Property Aggregation and Uncertainty with Links to the Management & Determination of Critical Design Features

Steven Whittle, and Ingrida Valiusaityte

Abstract—Within the domain of Systems Engineering the need to perform property aggregation to understand, analyze and manage complex systems is unequivocal. This can be seen in numerous domains such as capability analysis, Mission Essential Competencies (MEC) and Critical Design Features (CDF). Furthermore, the need to consider uncertainty propagation as well as the sensitivity of related properties within such analysis is equally as important when determining a set of critical properties within such a system.

This paper describes this property breakdown in a number of domains within Systems Engineering and, within the area of CDFs, emphasizes the importance of uncertainty analysis. As part of this, a section of the paper describes possible techniques which may be used within uncertainty propagation and in conclusion an example is described utilizing one of the techniques for property and uncertainty aggregation within an aircraft system to aid the determination of Critical Design Features.

Keywords—Complex Systems, Critical Design Features, Property Aggregation, Uncertainty.

I. INTRODUCTION

ENGINEERS at various stages, levels and domains of the design of complex systems, such as a modern fighter aircraft, make use of aggregated properties to obtain a wide ranging picture of the design (or requirements of the design) of a system, while lower level properties are used to describe the detailed design in a specific domain and sub-system. For example, in theory, the aerodynamicist has little concern initially for the material used to build the wing, the structural analyst is not interested in the drag coefficient of the wing, and the E-M engineer is happy to produce an aircraft geometry that is covered in Radar Absorbent Material (RAM) without concern for its ability to fly effectively. The e-m engineer is concerned only with reducing the Radar Cross Section of the aircraft (together with Electromagnetic Compatibility issues) not with the aerodynamic consequences of panel detachment nor the effect on the engines of such

S. Whittle is with the Systems Engineering Innovation Centre, BAE Systems, Loughborough University, Loughborough, LE11 3TU, UK (phone: +44-1509-635221; fax: +44-1509-635231; e-mail: s.whittle2@ Lboro.ac.uk).

I. Valiusaityte is with the Systems Engineering Innovation Centre, BAE Systems, Loughborough University, Loughborough, LE11 3TU, UK (e-mail: i.valiusaityte @Lboro.ac.uk).

panels being ingested. It is not until the higher level property of say survivability is considered that the need for efficient wings (to enable high speed due to reduced drag), low RCS (to reduce the probability of detection) and structural efficiency (to enable the aircraft to pull higher g turns), amongst other considerations, is recognized. At this stage the design is compromised (traded-off) in each of the different domains to enable the production of a robust, optimum aircraft system.

Unfortunately, this aggregation of properties results in somewhat fuzzy, ill-defined and immeasurable higher level attributes such as flexibility, lethality and "fitness for purpose". These should be compared with the well-defined measurable base level properties of, say, length, density and electric current. To ease the understanding of these high level attributes, often called "ilities" , it is the authors view that one needs to provide a hierarchical structure linking the system properties, together with the influence lower level properties have on the linked higher level ones. As an example related to Military aircraft design see Fig. 4. In its most rigorous form these relationships would be provided by an explicit complex mathematical model, but in practice, as one progresses up this property hierarchy, subjective views dominate. As a minimum the sensitivity between directly linked properties should be agreed with the relevant experts.

This breakdown / aggregation is now discussed in three different domains.

II. EXAMPLES OF PROPERTY AGGREGATION

A. Capability

The clear specification of system of systems performance or effectiveness is a pressing and difficult task given many customers transition from equipment to capability acquisition, and the current drive towards Networked Enabled Capability.

Understanding the properties of 'complex systems' with a large number of highly interconnected heterogeneous elements and how this relates to the provision of capability poses today a grand challenge for system research. Education and heath care systems, and complex transport systems –civil aircraft and rail networks – illustrate current limitations on predicting high-level system properties. It is not unusual for good-intentioned changes to organization, process or technology to lead to unpredicted detrimental effects, either on

other parts of the system, or the system as a whole. It is not difficult to find examples in the press of where existing 'systems' exhibit undesirable and/or unexpected characteristics. It is also common to see accounts of how attempts to improve and/or extend systems can fail spectacularly, for technical, operational or managerial reasons. This can be simply because of a lack of holistic (or systems) perspective and/or a lack of appropriate model-based analysis.

