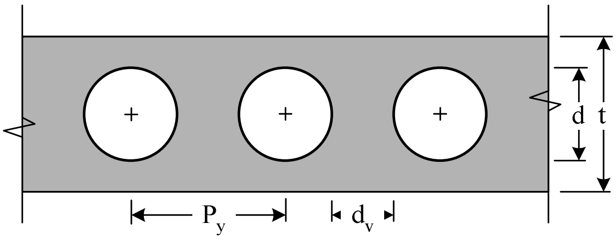

Fig. 1 Section of voided slab

III. Bending and Torsional Rigidities

Bending rigidities(D_x and D_y) in the longitudinal direction and in the transverse direction and torsional rigidities(D_{xy} and D_{yx}) can be obtained by treating the structure as equivalent stiffnesses.

For uncracked concrete voided slabs, [1] proposed the following equation for calculation of D_x and D_y .

$$D_x = E\left[\frac{t^3}{12} - \frac{\pi d^4}{64 P_y}\right] \quad (4)$$

$$D_y = \frac{Et^3}{12}\left[1 - 0.59(\frac{d}{t})^3(\frac{d}{P_x})\right] \tag{5}$$

Bending rigidities in the longitudinal direction is specified in the Ontario Highway Bridge Design Code (1975).

Elliott (1975) concludes that the spacing of voids has little effect on D_y and D_{xy}. These rigidities can be obtained by the following equation without incurring any significant error.

$$D_y = \frac{Et^3}{12}\left[1 - 0.95(\frac{d}{t})^4\right] \tag{6}$$

$$D_{xy(voided)} = \left[1 - 0.84(\frac{d}{t})^4\right] D_{xy(solid)} \tag{7}$$

[2] proposed the following equation for calculation of D_x, D_y and D_{xy}. It has been assumed that voided region is equivalent to a solid region of the same overall size but with reduced elasticity modulus in the longitudinal direction and in the transverse direction.

$$D_x = D_s \frac{I_x}{I} \tag{8}$$

$$D_y = D_s \frac{I_y}{I} \tag{9}$$

$$D_{xy} = \nu \sqrt{D_x D_y} \tag{10}$$

where I is moment of inertia of solid section, I_x is reduced moment of inertia in the longitudinal direction, I_y is reduced moment of inertia in the transverse direction and D is bending rigidity of solid region.

[3] proposed the following equation for calculation of D_x, D_y and D_{xy}.

$$D_x = \frac{Et^3}{12(1-\nu^2)}\left[1 - \frac{3\pi(d/t)^4(t/d_v)}{16(1+(d/d_v))}\right] \tag{11}$$

For certain range of d/t, n, t/t_3, the flexural rigidity in the transverse direction can be obtained by treating the voided slab as an equivalent Vierendeel frame.

$$D_y = \frac{Et^3}{12(1-\nu^2)}\left[1 - (\frac{d}{t})^3\right] \tag{12}$$

In case of without bending of the webs, D_y can be approximated in equation (12)

$$D_{xy} = \frac{Et^3}{12(1+\nu)}\left[\frac{3n(1+\frac{d}{d_v})(1+\frac{d}{t})(1-\frac{d^2}{t^2})}{4n(1+\frac{d}{d_v})(\frac{t}{d_v})(1-\frac{d^2}{t^2})}\right] \tag{13}$$

where n is the number of holes.

[5] have recommended formulate which give good comparisons with numerical and experimental studies. The equations for stiffness, D_x, D_y, D_{xy} and D_{yx}, are given by

$$D_x = \frac{EI_x}{P_y(1-\nu^2)} \tag{14}$$

$$D_y = \frac{Et^3}{12(1-\nu^2)}\left[1 - (\frac{d}{t})^4\right] \tag{15}$$

$$D_{xy} = \frac{Et^3}{24(1+\nu)}\left[1 - 0.85(\frac{d}{t})^4\right] \tag{16}$$

[6] proposed the following equation for calculation of D_x and D_y for orthotropic plate with rib.

$$D_x = \frac{EI_x}{P_y} \tag{17}$$

$$D_y = \frac{Et^3}{12(P_y - t_v + t_v(t/((t-d)/2)^3))} \tag{18}$$

Kim(2002) proposed the following equation for calculation of D_x and D_y. These rigidities can be obtained by the moment of inertia of voided section.

$$D_x = \frac{EI_x}{P_y} \tag{19}$$

$$D_x = \frac{EI_y}{P_x} \tag{20}$$

Park(2011) proposed the following equation for calculation of D_x and D_y. These rigidities can be obtained by the ratio of deflection.

$$D = \frac{E_R t^3}{12(1-\nu^2)} \tag{21}$$

where E_R is the deflection ratio of voided slab with solid slab.

