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Enhanced Particle Swarm Optimization Approach for Solving the Non-Convex Optimal Power Flow

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***Abstract*—**An enhanced particle swarm optimization algorithm (PSO) is presented in this work to solve the non-convex OPF problem that has both discrete and continuous optimization variables. The objective functions considered are the conventional quadratic function and the augmented quadratic function. The latter model presents non-differentiable and non-convex regions that challenge most gradient-based optimization algorithms. The optimization variables to be optimized are the generator real power outputs and voltage magnitudes, discrete transformer tap settings, and discrete reactive power injections due to capacitor banks. The set of equality constraints taken into account are the power flow equations while the inequality ones are the limits of the real and reactive power of the generators, voltage magnitude at each bus, transformer tap settings, and capacitor banks reactive power injections. The proposed algorithm combines PSO with Newton-Raphson algorithm to minimize the fuel cost function. The IEEE 30-bus system with six generating units is used to test the proposed algorithm. Several cases were investigated to test and validate the consistency of detecting optimal or near optimal solution for each objective. Results are compared to solutions obtained using sequential quadratic programming and Genetic Algorithms.

***Keywords*—**Particle Swarm Optimization, Optimal Power Flow, Economic Dispatch.

I. Introduction

POWER Engineers perform special tasks to optimally analyze, monitor, and control different aspects of power systems. The main required tasks are economic dispatch, unit commitment, state estimation, automatic generation control, and optimal power flow (OPF). The latter is regarded as the backbone tool that has been extensively researched since its first introduction [1].

OPF simply attempts to find the optimal settings of a given power system network that optimize a certain objective function while satisfying its power flow equations, system security, and equipment operating limits. Several control actions, some of which are generators' real power outputs and voltages, transformer tap changing settings, phase shifters, switched capacitors and reactors, are adjusted to achieve an optimal network setting based on the problem formulation. One of the main difficulties of the OPF problem is the nature of the control variables since some of them are continuous (real power outputs and voltages) and others are discrete (transformer tap setting, phase shifters, and reactive injections). The overall fuel cost function is by far the most common objective used in OPF studies. However, other traditional objectives are minimization of active power loss, bus voltage deviation, emission of generating units, number of control actions, and load shedding. Restructuring of the electric power industry has also introduced new objectives to the OPF problem like maximization the social welfare and individual supplier's profit [2;3].

Various non-classical optimization tools have emerged to cope with some of the traditional optimization algorithms' shortcomings. The main modern optimization techniques are genetic algorithm (GA), evolutionary programming (EP), artificial neural network (ANN), simulated annealing (SA), ant colony optimization (ACO), and particle swarm optimization (PSO). Most of these relatively new developed tools mimic a certain natural phenomenon in its search for an optimal solution like species evolution (GA and EP), human neural system (ANN), thermal dynamics of a metal cooling process (SA), or social behavior (ACO and PSO). They have been successfully applied to wide range of optimization problems in which global solutions are more preferred than local ones [4;5].

PSO has been previously used to solve the OPF problem. Researchers in references [6-8] have attempted to utilize PSO to solve the OPF problem considering different objective functions. In the aforementioned work, only continuous control settings were considered as optimization variables which restrict its applicability to real power systems. Authors in references [9-11] employed PSO to solve the OPF with the inclusion of both discrete and continuous optimization variables. Furthermore, they augmented the OPF objective function by adding penalty terms to transform the constrained OPF into unconstrained one. This approach usually suffers a major difficulty in how to properly select penalty factor values.

In this paper, a discrete PSO algorithm capable of handling both discrete and continuous optimization variables is proposed to solve the OPF problem. In the proposed

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algorithm, a mechanism for handling inequality constraints is developed to reinforce constraints avoiding the use of penalty factors or augmenting the objective function. Newton-Raphson iterative technique is incorporated within the PSO algorithm to minimize the power mismatch in the power flow equations. The objectives considered in this study are the conventional quadratic function and the augmented quadratic function.

II. Problem Formulation

The OPF aim is to optimize a certain objective subject to several equality and inequality constraints. It can be mathematically modeled as follows:

$$Min \quad F(x,u) \tag{1}$$

Subject to

$$g(x,u) = 0 \tag{2}$$

$$h_{\min} \le h(x,u) \le h_{\max} \tag{3}$$

where vector x denotes the state variables of a power system network that contains the slack bus real power output (P_{G1}), voltage magnitudes and phase angles of the load buses (V_{Lk}, θ_{Lk}), and generator reactive power outputs (Q_G). Vector u represents both integer and continuous control variables that consist of real power generation levels (P_{GN}) and voltage magnitudes ($|V_{GN}|$), transformer tap setting (T_k), and reactive power injections (Q_{Ck}) due to VAR compensations.