To obtain a solution too many of the above questions one needs to develop a framework of properties, functionality and architecture linked by existing, meta- or pseudo- models. This will enable a common understanding of requirements / capability and aid trade off.

The need to trade-off across all Lines of Development, through the lifecycle and at different abstractions from the geo-political and national level to equipment and onwards to system, sub-system and even component level, will require a functional linkage between properties to facilitate optimization, and cost models obviously provide an important input to this overall optimization problem.

Uncertainty obviously plays a large role in capability provision. Can we manage risk, to provide a sufficiently robust system, without allowing for the determination & propagation of uncertainty within, for example, models, data, and environmental bounds?

An example of the breakdown of capability is shown in Fig. 1 where the medium weight capability for the rapid deployment of troops within, say, a peacekeeping scenario is broken down at the high level via a bulls-eye target. The colour coding is representing the weighting between the different levels.
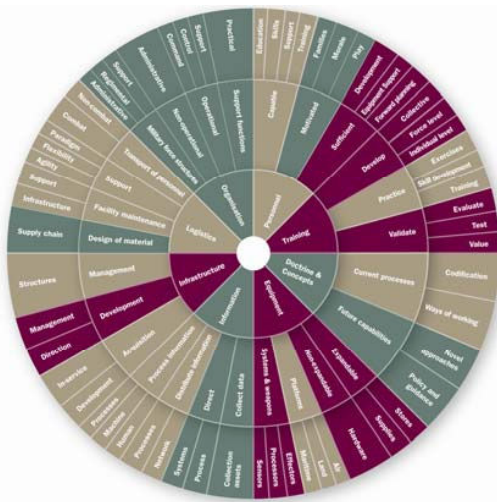

Fig. 1 Capability Bull's-eye[TM]

B. Mission Essential Competencies Related to Pilot Training

Mission Essential Competencies (MECs) [1], [2], [3], [4] have been pioneered in the U.S., although wider application has also occurred such as in the UK Composite Air Operations

(COMAO) experiments for training using distributed networks [5]. These MECs have been developed with the aim of capturing the dynamics of combat and according to Schreiber et al. [6] the MECs for an air-to-air mission process define which skills constitute a proficient fighter pilot in combat, that are readily applicable to a realistic environment. MECs are defined as "the higher order individual, team, and inter-team competencies that a fully prepared pilot, crew, or flight require for successful mission completion under adverse conditions in a non-permissive environment" [4]. The hierarchical structure of the MEC model consists of the MECs at the top level, the Supporting Competencies (SCs) at the next level and Knowledge and Skills (SKs) at the lowest level. MECs are broad in nature and difficult to measure. Supporting Competencies are more general than MECs, and reflect areas of competence needed in carrying out the MECs. Some supporting competencies are applicable across all MECs, and others are applicable for only a few MECs. A supporting competency can then be attained through a variety of SK requirements, which have a more suitable granularity for measurement. In addition to Skills & Knowledge, the MECs can also include critical experiences that have direct impact on a particular SK under operational-like conditions [2].

Intra-relationships of the MEC model depend on the scenario, although binary relationships can be generalised. The model in Fig. 2 shows the three levels of competencies, MECs, SCs and SKs, with unclear links between these levels as well as the complex interactions between the components of the same level, as is the case in the real world.

Given such conceptual linking, training needs can be easily determined by working backwards from the highest level (MECs) to the lowest level (SK) and then to specific training tasks (Experiences).

MECs for an air-to-air mission quoted by Symons et al. [2] are as follows:

- Organize Forces to Enable Combat Employment
- Detects Factor Groups in Area of Responsibility
- Intercept and Target Factor Groups
- Engage-Employ Ordnance & Deny Enemy Ordnance
- Assessment/Reconstitute-Initiate Follow on Actions
- Remain Oriented to Force Requirements
- Recognize Trigger Events that Require Shift in Phase

