

# Entropy Generation Analysis of Free Convection Film Condensation on a Vertical Ellipsoid with Variable Wall Temperature

Sheng-An Yang, Ren-Yi Hung, and Ying-Yi Ho

**Abstract**—This paper aims to perform the second law analysis of thermodynamics on the laminar film condensation of pure saturated vapor flowing in the direction of gravity on an ellipsoid with variable wall temperature. The analysis provides us understanding how the geometric parameter- ellipticity and non-isothermal wall temperature variation amplitude “ $A$ .” affect entropy generation during film-wise condensation heat transfer process. To understand of which irreversibility involved in this condensation process, we derived an expression for the entropy generation number in terms of ellipticity and  $A$ . The result indicates that entropy generation increases with ellipticity. Furthermore, the irreversibility due to finite temperature difference heat transfer dominates over that due to condensate film flow friction and the local entropy generation rate decreases with increasing  $A$  in the upper half of ellipsoid. Meanwhile, the local entropy generation rate enhances with  $A$  around the rear lower half of ellipsoid.

**Keywords**—Free convection; Non-isothermal; Thermodynamic second law; Entropy, Ellipsoid.

## I. INTRODUCTION

THERE are two types of techniques to enhance condensation heat transfer process and thus to increase the performance of condensers. They are passive and active enhancement techniques. The passive techniques do not require the application of the external power, whereas the active techniques require activator or power supply to bring about the enhancement. When we attempt to enhance a rate of condensation, what we usually do is the increasing condensing area via fins or the formation of very thin condensation. The later is achieved by using objects having favorable surface tension due to surface curvature. One of objects having favorable surface tension is the object of elliptical cross section or vertical ellipsoids. As for this kind of passive enhancement

of condensation heat transfer, several researches, such as Yang and Hsu [1] and Yang and Chen [2], Ali and McDonald [3], Karimi [4], and Memory et al. [5] confirmed that cylinders, fins, or extended surfaces of elliptical profiles with major axes aligned with gravity are superior to those of circular profiles.

All the above heat transfer analyses belong to the filed of energy analysis, i.e. first law analysis, but first law analysis does not account for the irreversibility or degradation of energy in the system. Second law analysis provides an effective technique for measuring and optimizing performance of a thermal system by accounting for the energy quality. Second law analysis of thermal systems is widely gaining acceptance over traditional energy methods in both industry and academia as it is developed into a set of standards for measuring the performance. Entropy generation is associated with thermodynamic irreversibility which is common in all types of heat transfer processes. Film condensation belongs to phase-change heat transfer, but little literature regarding its second-law analysis is investigated.

Adeyinka and Naterer [6] first investigated the physical significance of entropy generation in plate film condensation. Lin et al. [7] performed the second-law analysis on saturated vapor flowing through and condensing inside horizontal cooling tubes. They noted that in a tube case, an optimum Reynolds number exists at which the entropy generates at a minimum rate. Dung and Yang [8] presented the entropy generation minimization method to optimize a saturated vapor flowing slowly onto and condensing on an isothermal horizontal tube. They observed that entropy generation provides a useful parameter in the optimization of a two-phase system. More recently, we first conducted a study [9] on the local entropy generation rate of laminar free convection film condensation on an elliptical cylinder. That paper investigated how the geometric parameter-ellipticity affects local entropy-generation rate during film-wise condensation heat transfer process. The second law analysis of the film condensation outside ellipsoids still remains an unsettled question so far.

