

Stochastic Scheduling to Minimize Expected Lateness in Multiple Identical Machines

Ghulam Zakria, Zailin Guan, Yasser Riaz Awan*, Wan Lizhi

Abstract—There are many real world problems in which parameters like the arrival time of new jobs, failure of resources, and completion time of jobs change continuously. This paper tackles the problem of scheduling jobs with random due dates on multiple identical machines in a stochastic environment. First to assign jobs to different machine centers LPT scheduling methods have been used, after that the particular sequence of jobs to be processed on the machine have been found using simple stochastic techniques. The performance parameter under consideration has been the maximum lateness concerning the stochastic due dates which are independent and exponentially distributed. At the end a relevant problem has been solved using the techniques in the paper..

Keywords—Quantity Production Flow Shop, LPT Scheduling, Stochastic Scheduling, Maximum Lateness, Random Due Dates

I. INTRODUCTION

THERE are many techniques which have been put forward in literature to solve scheduling problems which include optimal search algorithms, mathematical programming, artificial intelligence (AI) based search techniques, rules or heuristics, and commercial finite capacity schedulers [1, 2, 3, 4, 5, 6]. Because of large amount of difficulty in scheduling problems, researchers have often focused on simplifying the scope of scheduling with the intention of making analysis more tractable.

There are many real world problems in which parameters like the arrival time of new jobs, failure of resources, and completion time of jobs change continuously. If the modifications are major then it is better to design solutions, which are robust to these changes. There are many approaches being put forward to solve uncertain problems. Redundancy based scheduling generates schedules with temporal slack so that unexpected events during execution can be tackled using that reserved slack [7]. Contingent scheduling anticipates

G.Zakria is a PhD candidate in the Department of Industrial and Manufacturing System Engineering, Huazhong University of Science and Technology (HUST), Wuhan, China. (E-mail: engr_industrious@yahoo.com). Dr. Zailin Guan is a Professor in the State Key Lab of Digital Manufacturing Equipment & Technology, HUST, Wuhan, China. His research interests are advanced planning & scheduling and modeling & simulation of manufacturing systems.

Corresponding author: Yasser Riaz Awan is a student in the State Key Lab of Digital Manufacturing Equipment & Technology, HUST, Wuhan, China. (E-mail: yz_awan@hotmail.com).

likely changes and generates multiple schedule each behaving optimally to a different anticipated change [8]. Probabilistic scheduling uses probabilities of possible events and then calculates the schedules which behave optimally to a given parameter [9].

We are concerned with the following stochastic scheduling problem. A set of n jobs are to be processed on a multiple identical machines, which are all available at time zero. The processing times P_i of job, where $i = 1, 2, \dots, n$, are independent random variables. Associated with each job i is a stochastic due date D_i . The objective of the scheduler is to find which jobs are to be processed on which machine and then find the order of the jobs to be processed one by one on the given machine so as to optimize certain performance measures. The performance measure, that we will focus on will be the *maximum lateness* L_{\max} , which is among most important and traditionally studied performance measures, defined as

$$L_{\max} = \max \{L_i, i = 1, 2, \dots, n\}, \quad (1)$$

Where,

$$L_{\max} = \max_{i=1, 2, 3, \dots, n} \{L_i, i = 1, 2, \dots, n\}, \quad (2)$$

Equation (2) represents the lateness of completing job i , and C_i is the completion time of job i under a given policy. If we remove all the objects of uncertainties in the problem it becomes deterministic which was first studied in 1955 [10]. Since then a lot of research has been focused on this topic [11, 12, 13]. Although the deterministic version of this problem has been a subject of widespread research the stochastic counterpart has been neglected all along.

In this paper, we first schedule the jobs to the specific machine center by LPT method using deterministic processing times [14] then we deal with stochastic due dates which are independent and exponentially distributed. The exponential distribution has been widely used to model waiting times and other uncertain times, and is permissible when the level of uncertainty is high [15].

This paper is organized as follows; The related literature review has been given in the next section. In Section 3 the assumption related with our research are given. In Section 4 the basic methods and techniques to solve the problem have been given. An associated problem has been solved in Section 5 whereas the conclusion has been given in Section 6.

II. LITERATURE REVIEW

The scheduling of jobs on multiple machines with deterministic processing times in quantity production has been usually done by LPT scheduling method. It produces a schedule in which different jobs are given to a certain machine center but it does not specify the specific order in which those jobs are to be produced. In LPT scheduling first we sort the jobs in descending order of their total processing time then we initialize the current finishing time of each processor to zero. After that the first job in the job list is assigned to the processor having minimum finishing time. If more than one processor has same finishing time, any one of them can be selected. The new finishing time of processor is found by adding old finishing time of that processor and the processing time of selected job. In the end we remove that job from the job list. This procedure is repeated until the task list is empty.