A. Objective Function

In this study, minimization of different fuel cost functions is considered to examine the performance of the proposed algorithm. Objective functions taken into considerations are the conventional quadratic function and the augmented quadratic function.

1) Fuel Cost:

The goal of the OPF problem is to find the best network settings that minimize the overall fuel cost function while imposing all network constraints. Traditionally, the overall fuel cost function for a number of thermal generating units can be modeled by a quadratic function (convex and differentiable) as follows:

$$F_1 = \sum_{i=1}^{i=N} (a_i + b_i P_i + c_i P_i^2) \text{ \$/hr} \tag{4}$$

However, this model neglects the valve point loading effect that introduces rippling effects to the actual fuel cost curve. Equation (4) is modified by adding an additional sine term to account for the valve effects in this manner [12]:

$$F_1 = \sum_{i=1}^{i=N} \left[a_i + b_i P_i + c_i P_i^2 + \left| e_i \sin(f_i (P_i^{\min} - P_i)) \right| \right] \text{ \$/hr} \tag{5}$$

This precise modeling creates challenges to most derivative-based optimization algorithms in finding the global solution since the objective is no longer convex nor differentiable every where.

B. Constraints

The OPF problem has two kinds of constraints:

1) Equality Constraints:

These are the sets of nonlinear power flow equations that govern the power system, i.e.

$$P_{G_i} - P_{D_i} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) = 0 \tag{6}$$

$$Q_{G_i} - Q_{D_i} + \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) = 0 \tag{7}$$

where P_{Gi} and Q_{Gi} are the real and reactive power outputs injected at bus i respectively, the load demand at the same bus is represented by P_{Di} and Q_{Di}, and elements of the bus admittance matrix are represented by $|Y_{ij}|$ and θ_{ij}.

2) Inequality Constraints:

These are the set of continuous and discrete constraints that represent the system operational and security limits like the bounds on:

1. The generators real and reactive power outputs;

$$P_{G_i}^{\min} \le P_{G_i} \le P_{G_i}^{\max}, \ i = 1, \ldots, G_N \tag{8}$$

$$Q_{G_i}^{\min} \le Q_{G_i} \le Q_{G_i}^{\max}, \ i = 1, \ldots, G_N \tag{9}$$

2. Voltage magnitudes at each bus in the network;

$$V_i^{\min} \le V_i \le V_i^{\max}, \ i = 1, \ldots, N \tag{10}$$

3. The discrete transformer tap settings;

$$T_i^{\min} \le T_i \le T_i^{\max}, \ i = 1, \ldots, T_N \tag{11}$$

4. The discrete reactive power injections due to capacitor banks;

$$Q_{C_i}^{\min} \le Q_{C_i} \le Q_{C_i}^{\max}, \ i = 1, \ldots, C_N \tag{12}$$

Note that P_{Gi}, Q_{Gi}, and V_i are continuous variables while T_i and Q_{Ci} are discrete ones.

5. The transmission lines loading;

$$S_{L_i} \le S_{L_i}^{\max}, \ i = 1, \ldots, L_N \tag{13}$$

III. The Proposed Hybrid Algorithm

Two scientists, Kennedy and Eberhart, first introduced PSO in 1995 as a new heuristic method [13]. The original PSO model was intended to handle only continuous nonlinear optimization problems. However, further improvement elevated the PSO capabilities to solve a wider class of problems. References [14-16] present more information on PSO recent developments. The concept behind this optimizer came from early attempts conducted by Eberhart and Kennedy to model the flocking behavior of many species, like birds or school of fish, in their food hunting. They realized that such species try to approach their target in an optimal manner which resembles finding the optimal solution to any mathematical optimization problem.