Since the current state of knowledge about second law analysis of free convection film condensation outside an ellipsoid is somewhat incomplete, this investigation into the entropy generation rate will thus help us achieve the complete thermodynamic analysis, including the first and second law

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## NOMENCLATURE

$A$	the wall temperature variation amplitude	$T_{sat}$	saturation temperature of vapor
$a$	semi-major axis of ellipse	$T_w$	wall temperature
$b$	semi-minor axis of ellipse	$U$	reference velocity component in $X$ direction
$B_o$	Bond number, $(\rho - \rho_v)ga^2 / \sigma$	$u$	velocity component in $X$ direction
$Br$	Brinkman number, $\mu u_0^2 / k\Delta T$	$v$	velocity component in $Y$ direction
$C_p$	specific heat of condensate at constant pressure	Greek symbols	
$D_e$	equivalent diameter of ellipsoid	$\delta$	thickness of condensate film
$e$	ellipticity of ellipse	$\delta^*$	dimensionless thickness of condensate film
$g$	acceleration due to gravity	$\theta$	angle measured from top of the ellipsoid
$h$	condensing heat transfer coefficient at angle $\phi$	$\mu$	absolute viscosity of condensate
$h'_{fg}$	latent heat of condensation corrected for condensate subcooling	$\rho$	density of condensate
$Ja$	Jakob number, $C_p(T_{sat} - T_w) / h'_{fg}$	$\rho_v$	density of vapor
$k$	thermal conductivity of condensate	$\sigma$	surface tension coefficient in the film
$\dot{m}$	condensate mass flow rate per unit length of ellipsoid	$\phi$	angle between the tangent to ellipsoid surface and the normal to direction of gravity
$N_F''$	film flow friction irreversibility	$\Omega$	dimensionless temperature difference, $\Delta T / T_s$
$N_H''$	heat transfer irreversibility	Subscripts	
$N_S''$	the entropy generation number	$sat$	saturation
$Nu$	local Nusselt number, $hD_e / k$	$V$	vapor
$S_{gen}'''$	the volumetric entropy generation rate,	Superscripts	
$S_o'''$	characteristic transfer rate	$*$	indicates dimensionless
$Ra$	Rayleigh number, $(\rho - \rho_v)\rho g D_e^3 / \mu^2$		

analysis. We will derive an expression for the entropy generation number, which accounts for the combined action of the specified irreversibility. Basically, this study makes good engineering sense to focus on the irreversibility of film condensation heat transfer and try to understand the function of the entropy generation mechanism.

## II. THERMAL ANALYSIS

Consider a vertical ellipsoid, with major axis “ $2a$ ” in the direction of gravity and minor axis “ $2b$ ”, situated in a quiescent pure vapor which is at its saturated temperature  $T_{sat}$ . Moreover, the wall temperature  $T_w$  may be non-uniform and below  $T_{sat}$ . Thus, condensation occurs on the wall and a continuous film of the liquid runs downward over the ellipsoid under the actions of the component of gravity, and of the surface tension forces.

Fig. 1 illustrates schematically a physical model and coordinate system where the curvilinear coordinates  $(x, y)$  are aligned along an ellipsoid surface and its normal. The following simplifications are made in the analysis:

- 1) The condensate film flow is laminar and steady-state.
- 2) The inertia effect is neglected.
- 3) The condensate film thickness is much smaller than the

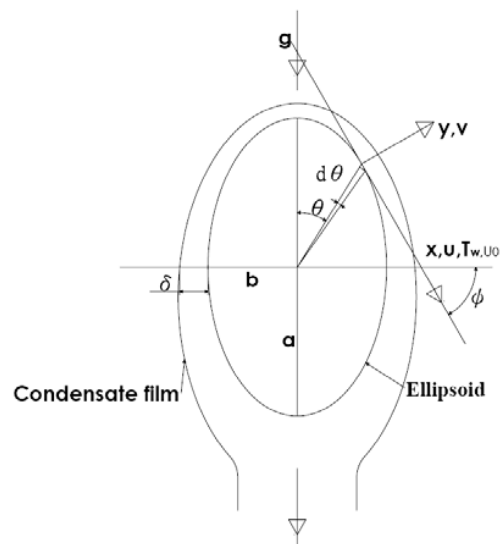


Fig. 1 Physical model and coordinate system for condensate film flow on an ellipsoid

curvature diameter.