To compute the maximum lateness in stochastic scheduling the following techniques have been used. If (i) denotes the index of the job scheduled at the i th position. Then a permutation policy can be represented as $\pi = ((1), (2), \dots, (n))$. The completion time of the i th completed job is then denoted by $C_{(i)}$, and $\mu_{(i)}$ is the corresponding rate of due date. The makespan (completion time of all jobs) is given by

$$C_{(n)} = \sum_{i=1}^n P_{(i)} = \sum_{i=1}^n P_i \quad (3)$$

which is independent of the sequence π . Let $F\pi(x)$ denote the cdf of L_{\max} under π . By (1.1) and (1.2) together with (2.1) and the independence between the due dates,

$$\begin{aligned} F_{\pi}(x) &= P_i(L_{\max} \leq x) = P_i(C_i - D_i \leq x, i=1, \dots, n) = \prod_{i=1}^n P_i(D_i \geq C_i - x) \\ &= \prod_{i=1}^n \exp\{-\mu_i(c_i - x)_+\} = \exp\left\{-\sum_{i=1}^n \mu_i(c_i - x)_+\right\} \quad (4) \end{aligned}$$

The following formula provides a representation of the performance measure under an arbitrary permutation policy, as well as an equivalent deterministic scheduling problem to the expected maximum lateness problem.

Under any permutation policy π ,

$$E[L_{\max}] = C_{(n)} - V(\pi), \quad (5)$$

Where,

$$V(\pi) = \frac{1}{\mu_{(n)}} - \sum_{j=1}^{n-1} \frac{\mu_j R_j}{\sum_{i=j}^n \mu_i \sum_{i=j+1}^n \mu_i} \quad (6)$$

With

$$R_j = \exp\left\{-\sum_{k=j+1}^n \sum_{i=k}^n \mu_i p_{(k)}\right\}, j=1, 2, 3, \dots, n-1 \quad (7)$$

Consequently, minimizing $E[L_{\max}]$ is equivalent to maximizing $V(\pi)$ in equation (6).

III. ASSUMPTIONS & NOTATIONS

A- Assumption

- i. All machines in the cell are identical.
- ii. Each job can be processed on any machine.
- iii. The processing time of each job is deterministic and known.
- iv. The random due dates are independent and exponentially distributed. The rate of distribution is also known.

B-Notations

P_i = processing time of job i

D_i = Random due date for job i

$C_{(i)}$ = Completion time of job i

$\mu_{(i)}$ = Rate of due date

π = Sequence of jobs

f_j = Completion time of machine j .

M_j = Machine j in the cell.

J_i = Job i in the sorted list

IV. METHODS AND TECHNIQUES

To first assign the jobs to their respective machine centers we will use LPT method and after that we will schedule the jobs in the particular order in which they are to be processed by stochastic techniques. The main performance parameter under consideration will be the maximum lateness.

A. LPT Method for assigning jobs to machines

Initialize the completion time f_j of each machine M_j to zero.
 Find the total processing time P_i of each job.
 Sort the jobs in descending order of total processing time P_i if any two jobs have same P_i sort them arbitrarily.
 Find the machine M_j with minimum f_j , if two shops have same value choose any one of them.
 Assign first job J_i in the sorted list to the choose machine M_j .
 Add processing time P_i of selected job J_i with the completion time of f_j of machine M_j .
 $f_j = f_j + P_i$
 Remove J_i from the list. Repeat it until the job list becomes empty.

B. Scheduling jobs

Now after we get the jobs which are to be processed on each machine we will find the particular order by the following technique.

Consider a case where $n=2$ jobs. Then the only possible sequences are $\pi_1 = \{1, 2\}$ and $\pi_2 = \{2, 1\}$. Now considering maximum lateness as performance measure and using equations 2.2, 2.4, 2.5, 2.6 for this particular case we get

$$E[L_{\max}^{\pi_1}] = p_1 + p_2 - \frac{1}{\mu_2} + \frac{\mu_1}{(\mu_1 + \mu_2)\mu_2} e^{-\mu_2 p_2}$$

and

$$E[L_{\max}^{\pi_2}] = p_1 + p_2 - \frac{1}{\mu_1} + \frac{\mu_2}{(\mu_1 + \mu_2)\mu_1} e^{-\mu_1 p_1}$$

Thus $E[L_{\max}^{\pi_1}] < E[L_{\max}^{\pi_2}]$ is equivalent to

$$-\frac{1}{\mu_2} + \frac{\mu_1}{(\mu_1 + \mu_2)\mu_2} e^{-\mu_2 p_2} < -\frac{1}{\mu_1} + \frac{\mu_2}{(\mu_1 + \mu_2)\mu_1} e^{-\mu_1 p_1}$$

That is,

$$\frac{1}{\mu_1} - \frac{1}{\mu_2} < \frac{\mu_1 \mu_2}{(\mu_1 + \mu_2)} \left[\frac{e^{-\mu_1 p_1}}{\mu_1^2} - \frac{e^{-\mu_2 p_2}}{\mu_2^2} \right]$$

Simple computation yields

$$\frac{\mu_1^2}{1 - e^{-\mu_1 p_1}} > \frac{\mu_2^2}{1 - e^{-\mu_2 p_2}}$$

From this equation we conclude that the jobs should be scheduled in the decreasing order of

$$\frac{\mu_i^2}{1 - e^{-\mu_i p_i}}, i = 1, 2, 3, \dots, n.$$

V. PROBLEM

Consider nine jobs $J_1, J_2 \dots J_9$ each has to pass through one operation in a machine cell having three similar machines. Rate of due dates and processing times for each job is given in table 1.

