

# Influence of Heat Transfer on Stability of Newtonian and Non-Newtonian Extending Films

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**Abstract**—The stability of Newtonian and Non-Newtonian extending films under local or global heating or cooling conditions are considered. The thickness-averaged mass, momentum and energy equations with convective and radiative heat transfer are derived, both for Newtonian and non-Newtonian fluids (Maxwell, PTT and Giesekus models considered). The stability of the system is explored using either eigenvalue analysis or transient simulations. The results showed that the influence of heating and cooling on stability strongly depends on the magnitude of the Peclet number. Examples of stabilization or destabilization of heating or cooling are shown for  $Pe \ll 1$ , and  $Pe = O(1)$  cases, for Newtonian and non-Newtonian flows.

**Keywords**—Extended films, stability, eigen-analysis for stability, transient response, polymer instability, Non-Newtonian fluids.

## I. INTRODUCTION

TO generate thin filaments, a molten polymer is pumped through a circular opening in a metal plate and drawn down by axial tension. The thread then cools, solidifies and is extracted, see Matovich et al. [1]. The study of the stability of this process is the focus of our work, particularly as it relates to the velocity ratio, or the ratio of the velocity of the film at the exit to the velocity at the entrance as shown below.

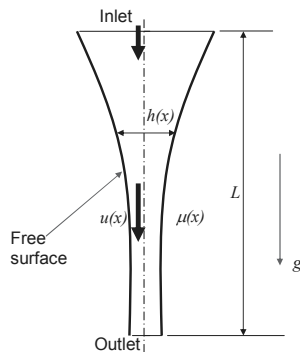


Fig. 1 Problem definition

The film will get unstable when a critical velocity ratio beyond which the perturbations will amplify in time. The

preliminary work started in an IMA workshop Mathematical Modeling in Industry XII in 2008 [2]. Since then, the work is expanded to a range of non-isothermal conditions and recently to non-Newtonian fluids, Amosov, Boratav, Zheng 2009 [3], Boratav & Zheng 2010 [4], Boratav, Zheng, Zhou 2011 [5]. Some other references in literature can be found in: Pearson et al. 1969 [6], Yeow 1974 [7], German et al. 2006 [8], Howell 1996 [9], Suman et al. 2008 [10], Zhou & Kumar 2010 [11].

The organization of the paper is as follows. In Section II, the mathematical formulation is presented. In Section III, problem solution is given. Section III A gives results from the steady and eigen solutions. In Section III B the results from the transient solutions are presented. Section IV summarizes the conclusions.

## II. PROBLEM FORMULATION

The problem sketch is shown in Fig. 1, where  $x$  is the down-the-draw direction,  $L$  is the length of the domain,  $h(x)$ ,  $u(x)$ ,  $\mu(x)$  are the thickness, (averaged across thickness) velocity and viscosity respectively. Gravity is applied in the down-the-draw direction. All the variables are functions of only the down-the-draw coordinate  $x$ , i.e., they are thickness averaged quantities and cross-draw dependence are neglected. It is assumed that thickness  $h(x)$  is much smaller than the length of the domain,  $L$  in all our derivations that use this small value expansion.

The governing equations of the 1D viscous sheet problem are given in the following non-dimensional form, namely, the mass and momentum conservation

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} = 0 \quad (1)$$

$$\text{Re} \frac{\partial(hu)}{\partial t} = \frac{\partial}{\partial x} \left( 4h\mu \frac{\partial u}{\partial x} - \text{Re} u^2 \right) + \text{St} \cdot h \quad (2)$$

where

$$P_{xx} = 4\mu(T) \frac{\partial u}{\partial x} \quad (3)$$

is the dimensionless stress with  $\mu(T)$  being the dimensionless temperature-dependent viscosity, and

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$$\text{Re} = \frac{\rho UL}{\mu_c} \quad \text{St} = \frac{\rho g L^2}{\mu_c U} \quad (4)$$

are the Reynolds number (which represents the ratio of inertial to viscous forces) and Stokes number (which represents the ratio of gravity to viscous forces) respectively.  $\rho$  is the density,  $\mu_c$  is the characteristic viscosity,  $g$  is the gravitational acceleration, and  $L$  and  $U$  are the characteristic length, velocity respectively. The energy equation is given by:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{1}{\text{Pe}} \left( \frac{\partial^2 T}{\partial x^2} + A \right)$$

$$A = \frac{2}{h} \left( \frac{1}{2} \frac{\partial h}{\partial x} \frac{\partial T}{\partial x} - \text{Bi} (T - T_{\text{amb}}(x)) - \text{Cr} (T^4 - T_{\text{inf}}^4(x)) \right) \quad (5)$$

where the non-dimensional numbers appearing in the energy equation are defined below:

$$\text{Pe} = \frac{\rho C_p UL}{k} \quad \text{Bi} = \frac{\alpha L}{k \delta} \quad \text{Cr} = \frac{\varepsilon \sigma L T_c^3}{k \delta} \quad (6)$$