- 4) Viscous dissipation is ignored.
- 5) Compared with the normal conduction, the streamwise conduction is negligible.

Based on the above simplifications, the condensate film governing equations for conservations of mass, momentum, and energy are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\mu \left( \frac{\partial^2 u}{\partial y^2} \right) = -(\rho - \rho_v) g [\sin \phi + Bo(\phi)] \quad (2)$$

$$k \left( \frac{\partial^2 T}{\partial y^2} \right) = 0 \quad (3)$$

, subject to the following boundary conditions:

$$y = 0 ; u = 0 ; T = T_w \quad (4)$$

$$y = \delta ; \frac{\partial u}{\partial y} = 0 ; T = T_{sat} \quad (5)$$

On account of varying radius of surface curvature, the surface tension forces can be derived here, as expressed in Yang and Chen [2]:

$$Bo(\phi) = \frac{1}{Bo} \frac{3e^2}{2a^2} \left( \frac{1-e^2 \sin^2 \phi}{1-e^2} \right)^2 \sin(2\phi) \quad (6)$$

Integrating (2) and (3) with the use of the boundary conditions gives, respectively.

$$u(y) = \frac{(\rho - \rho_v)}{\mu} g \delta^2 [\sin \phi + Bo(\phi)] \left( \frac{y}{\delta} - \frac{1}{2} \frac{y^2}{\delta^2} \right) \quad (7)$$

$$T = \left( \Delta T \frac{y}{\delta} \right) + T_w \quad (8)$$

Using (7), we can obtain the mass flow rate of condensate film.

$$\begin{aligned} \dot{m} &= \int_0^\delta \rho u 2\pi r \sin \theta dy \\ &= \rho(\rho - \rho_v) \frac{g \delta^3}{3\mu} [\sin \phi + Bo(\phi)] 2\pi r \sin \theta \end{aligned} \quad (9)$$

An energy balance at the condensate-vapor interface, as in the Nusselt-Rohsenow condensation theory, gives

$$h'_{fg} \frac{d\dot{m}}{dx} = k \frac{\Delta T}{\delta} 2\pi r \sin \theta \quad (10)$$

where,  $h'_{fg} = h_{fg} (1 + 0.68 C_p \Delta T / h_{fg})$  is the modified latent heat of condensation proposed by Rohsenow [10] to account for convection in the film. In order to derive the local film thickness  $\delta$  at the circumferential arc length  $x$  in terms of  $\phi$ , we can substitute (9) into (10) and obtain

$$\frac{\rho(\rho - \rho_v)}{3\mu r \sin \theta} h'_{fg} \frac{d}{dx} \left\{ g \delta^3 [\sin \phi + Bo(\phi)] r \sin \theta \right\} = k \frac{\Delta T}{\delta} \quad (11)$$

To solve the above equation, it is convenient at this point to express  $dx$  and  $r \sin \theta$  in terms of  $e$  and  $\phi$ . the differential streamwise length can be written as proposed by Yang [11]

$$dx = \frac{rd\theta}{\cos(\phi - \theta)} \quad (12)$$

Next, by using the geometric relationship for tangent to the surface

$$\tan \phi = \frac{\tan \theta}{(1 - e^2)} \quad (13)$$

And in Yang [11]:

$$r = a[(1 - e^2)/(1 - e^2 \cos^2 \theta)]^{0.5} \quad (14)$$

And with the help of (14), one may obtain the following expressions:

$$dx = a(1 - e^2)(1 - e^2 \sin^2 \phi)^{3/2} \quad (15)$$

Once the wall temperature distribution  $T_w(\phi)$  is specified or fitted by experimental data, the mean wall temperature is really available as

$$\overline{T_w} = 2 \left[ \frac{a}{D_e} (1 - e^2) \right]^2 \int_0^\pi T_w(\phi) \sin \phi \cdot [(1 - e^2 \sin^2 \phi)^{-2}] d\phi \quad (16)$$

and express the temperature difference across the film as

$$T_{sat} - T_w = (T_{sat} - \overline{T_w}) F_t(\phi) = \Delta T F_t(\phi) \quad (17)$$

where  $\Delta T = T_{sat} - \overline{T_w}$ .