TABLE I
JOBS DATA

Jobs J_i	Processing times P_i	Rate of Due dates μ_i
J_1	11	0.528
J_2	13	0.749
J_3	16	0.129
J_4	15	0.679
J_5	19	0.921
J_6	11	0.425
J_7	11	0.627
J_8	9	1.434
J_9	18	0.549

There are three machines M_1, M_2, M_3 . We have to select which jobs are to be assigned to which machine. We use LPT.

Using LPT Technique

Sort the jobs in descending order of their processing times P_i . Sorted jobs are shown in table 2.

TABLE II
SORTED LIST OF JOBS IN DESCENDING ORDER OF THEIR PROCESSING TIMES

Jobs J_i	Processing times	Rate of Due dates μ_i
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	P_i	
J ₅	19	0.921
J ₉	18	0.549
J ₃	16	0.129
J ₄	15	0.679
J ₂	13	0.749
J ₁	11	0.528
J ₆	11	0.425
J ₇	11	0.627
J ₈	9	1.434

Initialize flow time of machines to zero. Place first job in the sorted list i.e., J₅ on the machine having minimum flow time. All machines have same flow time $f_i = 0$. Place the selected job to any one of the machine.

$$f_1 = f_1 + 19 = 0+19$$

$$f_1 = 19$$

Compare f_1, f_2, f_3 . Select one of them having minimum value. i.e., f_2 or f_3 .

Place next job J₉ on the selected machine.

$$f_2 = f_2 + 18 = 0+18$$

$$f_2 = 18$$

Compare f_1, f_2, f_3 . f_3 has minimum value, place next job J₃ to this machine.

$$f_3 = f_3 + 16 = 0+ 16$$

$$f_3 = 16$$

Compare f_1, f_2, f_3 . f_3 has minimum value, place next job J₄ to machine M₃ and find its flow time

$$f_3 = 16+15$$

$$f_3 = 31$$

Compare $f_1, f_2, f_3, f_1 = 19, f_2 = 18, f_3 = 31$. f_2 is minimum, place next job J₂ to machine M₂.

$$f_2 = 18 + 13 = 31.$$

Now f_1 has minimum value, assign next job, J₁ to machine M₁.

$$f_1 = f_1 + 11 = 19+11=30.$$

Again f_1 has minimum value, assign next job J₆ to machine M₁.

$$f_1 = f_1 + 11 = 30 + 11 = 41$$

Now,

$f_2 = f_3 = 31$ and $f_1 = 41$, place next job J₇ to any one of the machines M₂ or M₃.

$$f_2 = f_2 + 11 = 31+11 = 42$$

Now f_3 is smaller, place next job J₈ on M₃

$$f_3 = f_3 + 8 = 31 + 8 = 39.$$

Jobs allocated to each machine are shown in table 3.

TABLE III
JOBS ALLOCATED TO EACH MACHINE

M ₁	M ₂	M ₃
J ₅	J ₉	J ₃
J ₁	J ₂	J ₄
J ₆	J ₇	J ₈


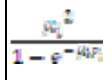
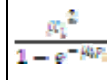
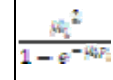
Now after we get the jobs which are to be processed on each machine we will find the particular order by following technique. The jobs would be scheduled in the decreasing order of 

TABLE IV

JOBS ALLOCATED TO EACH MACHINE WITH THEIR RESPECTIVE VALUE.

M ₁	M ₂	M ₃
		

J ₅	0.848	J ₉	0.301	J ₃	0.0182
J ₁	0.279	J ₂	0.561	J ₄	0.461
J ₆	0.182	J ₇	0.343	J ₈	2.056

Final job sequence according to given algorithm is shown below in table 5.

TABLE V
FINAL JOBS SEQUENCE ALLOCATED TO EACH MACHINE

Machines	Job sequence
M ₁	J ₅ , J ₁ , J ₆
M ₂	J ₂ , J ₇ , J ₉
M ₃	J ₈ , J ₄ , J ₃

VI. CONCLUSION

This paper tackles the problem of scheduling jobs with random due dates on multiple identical machines in a stochastic environment. The performance parameter under consideration has been the maximum lateness. Algorithms have been developed to produce a near optimal result in given conditions. In addition, an analytic solution in a special case has been investigated, which furthers the cause and understanding of the given research.

The solution developed however does not consider the effect of random processing time. In addition, if the due dates followed another distribution apart from the exponential distribution than the problem becomes much more complicated. The issue of multiple different machines with jobs having to be processed on multiple machines is also another extension. Additional research efforts are clearly essential to widen the investigations to these problems.

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