Fig. 2 shows the top level breakdown of these MECs into supporting competencies and further into required skills and knowledge [2].
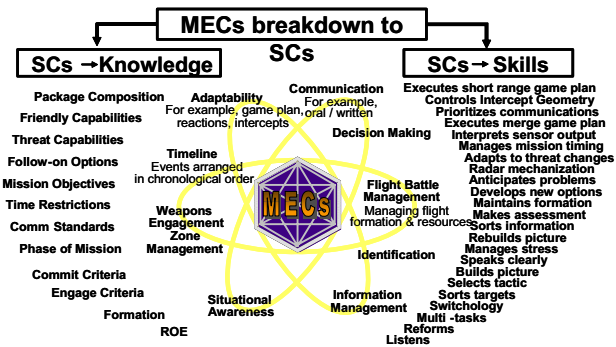
Fig. 2 Pilot MEC Breakdown

Fig. 4 CDF Property Breakdown

C. Critical Design Features

CDFs form a sub-set of the customer's key requirements for the system at the highest level of the product breakdown structure where criticality is ranked against the risk of hitting a particular key requirement. In this sense such attributes and measures of the system are dependant on the uncertainty of sub-system property values and the sensitivity between different levels of properties. Once again a breakdown of high level properties is required, generically shown in Fig. 3, a specific example of which is shown in Fig. 4. This domain is discussed in greater detail in section IV.
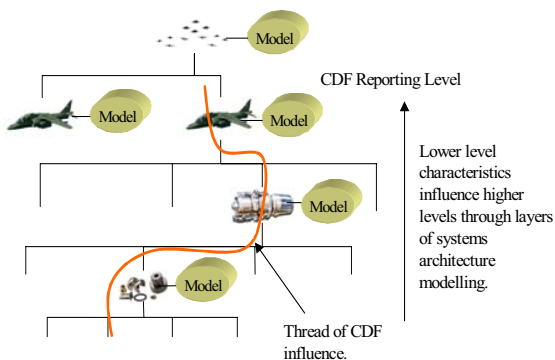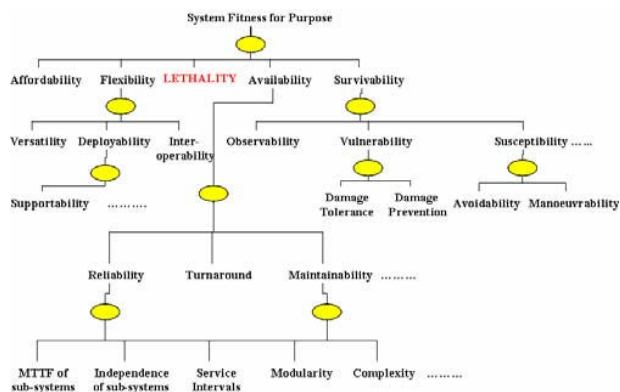

Fig. 3 Generic Breakdown

III. UNCERTAINTY ANALYSIS

The need to perform uncertainty analysis across a property hierarchy has been described in the previous section. Various approaches exist to aggregate uncertainty, the choice made often being dependent on the type of hierarchical breakdown. For example, a more subjective breakdown could utilize Dempster-Shafer belief functions [7] or fuzzy analysis [8], while a more quantitative breakdown could better make use of a probabilistic Taylor series expansion or Monte Carlo approach. The remaining part of this section documents some possible techniques.

A. Monte Carlo Simulation

Monte Carlo Simulation can be regarded as the observation of random numbers chosen in such a way that they directly simulate the physical random process of the original problem. In this respect samples of the uncertain parameters **x** are generated from the PDF **p(x)** and then these samples are used to determine the response **y(x)**. This approach tends to be computationally intensive and occasionally unfeasible for large complex systems.

B. Taylor Series Expansion

If $z = f(x_1,...x_n)$ then it can be shown using a Taylor series expansion [9] that

Expected value of z =

$$E(z) = f(\mu_1, \mu_2, ... \mu_n) + \frac{1}{2}\left[\sum_{i=1}^{n}\left(\frac{\partial^2 z}{\partial x_i^2}\right)\sigma_i^2 + \sum_{\substack{i=1 \\ i \neq j}}^{n}\sum_{j=1}^{n}\left(\frac{\partial^2 z}{\partial x_i \partial x_j}\right)\sigma_{ij} + ...\right] \quad (1)$$

and variance =

$$\sigma_z^2 = E\left[(z - E(z))^2\right] = \sum_{i=1}^{n}\left(\frac{\partial z}{\partial x_i}\right)_\mu^2 \sigma_i^2 + \frac{1}{2}\sum_{\substack{i=1 \\ i \neq j}}^{n}\sum_{j=1}^{n}\left(\frac{\partial z}{\partial x_i}\right)\left(\frac{\partial z}{\partial x_j}\right)\sigma_{ij} + ... \quad (2)$$

where σ_i is the standard deviation of x_i and σ_{ij} is the co-variance of x_i & $x_j = E[(x_i-\mu_i)(x_j-\mu_j)]$.