Representative numerical results for the common axisymmetric case that involves the cosine distribution of non-isothermal wall temperature variation are given as

$$F_t(\phi) = 1 - A \cos(\phi) \quad (18)$$

Here, the non-isothermally function is adopted from the experiment of Lee et al. [12] for circular tube. Note that  $0 \leq A \leq 1$  and the amplitude  $A$  depends largely on the ratio of the outside-to-inside heat transfer coefficients.

Inserting (12) through (18) into (11), and introducing the transformation method, we can obtain dimensionless local condensate liquid film thickness as follows.

$$\begin{aligned} \delta^* &= \delta \left[ \frac{2ak\mu\Delta T}{\rho(\rho - \rho_v)gh'_{fg}} \right]^{-\frac{1}{4}} \\ &= \left\{ \sin \phi [\sin \phi + Bo(\phi)] \right\}^{-\frac{1}{3}} (1 - e^2 \sin^2 \phi)^{\frac{1}{6}} \\ &\quad \cdot \left[ 2(1 - e^2) \int F_t(\phi) [\sin \phi + Bo(\phi)]^{\frac{1}{3}} \sin^{\frac{4}{3}} \phi (1 - e^2 \sin^2 \phi)^{-\frac{13}{6}} d\phi \right]^{\frac{1}{4}} \end{aligned} \quad (19)$$

As in Nusselt [13] theory, interpreting the result of model is straightforward by employing the usual idea of a local heat transfer coefficient as follows:

$$Nu = \frac{hD_e}{k} = \left[ \frac{Ra}{Ja} \right]^{\frac{1}{4}} / \delta^* \quad (20)$$

where,

$$\begin{aligned} Ra &= \frac{\rho(\rho - \rho_v)g \text{Pr} D_e^3}{\mu^2} \\ Ja &= \frac{C_p(T_{sat} - T_w)}{h'_{fg}} \end{aligned}$$

$$U_0 = \frac{1}{2} \frac{(\rho - \rho_v)g}{\mu} D_e^2$$

According to Bejan [14], together with the fifth item of above-mentioned assumptions, the entropy generation rate for convection heat transfer can be written as

$$S_{gen}''' = \frac{k}{T_s^2} \left( \frac{\partial T}{\partial y} \right)^2 + \frac{\mu}{T_s} \left( \frac{\partial u}{\partial y} \right)^2 \quad (21)$$

On the right-hand side of (21), the first term and the second term represents the entropy generation due to heat transfer and due to condensate film flow friction, respectively. Substituting (7) and (8) into (21), and assuming  $T = T_{sat} + T - T_{sat} \approx T_{sat} = T_s$  yield

$$S_{gen}'' = \frac{k}{T_s^2} \left( \frac{\Delta T}{\delta} \right)^2 + \frac{\mu}{T_s} \frac{4U_0^2}{D^4} \delta^2 [\sin \phi + Bo(\phi)]^2 \left( 1 - \frac{y}{\delta} \right)^2 \quad (22)$$

Next, entropy generation number ( $N_S'''$ ) is the ratio of the volumetric entropy generation rate ( $S_{gen}'''$ ) to a characteristics transfer rate ( $S_o'''$ ).