A swarm consists of number of particles that evolve or fly throughout the problem hyperspace to search for optimal or near optimal solution. The coordinates of each particle represent a possible solution with two vectors associated with it, the position (X_i) and velocity (V_i) vectors. In N-dimensional search space, $X_i = [x_{i1}, x_{i2}, ..., x_{iN}]$ and $V_i = [v_{i1}, v_{i2}, ..., v_{iN}]$ are the two vectors associated with each particle *i*. During their search, particles interact with each others in a certain way to optimize their search experience. Different variants of the particle swarm paradigms were proposed but the most general one is the *gbest* model where the whole population is considered as a single neighborhood throughout the optimization process [17]. Throughout their flying experince, the particle with the best solution shares its position coordinates (*gbest*) information with the rest of the swarm. Then, each particle updates its coordinates based on its own best search experience (*pbest*) and *gbest* according to the following equations:

$$v_i^{k+1} = wv_i^k + c_1 r_1 (pbest_i^k - x_i^k) + c_2 r_2 (gbest^k - x_i^k) \quad (14)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (15)$$

where

- c_1 and c_2 are two positive constants, they keep balance between the particle's individual and social behavior when they are set equal;
- r_1 and r_2 are two randomly generated numbers with a range of [0,1] added in the model to introduce stochastic nature;
- *w* is the inertia weight and it is a linearly decreasing function of the iteration index:

$$w(k) = w_{max} - \left(\frac{w_{max} - w_{min}}{Max.\ Iter.} \right) \bullet k \quad (16)$$

- *k* is the iteration index.

The developed hybrid approach combines PSO technique with Newton-Raphson based power flow program in which the former technique is used as a global optimizer to find the best combinations of the mixed type control variables while the latter serves as a solver to the nonlinear power flow equations. The PSO employs a population of particles or possible solutions to explore the feasible solution hyperspace in its search for an optimal solution. Each particle's position is used as a feasible initial guess for the power flow subroutine. This method of multiple initial solutions can provide better chance of detecting an optimal solution to the power flow equations that would globally minimize the objective function.

A. Constraints Handling Methods

Various methods were proposed to handle constraints in evolutionary computation optimization algorithms just like in the case of the PSO. The most common used constraint handling methods are [18]:

1. Feasible solution preservation: in this approach, intial solutions are placed in the feasible search space and maintain within by using an update mechanism that generates only feasible solutions.
2. Rejection of infeasible solution: this method rejects any solution that violates the feasible search space.
3. Penalty function: in which a penalty factor is added to the objective once any constraint violation occurs.
4. Solution repair method: this approach converts the infeasible solution to a feasible one by performing special operations.

Deciding on the proper constraints handling method is highly reliant on the problem's nature. Reference [18] indicates that in the solution repair method, the process of reinstating the infeasible solution to a feasible one can be as challenging as solving the original problem. Disadvantages of the penalty function method are presented in the introduction part of this paper.

1) Hybrid Constraints Handling Mechanism

A new constraint handling technique that combines the preserving feasible solution and infeasible solution rejection methods is proposed to handle different constraints imposed on the OPF problem. In the developed mechanism, each element of the particle's position vector is randomly initialized within its feasible range. After measuring the fitness of each particle and updating its position vector, the new position is checked for feasibility. If a given element of the position vector exceeds its restrictions, this position is discarded and its value is restored to the best position achieved by that particle, i.e. *pbest*. This reinstatement operation keeps that infeasible particle *alive* as a possible candidate that could find the optimal solution instead of a complete rejection that eliminates its potential in the swarm.

IV. Simulation Results and Discussion

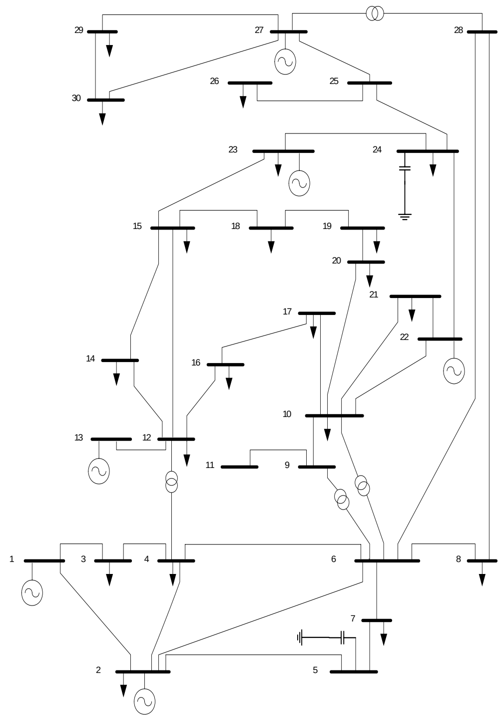

Fig. 1 IEEE 30-bus standard test system

Simulation tasks were carried out in Matlab® computing environment and the standard IEEE 30-bus test system was used to validate its potential. It consists of six generating units interconnected with 41 branches of a transmission network to serve a total load of 189.2 MW and 107.2 Mvar as shown in Fig. 1. Detailed description of the system's data is presented in [19].