If the $x_i's$ are independent and z is approximately linear in terms of the $x_i's$ over a high percentage of the distribution (say $\pm 2\sigma$) then we may also neglect the higher order terms, so

$$\mu_z \approx z(\mu_1, \mu_2, ..., \mu_n) \quad and \quad \sigma_z^2 \approx \sum_{i=1}^{n} \left(\frac{\partial z}{\partial x_i} \right)^2 \sigma_i^2 \quad (3) \ \& \ (4)$$

In the important special case Z=XY a more accurate solution is available if X & Y are independent, namely

$$E(z) = \mu_z = \mu_x \mu_y \quad (5)$$

and

$$\sigma_z^2 = \mu_y^2 \sigma_x^2 + \mu_x^2 \sigma_y^2 + \sigma_x^2 \sigma_y^2 \quad (6)$$

C. Quantifying Subjective Views

Uncertainty may be represented by a mean and standard deviation for the properties under consideration. If the distribution of these values is shown or assumed to be of a specific kind, for example, a normal distribution, the mean and standard deviation can be related to the confidence one has in specific values lying in a given range. For example if a property's distribution is assumed normal and one is 68% confident that its value lies in the range (a,b) then μ should be taken as (a+b)/2 and σ = (b-a)/2. Subjective views can then be ascertained by the expert providing data relating to how confident he/she is that the value will lie between certain limits.

D. Interval Analysis

With this approach the input properties **x** are not described by means and standard deviations together with specific distributions but by simple bounds $\underline{x}_i \leq x_i \leq \overline{x}_i$ for i = 1,,,m where $\underline{x}_i$ and $\overline{x}_i$ represent the lower and upper bounds on x_i. The approach is to determine the bounds on the outputs **y(x)** using interval analysis. Two approaches are commonly used when using simple interval arithmetic to compute the ranges of the output and the second is to use optimization (or more correctly anti-optimization) to determine the least favourable output response using the bounds on the input properties. The optimization approach can be further expanded by consideration of convex modelling in which the hyper-cube representation of the bounds on the input properties are more accurately modelled by a convex surface inscribed within the hyper-cube, hence removing its vertices from the input space.

1) Interval Arithmetic

The first approach uses the following relationships involving simple arithmetic operations applied to intervals
[a,b]+[c,d]=[a+c,b+d]

[a,b]-[c,d]=[a-d,b-c]

[a,b]x[c,d]=[min(ac,ad,bc,bd),max(ac,ad,bc,bd)]=[ac,bd]   if a,b,c,d≥0

[a,b]/[c,d]=[a,b]x[1/d,1/c]=[min(a/d,a/c,b/d,b/c),max(a/d,a/c,b/d,b/c)]=[a/d,b/c] if a,b,c,d≥0,   provided 0 ∉ [c,d]

α[c,d]=[αc, αd] if $\alpha \geq 0$ or [αd, αc] if α<0.

2) Interval Analysis - Optimization

If the output y is scalar then its bounds may be determined by an optimization routine with the function y (or its inverse if employing a minimization routine) as the objective/cost function and the bounds $\underline{x}_i \leq x_i \leq \overline{x}_i$ for i = 1,,,m acting as constraints. This approach [10] will allow one to obtain the worst case response/functional evaluation given the uncertainty of the input properties. The approach could be obviously extended to the case of multiple outputs **y** at the expense of possible prohibitive processing time. Obviously, if the functions and bounds are linear then linear programming techniques may be used to solve the resulting optimization problem.