$$N_S''' = \frac{S_{gen}'''}{S_o'''} \quad (23)$$

where,

$$S_o''' = \frac{k(\Delta T)^2}{D_e^2 T_s^2} \quad (24)$$

Further, by introducing the following dimensionless parameters

$$Br = \frac{\mu U_0^2}{k \Delta T} \quad (25)$$

$$\Omega = \frac{\Delta T}{T_s} \quad \text{and} \quad \eta = \frac{y}{\delta} \quad (26)$$

the entropy generation number can be expressed as:

$$N_S''' = \frac{(Ra/Ja)^{0.5} F_i^2(\phi)}{(\delta^*)^2} + \frac{Br}{\Omega} \frac{(\delta^*)^2}{(Ra/Ja)^{0.5}} [\sin \phi + Bo(\phi)]^2 (1 - \eta)^2 \quad (27)$$

$$= N_H''' + N_F'''$$

Notably,  $N_H'''$  denotes the dimensionless entropy generation due to heat transfer irreversibility

$$N_H''' = \frac{(Ra/Ja)^{0.5} F_i^2(\phi)}{(\delta^*)^2} \quad (28)$$

$N_F'''$  denotes the dimensionless entropy generation due to fluid friction irreversibility at the wall ( $y=0$ ) and is evaluated as follows.

$$N_F''' = \frac{Br}{\Omega} \frac{(\delta^*)^2}{(Ra/Ja)^{0.5}} [\sin \phi + Bo(\phi)]^2 (1 - \eta)^2 \quad (29)$$

To understand which of the condensate flow friction or heat transfer dominates, we introduce a criterion known as the

irreversibility distribution ratio in the following equation:

$$\psi = \frac{N_F'''}{N_H'''} \quad (30)$$

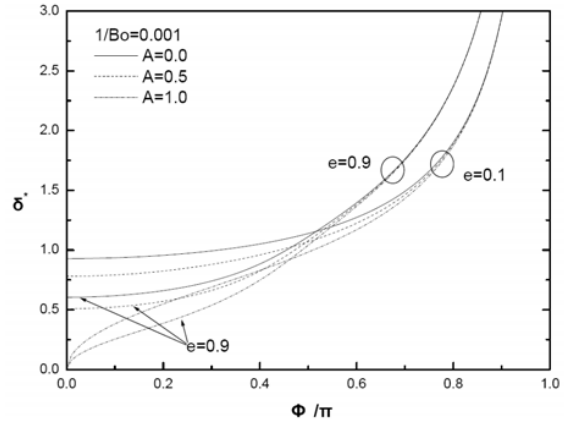


Fig. 2 The condensation film profile for varying  $A$  and ellipticity

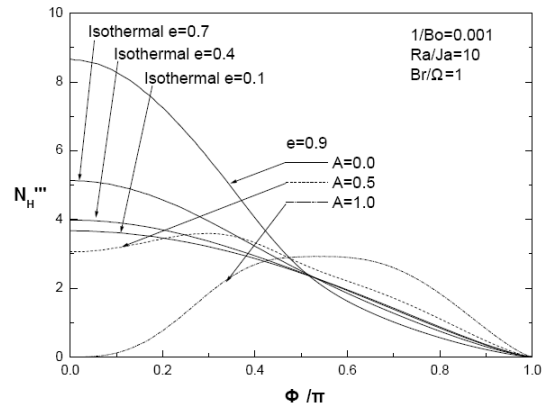


Fig. 3 Irreversibility due to heat transfer for varying  $A$  and ellipticity

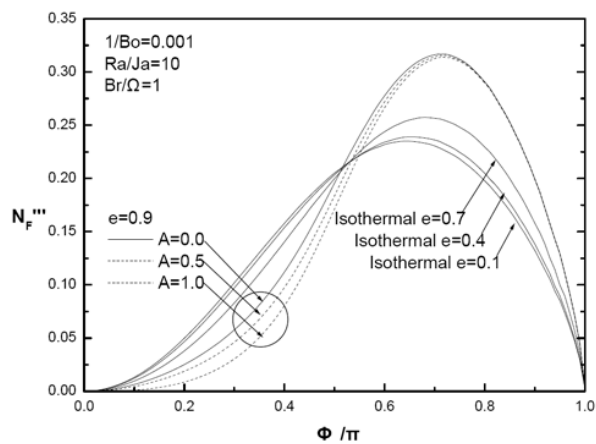


Fig. 4 Irreversibility due to film flow friction for varying  $A$  and ellipticity

### III. RESULTS AND DISCUSSION

Fig. 2 shows the condensation film profile for varying ellipticity and  $A$ . It demonstrates that the dimensionless condensate film thickness  $\delta^*$  decreases slightly as  $A$  increases.