In the case of non-Newtonian material, the stress-strain relation is no longer linear. The upper-convected Maxwell (UCM) model is the simplest yet realistic viscoelastic model that has been extensively used in the literature Bird et al. [12]. In non-dimensional form, the equations for the two stress components can be written as

$$\sigma_{xx} + \text{De} \mu(T) \left( \frac{\partial \sigma_{xx}}{\partial t} + u \frac{\partial \sigma_{xx}}{\partial x} - 2 \sigma_{xx} \frac{\partial u}{\partial x} \right) = 2 \mu(T) \frac{\partial u}{\partial x}$$

$$\sigma_{yy} + \text{De} \mu(T) \left( \frac{\partial \sigma_{yy}}{\partial t} + u \frac{\partial \sigma_{yy}}{\partial x} + 2 \sigma_{yy} \frac{\partial u}{\partial x} \right) = -2 \mu(T) \frac{\partial u}{\partial x} \quad (7)$$

where  $\text{De} = \lambda / \text{tf}$  time and the flow time. The relaxation time  $\text{tf}$  of a viscoelastic material is the ratio of its viscosity and its elastic modulus  $\lambda = \mu / G$  where  $\mu$  is the viscosity and  $G$  is the elastic modulus. The characteristic flow time  $\text{tf}$  can be related to the dimensionless time  $L/U_0$ , where  $L$  is the distance from the entrance ( $x = 0$ ) to the exit ( $x = L$ ) and  $U_0$  is the fluid velocity at the entrance. To further simplify the problem, we consider the relaxation time of the glass is constant from the entrance to the exit. Thus, the Deborah number remains constant in the computational domain. Also it is worth noting that in the limit of vanishing  $\text{De}$ , the UCM model reduces to Newtonian.

We also considered two more viscoelastic models, PhanThien-Tanner (PTT) and Giesekus models in our work. For brevity, the governing equations for these two models will not be presented.

The boundary conditions for the governing equations are given as the following:

$$h(x=0)=1, u(x=0)=1, T(x=0)=1, \sigma_{xx} + \sigma_{yy} = 0$$

$$u(x=1) = dr \quad (8)$$

where  $dr$  is the draw ratio, defined as the ratio of the outlet velocity to the inlet velocity.

Note that In addition to the boundary conditions that are laid out for the Newtonian system, an extra boundary condition for the viscoelastic system is needed since the non-newtonian system has two more equations and the momentum equation has changed its type from 2nd order to 1st order. A stress relation at  $x = 0$ ,  $\sigma_{xx} + \sigma_{yy} = 0$ , for the extra boundary condition is used in our study. Such a stress relation has been used in viscoelastic fiber draw studies, for example, see [13], [11].

For the stability analysis, we pursue two paths: (1) Eigen-analysis, (2) Transient analysis. For the eigen-analysis, we solve the system numerically to attain a steady state, after which all the variables are perturbed pursuing a standard normal mode analysis. The minimum of real parts of eigenvalues are checked to see if positive (unstable) or negative (stable).

For the transient analysis, we solve the system numerically to attain a steady state after which we perturb one of the variables, for example the velocity at the outlet, for a finite time. After the perturbation, the system is let to evolve and the transient response of variables is followed in time. For example, the time oscillations of outlet thickness will show if they grow in time (unstable) or decay (stable).

### III. RESULTS

A steady state solution is first obtained with given parameters and process conditions numerically. Then, the stability of the steady state to infinitesimal disturbances is determined by either (i) linearizing the system of partial difference equations, leading to an eigenvalue problem or (ii) the transient response to time-dependent perturbations of the system is obtained. COMSOL Multiphysics 3.5 software is used in our numerical computations, with our scripting the problem.

Considering the above-given energy equation (5), in this analysis, the radiation term will be omitted just for brevity. Inclusion of that term will not influence the main points that will be presented in this work.

There are three regimes that can be encountered depending on the magnitude of the Peclet number, which is the ratio of the right hand side to the left hand side (the advection term). Namely:

$$\begin{array}{ll} \text{Pe} \gg 1 & \text{Advection dominated} \rightarrow \text{isothermal} \\ \text{Pe} \sim O(1) & \text{Coupled} \\ \text{Pe} \ll 1 & \text{Advection insignificant} \end{array}$$

The spatial variation of the ambient temperature  $T_{amb}$  (which is specified as an input) can be related to the viscosity using the following expressions:

$$\frac{\mu(T_{amb}(x))}{\mu_c} = \exp(qx)$$

$$\mu(T_{amb}(x)) = 10^{\frac{A + \frac{B}{T_{amb}(x) - T_0}}{T_c - T_0}}$$

$$\mu_c = 10^{\frac{A + \frac{B}{T_c - T_0}}{T_c - T_0}} \quad (8)$$

where the relation between the viscosity and temperature with fluid-dependent properties A, B and  $T_0$  is known as the Vogel-Fulcher-Tammann (VFT) equation in literature.