3) Convex Modelling

The bounds on the input parameter **x** can be said to lie within a hyper-cube whose faces are determined by the lower and upper limits of each individual x. For example for a 3 parameter system, bounds $\underline{x}_i \leq x_i \leq \overline{x}_i$ for i = 1,2,3 define a cuboid in 3 dimensional space. In practice the upper or lower bounds on the parameters are unlikely to occur simultaneously and hence an object inscribed within the hyper-cube, removing the vertices from the input space, may well provide a more suitable bound than the hyper-cube. Expanding on the previous example, results in an ellipsoidal input space as a more suitable bound for the input parameters. In general, any value in m-space may be used, but in practice the volume is assumed to be convex and represented by the equation $\underline{x}^T \underline{\underline{A}} \underline{x} \leq \alpha^2$ where $\underline{\underline{A}}$ is a positive definite matrix and α is a real number. Having defined the bounds on the input parameters as above, the corresponding bounds on the solution **y** may be found by use of an optimization routine as was the case with interval analysis optimization.

The major problem with these approaches is that the predicted bounds on the output **y** can be very conservative since there is no probabilistic information utilized relating to specific parts of the range.

Other approaches such as Dempster-Shafer Theory and Fuzzy modelling are discussed within [3] and [4]. In particular Fuzzy analysis is presently being used by the authors to model Mission Essential Competencies mentioned earlier in this paper [11].

IV. CRITICAL DESIGN FEATURES

Since the analysis of critical design features requires, by its nature, a determination of uncertainty; a means of analyzing, combining and propagating this uncertainty is required to help perform a trade-off analysis. This information can then be used to show that the performance reaches the required levels (i.e. the critical design features are met) with a stated confidence, or alternatively to indicate how best to direct effort on modifying those lowest level properties which influence the CDFs to greatest effect to improve overall performance at minimum cost.

Failure to represent the uncertainty in the parameters, results in immeasurable uncertainty in the values of the CDFs, and hence provides limited information on how best to improve overall performance and reduce uncertainty in the specific architecture.

Although this uncertainty analysis can be accomplished to some extent by the use of possibilisitic / subjective approaches, using techniques such as interval analysis, fuzzy modelling and belief functions, the remaining part of this paper will discuss only the use of probability density functions to describe the input uncertainty. If only subjective data is available, e.g. input property values are expected to lie between certain values with a given confidence, then one could use a uniform, beta or truncated Gaussian distribution or another distribution whose PDF is zero outside the bounds stated. Alternatively, expert views can be captured by them indicating confidence intervals e.g. 80% confident that input data will lie between specific bounds. This will allow one, if the form of the distribution is known, to transform the confidence interval data to mean and variance statistics.

If the input is available as experimental data then "tests of fit" and univariate estimation analysis can be used to determine the confidence in the PDF's of the initial properties.

The mean and variance of the input data can then be aggregated stochastically by using the central limit theorem (in respect of summation of sufficient distributions with the necessary independence) to prove the output is normally distributed, and utilization of a Taylor series expansion (Eqs. 1 & 2) to determine an estimate of the mean and variance of that output. If the central limit theorem cannot be used then the utilization of the Bienaymé-Chebyshev inequality [12] should be considered.

It should be noted that "fitness for purpose" of the design equates to meeting all the "design for" objectives that define purpose e.g. affordability, lethality, availability and that product maturity will increase progressively if:

- CDFs are recognized at outset
- CDFs drive the engineering plan
- CDFs are stable
- CDFs are partitioned and flowed down to sub-systems

and achieving a CDF will require different, possibly numerous enabling sub CDFs, in each sub-system.

The objectives of the "Critical Design Features Management" process are then:

o To provide techniques to assist in the identification of CDFs
o To point to the area of design which, if appropriately modified, would resolve the 'criticality' in a CDF without raising further problems.
o To feed into the "Create Candidate Solutions" process information to allow modification and refinement of design options so as to resolve criticality.

A. Dumb Bomb Drop CDF Case Study

Although property error data and sensitivities between the aggregated properties within this case study are classified and hence undisclosed, the approach used to determine CDFs is described in this section.

The reader is referred to Fig. 4 which indicates the linkage between overall system fitness for purpose through the lethality property to the lowest level properties related to a dumb bomb drop (e.g. Inertial Navigation System Inaccuracy, Air Data Computer Fidelity). These lowest level properties are defined to be those independent properties which are not related to any lower level properties within the system of interest.