Figs. 3, 4, and 5 indicate the variation of dimensionless entropy generation numbers  $N_H''', N_F''', N_S'''$  with  $\phi/\pi$  under the surface tension effects for various ellipticities. Firstly, Fig. 3 indicates that the dimensionless entropy number due to heat transfer declines with the film thickness. This may account for the fact that the finite temperature difference heat transfer via thinner film will cause the higher irreversibility. Secondly, Fig. 4 demonstrates that the dimensionless entropy number due to film flow friction varies significantly with the square of  $(\sin \phi + Bo(\phi))$ . Note that if we ignore the effect of surface tension,  $Bo(\phi)$ , the maximum value of this entropy generation will occur at the mid of ellipsoid. Finally, it is clear that the local dimensionless entropy number  $N_S'''$  is similar to  $N_H'''$  because the entropy generation due to heat transfer dominates that due to film flow friction, as seen in Fig. 5. Besides, Fig. 5 also confirms the local entropy generation rate increases with ellipticity.

Fig. 6 shows entropy generation rate versus  $1/Bo$  and  $Ra/Ja$  for  $e=0.7$ . The entropy generation number is markedly affected by the non-isothermal wall temperature variation. This may account for the larger temperature differences. From (27), one may clearly see that the higher value of  $Ra/Ja$  produces more entropy generation because the heat transfer irreversibility varies as square root of  $Ra/Ja$ . Additionally, the higher value of  $1/Bo$  yields more entropy generation because of film flow friction.

Fig. 7 shows entropy generation rate versus  $Br/\Phi$  and  $A$  for  $e=0.9$ . From (27), one may clearly see that the higher value of  $Br/\Phi$  produces more entropy generation. The local entropy generation increases slightly with the increase in Brinkman number. This can be explained as the fact that the condensate film flow friction plays an insignificant role in the entropy generation rate.

Finally, Fig. 8 indicates the dependence of the irreversibility distribution ratio with  $A$ . The irreversibility distribution ratio for the case  $A=0$  is larger than that for the case  $A=1$ . This may account for the more contribution to irreversibility caused by larger temperature difference heat transfer. When  $\psi < 1$ , heat transfer irreversibility dominates over the flow friction irreversibility and vice versa for  $\psi > 1$ . Increasing non-isothermal wall temperature variation amplitude will enhance the heat transfer irreversibility due to finite temperature difference heat transfer. Hence, the irreversibility distribution ratio for isothermal wall case is larger than that for the non-isothermal case.