Note that the original system has two capacitors banks installed at bus 5 and 24 with ratings of 19 and 4 Mvar respectively. The following cases were considered:

Case study 1: The smooth fuel cost function in (4) is minimized considering only the continuous control variables, i.e. real power outputs and voltages at voltage-controlled buses. A comparison of results obtained using the PSO to those obtained using MATPOWER, MATLAB-based software that uses sequential quadratic programming (SQP) to solve the OPF, and GA are shown in Table I. MATPOWER is capable of solving the OPF when the objective is represented in polynomial form and is only capable of handling continuous variables. Results clearly indicate that PSO achieved better solution when compared to other methods.

TABLE I
PERFORMANCE OF SQP, PSO AND GA FOR CASE 1

	Fuel Cost ($/hr)		
Method	SQP	PSO	GA
P_{g1}	41.5400	43.6106	42.0125
P_{g2}	55.4000	58.0603	56.1285
P_{g13}	16.2000	17.5555	16.8002
P_{g22}	22.7400	22.9976	22.8546
P_{g23}	16.2700	17.0561	16.7745
P_{g27}	39.9100	32.5670	37.7854
V_1	0.9820	1.0000	0.9975
V_2	0.9790	0.9996	0.9874
V_{13}	1.0640	1.0594	1.0624
V_{22}	1.0160	1.0116	1.0125
V_{23}	1.0260	1.0214	1.0247
V_{27}	1.0690	1.0372	1.0474
Objective	576.8920	575.4108	576.2471

Case study 2: Since PSO is capable of handling optimization problems in which the objective is not required to be convex or differentiable, the fuel cost function is augmented with an additional sine term as in (5). This addition increases the degree of non-smoothness of the objective function significantly. As a result, more number of particles is needed to explore this complex solution hyperspace efficiently. Table II tabulated the results obtained using different swarm's size. Increasing the swarm's size improved the PSO performance in achieving better results at the expense of computational time.

V. CONCLUSION

The investigations presented in this paper examine the applicability of PSO in solving the OPF problem under different formulations and considering different objectives. The obtained results are partially compared to the outcomes of other optimization techniques. A hybrid constraint handling strategy is proposed to preserve only feasible solutions without the need to use penalty factors. The PSO is combined with Newton-Raphson algorithm to form a hybrid optimizer that can solve the discrete OPF problem with the inclusion of valve loading effects. This highlights the PSO capability of handling optimization problems with more complex modeling of system objectives and/or constraints. PSO performance and robustness in its search for optimal solution is highly dependant on its parameters tuning and the shape of the objective function. Objective functions with smooth shapes tend to require less number of particles and iterations to converge to the optimal solution while the ones with rough surface would require more number of particles and iterations to reach the same quality of solution.

TABLE II
RESULTS OF CASE 2 UNDER DIFFERENT SWARM'S SIZE

	20	30	100
P_{g1}	47.0677	47.0947	47.1264
P_{g2}	42.9114	42.3594	71.3658
P_{g13}	8.7898	35.9024	8.9719
P_{g22}	44.7279	37.3588	37.3909
P_{g23}	8.9825	8.8255	8.9935
P_{g27}	42.0437	20.9590	20.7771
V_1	1.0000	1.0000	1.0000
V_2	1.0988	1.0086	1.0965
V_{13}	1.0911	1.0166	1.0365
V_{22}	1.0866	1.0815	0.9821
V_{23}	1.0480	1.0568	1.0484
V_{27}	1.0289	1.0800	1.0876
Q_{C5}	33.0000	16.0000	29.0000
Q_{C24}	35.0000	15.0000	12.0000
T_{6-9}	1.0400	1.0100	1.0200
T_{6-10}	1.0100	1.0000	0.9900
T_{4-12}	1.0400	0.9900	1.0200
T_{27-28}	0.9900	1.0300	1.0400
Objective	658.4158	645.3329	615.2496