Here we give examples from eigen-analysis and transient response for different regimes:

#### A. Steady Solutions and Eigen-Analysis for Two Regimes

We will consider the second ( $Pe=O(1)$ , and small  $Pe$ ) regimes in this section. The large  $Pe$  regime results have been presented before [14]. The first plot below shows the fluid temperature distribution along the flow direction for different values of  $q$ , which adjusts the magnitude and the type (heating or cooling) of ambient transfer. The second plot shows the stability diagram, namely, the relation between heating/cooling and the critical draw ratio. The selected parameters are given on the plot. For this  $Pe = O(1)$  regime, heating destabilizes and cooling stabilizes. This is contrary to what we have found for the large  $Pe$  regime [14].

The other example is given below for the small  $Pe$  regime. The temperature solutions show that the fluid temperatures approach the ambient (dashed) distribution for large (negative or positive) values of  $q$ . The eigen-analysis shows that heating is stabilizing and cooling is destabilizing, opposite to the  $Pe=O(1)$  regime shown above.

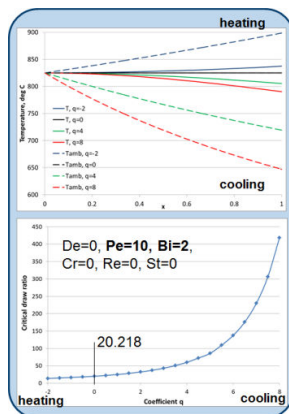


Fig. 2 Temperature solutions, ambient temperature and stability diagram for  $Pe=O(1)$  regime

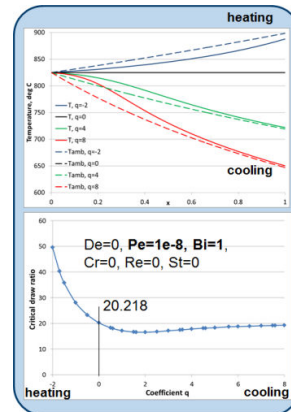


Fig. 3 Temperature solutions, ambient temperature and stability diagram for  $Pe \ll 1$  regime

#### B. Steady Solutions and Transient Results for Two Regimes

For the two regimes described above, we also obtained transient solutions. We chose two different  $q$  values which are:  $q = -2$ , and  $q = 4$ . The plot below shows the thickness variations in time for both  $q = -2$  and  $q = 4$  cases, and for the Newtonian case ( $De = 0$ ) and one non-Newtonian case ( $De = 6$  e-3). Also shown in this plot is the perturbation velocity (exerted in finite time) and the outlet velocity values are also shown on the y-axis on the right hand side.

It is seen for the heating case ( $q = -2$ ), the non-Newtonian effects are destabilizing, whereas for the cooling case ( $q = 4$ ), the viscoelasticity stabilizes. This is for the  $Pe = O(1)$  regime.

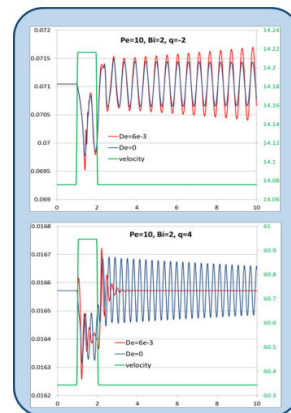


Fig. 4 Transient response for  $Pe = O(1)$  regime, for two different ambient temperatures (heating and cooling), one Newtonian and one non-Newtonian case are presented

Next, for the small  $Pe$  regime, we repeat the above analysis. For both  $q = -2$  and  $q = 4$  cases, the non-Newtonian effects are stabilizing in this regime as seen below.

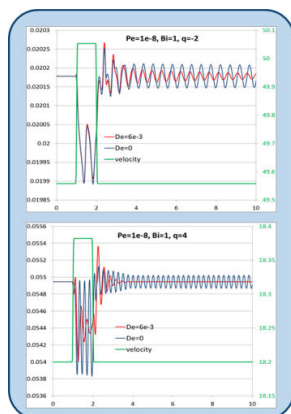


Fig. 5 Transient response for  $Pe \ll 1$  regime, for two different ambient temperatures (heating and cooling), one Newtonian and one non-Newtonian case are presented

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