In general assuming that $p_1 \ldots p_m$ are the lowest level properties and $p_{m+1} \ldots p_n$ are the aggregated properties which are arranged such that

$$p_{m+l+1} = f_{m+l+1}(p_1 \cdots p_m, p_{m+1} \cdots p_{m+l}) \quad \text{where}$$

$l \in \mathrm{N}[0, n-1]$ then to first order

$$\Delta p_{m+l+1} = \frac{\partial f_{m+l+1}}{\partial p_1} \Delta p_1 + \ldots \frac{\partial f_{m+l+1}}{\partial p_m} \Delta p_m + \frac{\partial f_{m+l+1}}{\partial p_{m+1}} \Delta p_{m+1} + \ldots \frac{\partial f_{m+l+1}}{\partial p_{m+l}} \Delta p_{m+l}$$

That is $\quad \underline{\underline{S}} \Delta \underline{p} = \underline{\underline{S}}' \Delta \underline{p}' \quad$ where $\qquad$ (7)

$$\underline{\underline{S}} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ -s_{m+2,m+1} & 1 & 0 & \cdots & 0 \\ -s_{m+3,m+1} & -s_{m+3,m+2} & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -s_{m+n,m+1} & -s_{m+n,m+2} & \cdots & -s_{m+n,m+n-1} & 1 \end{pmatrix},$$

$$\underline{\underline{S}}' = \begin{pmatrix} s_{m+1,1} & s_{m+1,2} & \cdots & s_{m+1,m} \\ s_{m+2,1} & s_{m+2,2} & \cdots & s_{m+2,m} \\ \cdots & \cdots & \cdots & \cdots \\ s_{m+n,1} & s_{m+n,2} & \cdots & s_{m+n,m} \end{pmatrix}, \quad \Delta \underline{p} = \begin{pmatrix} \Delta p_{m+1} \\ \Delta p_{m+2} \\ \cdots \\ \Delta p_{m+n} \end{pmatrix},$$

$$\Delta \underline{p}' = \begin{pmatrix} \Delta p_1 \\ \Delta p_2 \\ \cdots \\ \Delta p_m \end{pmatrix} \quad \text{and} \quad s_{i,j} = \frac{\partial f_i}{\partial p_j}$$

From (1) and (2) we have:

$$E(\Delta \underline{p}) = \underline{\underline{S}}^{-1} \underline{\underline{S}}' E(\Delta \underline{p}') \quad \text{and} \qquad (8)$$

$$Var(\Delta \underline{p}) = \left(\underline{\underline{S}}^{-1} \underline{\underline{S}}' \right)^{\langle 2 \rangle} Var(\Delta \underline{p}') \qquad (9)$$

Where $\underline{\underline{A}}^{\langle 2 \rangle} =$ Matrix $\underline{\underline{A}}$ with each element squared.

In the dumb bomb drop example the lowest level properties (bomb release timing, attitude and velocity, HUD positioning, windshear and sideslip etc.) will have errors with mean $E(\Delta \underline{p'})$ and variance $Var(\Delta \underline{p'})$ which may be determined from experimental data or expert opinion. This data together with the sensitivity matrices $\underline{\underline{S}}$ and $\underline{\underline{S'}}$, which in practice are determined by calculation, subjective view of experts or perturbation analysis, allow one to determine the mean and variance of the aggregated properties (e.g. lethality or even overall fitness for purpose). This analysis will then allow one to determine the uncertainty of higher level properties by utilizing the central limit theorem (independence of the lowest level properties is assumed) to ascertain the normality of error in these aggregated properties and hence aid the determination and management of CDFs by the following process.

1. Determine model fidelity
2. Perform sensitivity analysis
3. Establish CDFs
4. Identify trade opportunities and generate solution options
   - Perform trade-off analysis
   - Determine the effect of uncertainties in the analysis
   - Modify critical parameters (properties)
   - Consider new architectures
5. Repeat the above at the next design evolution
6. Evaluate change option and select preferred option
7. Initiate re-negotiation of requirements if necessary.

A suitable process, utilizing a Bayesian Belief Network, for determining model fidelity (item 1 above) is described in [13].

Items 2 and 3 are completed by the analysis indicated in the earlier case study which also indicates suitable trade-off via property modifications (item 4).

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