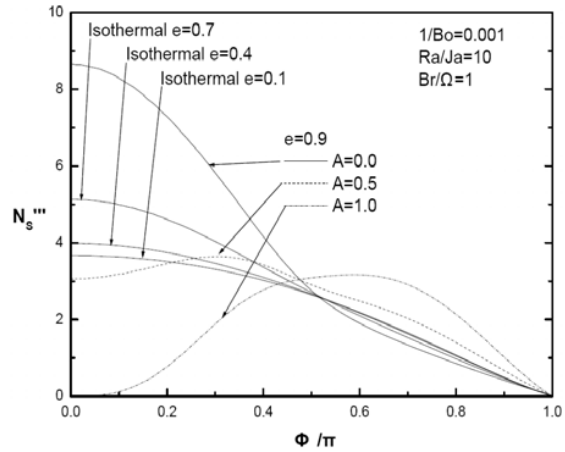


Fig. 5 Local entropy generation for varying  $A$  and ellipticity

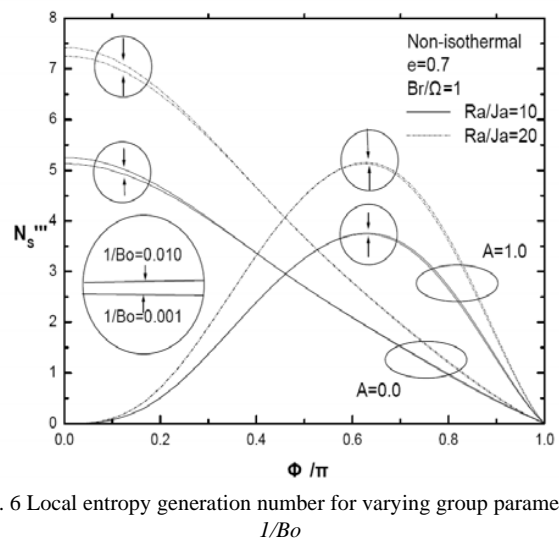


Fig. 6 Local entropy generation number for varying group parameters  $1/Bo$

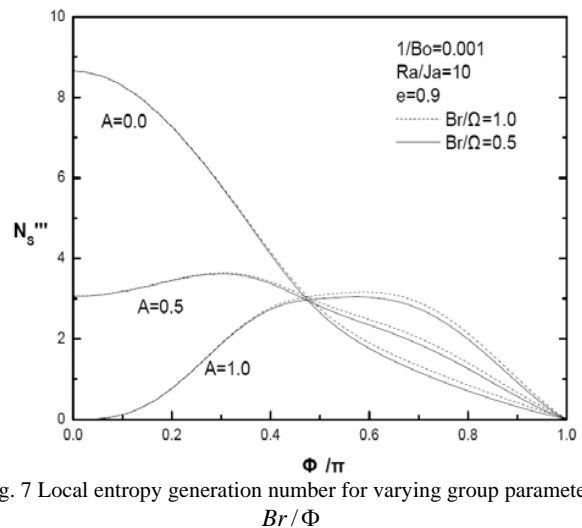


Fig. 7 Local entropy generation number for varying group parameters  $Br/\Phi$

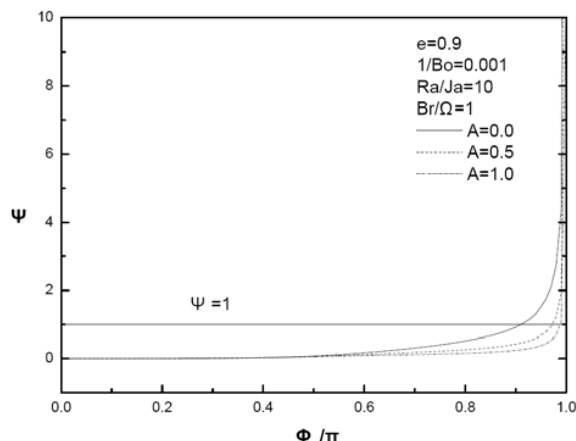


Fig. 8 The dependence of the irreversibility distribution ratio with  $A$

#### IV. CONCLUDING REMARKS

This work performed the entropy generation analysis of the laminar film condensation on an ellipsoid under the effect of the non-isothermal wall temperature variation for various ellipticities. The conclusions from this study can be summarized as follows:

- 1.) The local entropy generation increases with the decreases in Bond number.
- 2.) The local entropy generation increases with Brinkman number.
- 3.) The local entropy generation rate reduces with increasing wall temperature variation amplitude.
- 4.) The local entropy generation rate enhances with the wall temperature variation amplitude around the rear lower half of ellipsoid perimeter.
- 5.) Compared to entropy generation due to film flow friction, entropy generation due to heat transfer is generally dominant in most cases.
- 6.) The entropy generation Increases with the ellipticity of ellipsoid.

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