Table I presents the rigidities of voided slab that have suggested by literature research.

TABLE I
RIGIDITIES OF VOIDED SLAB

Unit: kN/mm^2

	Aster	Elliott	Jofriet	Pama
D_x	1.637E+07	1.637E+07	1.684E+07	1.684E+07
D_y	1.636E+07	1.569E+07	1.563E+07	1.461E+07
	Elliott & Clark	Ugural	Kim	Park etc.
D_x	1.684E+07	1.637E+07	1.637E+07	1.298E+07
D_y	1.605E+07	3.525E+02	1.520E+07	1.298E+07

IV. EVALUATION OF RIGIDITIES

Static defections are compared in order to verify the rigidities proposed by numerous investigators. Numerical example voided slab is shown in Fig 2.

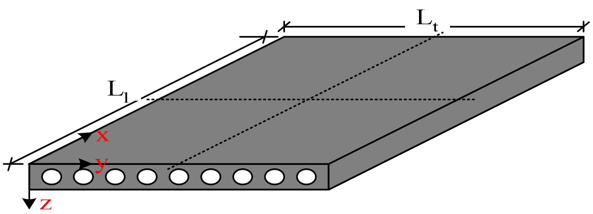

Fig. 2 Numerical example voided slab

The dimension of numerical example voided slab is 3000×4000×210 and the properties are shown in Table II. With the slab fixed supported at four edges, the concentrated load is applied at center point of example slab.

TABLE II
PROPERTIES OF NUMERICAL EXAMPLE VOIDED SLAB

L_l (mm)	L_t (mm)	d (mm)	t (mm)	P_y (mm)
4200	3150	120	210	210
t_v (mm)	E (kN/mm2)	ν	n	E_R
100	2.263E+01	0.167	9	0.70

The results are presented in the form of deflection as shown in Fig. 3 and Fig. 4 which are the deflection on the longitudinal and transverse center line. By comparisons with the deflection, the validity of the proposed method for rigidities calculation is demonstrated.

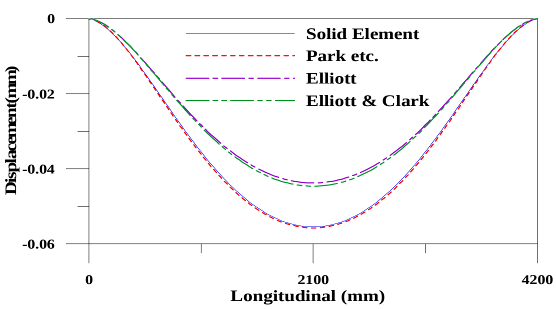

Fig. 3 Static deflection on the longitudinal line-self weight

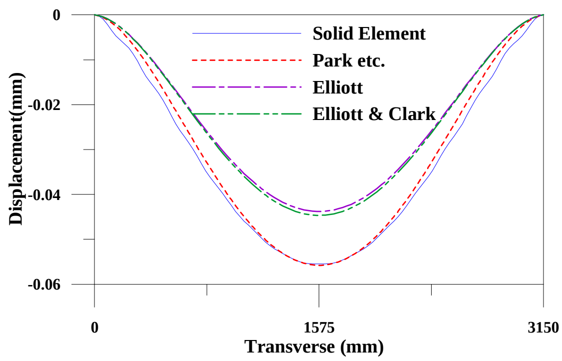

Fig. 4 Static deflection on the transverse line-self weight

Deflections of the proposed methods are in remarkably close agreement with those obtained by using 3D solid element. It is noted that the analysis utilizing simplified method produces results similar to those obtained by using 3D solid element. Thus it can be concluded that the analysis method with simplified idealization possess any advantage. Those are considerably more efficient so far as computer time is concerned. These simplified methods are apparently readily applicable to design office use.

V. CONCLUSIONS

As there is an infinite number of possible void shape and sizes, a comprehensive tabulation of rigidities is not possible. To determine rigidities parameters of a general cross-section, the simplified method can be used. The various methods that have been used during so years for analysis of voided slabs. The equivalent plate parameters (D_x , D_y and D_{xy}) could have significant effects on the accuracy of the analysis. These parameters should, therefore, model the actual structures as closely as possible without their requiring complex calculations. The methods appear to be the most appropriate for calculating the various equivalent plate parameters. From this study the following conclusions can be drawn concerning the choice of elastic rigidities of circularly voided slabs for use in orthotropic plate theory. With this suggested expressions of bending, coupling and torsional rigidities, the deflections and stress resultants in circularly voided slabs predicted by orthotropic plate theory show good agreement with results obtained by using 3D solid element

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