REFERENCES

[1] R. B. Squires, "Economic dispatch of generation directly from power system voltages and admittances," AIEE Transactions on Power Apparatus and Systems, vol. PAS-79, no. 3, pp. 1235-1245, 1961.
[2] J. D. Weber and T. J. Overbye, "An individual welfare maximization algorithm for electricity markets," IEEE Transactions on Power Systems, vol. 17, no. 3, pp. 590-596, 2002.
[3] Y. He, Y. H. Song, and X. F. Wang, "Bidding strategies based on bid sensitivities in generation auction markets," IEE Proceedings-Generation, transmission and Distribution, vol. 149, no. 1, pp. 21-26, 2002.
[4] Y. Valle, G. K. Venayagamoorthy, S. Mohagheghi, J. Hernandez, and R. G. Harley, "Particle Swarm Optimization: Basic Concepts, Variants and Applications in Power Systems," IEEE Transactions on Evolutionary Computation, vol. 12, no. 2, pp. 171-195, 2008.
[5] M. R. AlRashidi and M. E. El-Hawary, "Applications of computational intelligence techniques for solving the revived optimal power flow

problem," Electric Power Systems Research, vol. 79, no. 4, pp. 694-702, 2009.

[6] C. Wang, H. Yuan, Z. Huang, J. Zhang, and C. Sun, "A modified particle swarm optimization algorithm and its application in optimal power flow problem," Proceedings of 2005 International Conference on Machine Learning and Cybernetics., vol. 5, pp. 2885-2889, Guangzhou, China, 2005.

[7] R. Ma, P. Wang, H. Yang, and G. Hu, "Environmental/Economic Transaction Planning Using Multiobjective Particle Swarm Optimization and Non-Stationary Multi-Stage Assignment Penalty Function," 2005 IEEE/PES Transmission and Distribution Conference and Exhibition: Asia and Pacific, pp. 1-6, Dalian, China, 2005.

[8] B. Zhao, C. X. Guo, and Y. J. Cao, "Improved particle swarm optimization algorithm for OPF problems," IEEE/PES Power Systems Conference and Exposition, pp. 233-238, New York, USA, 2004.

[9] P. N. Biskas, N. P. Ziogos, A. Tellidou, C. E. Zoumas, A. G. Bakirtzis, and V. Petridis, "Comparison of two metaheuristics with mathematical programming methods for the solution of OPF," IEE Proceedings-Generation, transmission and Distribution, vol. 153, no. 1, pp. 16-24, 2006.

[10] S. He, J. Y. Wen, E. Prempain, Q. H. Wu, J. Fitch, and S. Mann, "An improved particle swarm optimization for optimal power flow," International Conference on Power System Technology, vol. 2, pp. 1633-1637, The Pan Pacific, Singapore, 2004.

[11] M. A. Abido, "Optimal power flow using particle swarm optimization," International Journal of Electrical Power & Energy Systems, vol. 24, no. 7, pp. 563-571, 2002.

[12] D. C. Walters and G. B. Sheble, "Genetic algorithm solution of economic dispatch with valve point loading," IEEE Transactions on Power Systems, vol. 8, no. 3, pp. 1325-1332, 1993.

[13] R. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," Proceedings of the Sixth International Symposium on Micro Machine and Human Science, pp. 39-43, Nagoya, Japan, 1995.

[14] H. Xiaohui, S. Yuhui, and R. Eberhart, "Recent advances in particle swarm," Proceedings of 2004 Congress on Evolutionary Computation, vol. 1, pp. 90-97, 2004.

[15] R. C. Eberhart and Y. Shi, "Guest Editorial Special Issue on Particle Swarm Optimization," IEEE Transactions on Evolutionary Computation, vol. 8, no. 3, pp. 201-203, 2004.

[16] J. Kennedy and R. C. Eberhart, Swarm Intelligence. San Francisco: Morgan Kaufmann, 2001.

[17] Y. Shi and R. Eberhart, "A modified particle swarm optimizer," IEEE World Congress on Computational Intelligence, pp. 69-73, Alaska, USA, 1998.

[18] G. Coath and S. K. Halgamuge, "A comparison of constraint-handling methods for the application of particle swarm optimization to constrained nonlinear optimization problems," The 2003 Congress on Evolutionary Computation, vol. 4, pp. 2419-2425, Canberra, Australia, 2003.

[19] M. R. AlRashidi and M. E. El-Hawary, "Hybrid Particle Swarm Optimization Approach for Solving the Discrete OPF Problem Considering the Valve Loading Effects," IEEE Transactions on Power Systems, vol. 22, no. 4, pp. 2030-2038, 2007.