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Study of Characteristics of Multi-Layer Piezoelectric Transformers by using 3-D Finite Element Method

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***Abstract*—**Piezoelectric transformers are electronic devices made from piezoelectric materials. The piezoelectric transformers as the name implied are used for changing voltage signals from one level to another. Electrical energy carried with signals is transferred by means of mechanical vibration. Characterizing in both electrical and mechanical properties leads to extensively use and efficiency enhancement of piezoelectric transformers in various applications. In this paper, study and analysis of electrical and mechanical properties of multi-layer piezoelectric transformers in forms of potential and displacement distribution throughout the volume, respectively. This paper proposes a set of quasi-static mathematical model of electro-mechanical coupling for piezoelectric transformer by using a set of partial differential equations. Computer-based simulation utilizing the three-dimensional finite element method (3-D FEM) is exploited as a tool for visualizing potentials and displacements distribution within the multi-layer piezoelectric transformer. This simulation was conducted by varying a number of layers. In this paper 3, 5 and 7 of the circular ring type were used. The computer simulation based on the use of the FEM has been developed in MATLAB programming environment.

***Keywords*—**Multi-layer Piezoelectric Transformer, 3-D Finite Element Method (3-D FEM), Electro-mechanical Coupling, Mechanical Vibration

I. Introduction

PIEZOELECTRIC transformers are electronic devices made from piezoelectric material such as BaTiO3, PZT, PbN206, PT, PLZT and PMN. Piezoelectric transformer is typically to convert electrical input voltage from one level to one another by using mechanical vibration of medium material. At resonance frequency, voltage input transfers its energy through piezoelectric material to generate vibration. This process is similar to that of actuators. This vibration can be recovered and be converted to electrical energy at the output layer with a specific voltage gain acting as transducers. In this paper, radial mode vibration [1] of a circular ring or shortly called the ring-dot type where its applications in ballast electronic has been increasingly found is selected for study. Finite Element Method (FEM) is one of the most popular numerical methods used for computer simulation. The key advantage of the FEM over other numerical methods in engineering applications is the ability to handle nonlinear, time-dependent and complex geometry problems. Therefore, this method is suitable for solving the problem of potentials and displacements distribution of the piezoelectric transformers. To utilize the advantages of the 3-D Finite Element Method (3-D FEM) for handling the electro-mechanical coupling problems, 3-D FEM model development and problem formulation need to be defined in electro-mechanical coupling problems of piezoelectric transformer.

In this paper, a set of quasi-static mathematical model of electro-mechanical coupling for piezoelectric transformer is briefed in Section II. Section III is to illustrate the utilization of the 3-D FEM by using Galerkin approach for the electro-mechanical modeling described in Section II. The domain of study with the 3-D FEM can be discretized by using linear tetrahedron elements. Section IV gives simulation results when consider multi-layer piezoelectric transformers of which 3, 5, and 7 layers of the circular ring type are used. This section also gives some discussion and points out the influence of different layers. Study and analysis of electrical and mechanical properties in forms of potential and displacement distribution are determined throughout the volume. The simulation conducted herein is based on the 3-D FEM method given in Section III. All the programming instructions are coded in MATLAB program environment with graphical representation for potentials and displacements. The last section gives conclusion.

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II. Modeling Of Electro-Mechanical Coupling For A Piezoelectric Transformer

Piezoelectric transformer can be described by using mathematical models to exhibit electro-mechanical coupling among stress tensor (**T**), strain tensor (**S**), electric field (**E**), and electric displacement (**D**) as in (1) and (2) [2], [3].

$$\mathbf{T} = c^{E}\mathbf{S} - e^{\mathrm{T}}\mathbf{E} \qquad (1)$$

$$\mathbf{D} = e\mathbf{S} + \varepsilon^{S}\mathbf{E} \qquad (2)$$

where c^E is the elastic stiffness tensor at constant electric field, ε^S is the dielectric permittivity tensor at constant strain, and e is piezoelectric stress tensor.

$$\nabla \cdot \mathbf{T} = \rho \ddot{u} \quad \text{with } \mathbf{S} = Bu \tag{3}$$

$$\nabla \cdot \mathbf{D} = 0 \quad \text{with } \mathbf{E} = -\nabla \Phi \tag{4}$$

And (3) and (4) are momentum balance and electric balance equations, respectively, in which ρ, B, u, and Φ are mass density, first spatial derivatives of the interpolation, mechanical displacement, and electric potential, respectively.

$$B = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix}$$

This paper has considered the time-harmonic system [4], therefore,

$$\ddot{u} = -\omega^2 u$$

…, where ω is the angular frequency.

As described, a set of quasi-static mathematical model of electro-mechanical coupling for piezoelectric transformer by using a set of partial differential equations is collected as in (5) and (6).

$$c^E(\nabla \cdot Bu) + \rho\omega^2 u + e^{\mathrm{T}}(\nabla \cdot \nabla\Phi) = 0 \tag{5}$$

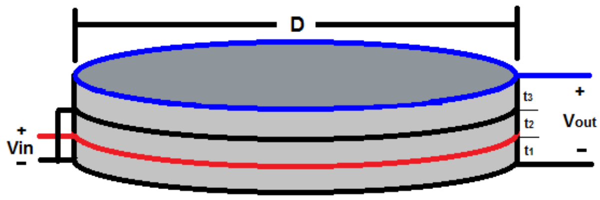
$$e(\nabla \cdot Bu) - \varepsilon^S(\nabla \cdot \nabla\Phi) = 0 \tag{6}$$

Analytically, there is no simple exact solution of the above equation. Therefore, in this paper the 3-D FEM is chosen to be a potential tool for finding approximate potential and displacement solutions for the quasi-static partial differential equation described as in (5) and (6) [5], [6].

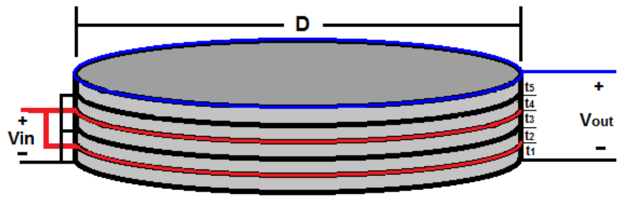
III. 3-D FEM For The Piezoelectric Transformer

A. Discretization

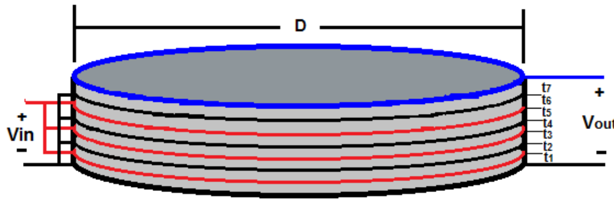
This paper uses PZT powder to form circular ring multi-layer piezoelectric transformers of 3, 5, and 7 layers for test as described in Fig. 1. The test specimens have 30 mm of diameter (D) and 3 mm of the total thickness (t). The domain of study with the 3-D FEM can be discretized by using linear tetrahedron elements. This can be accomplished by using Solidworks for 3-D grid generation. Fig. 2 displays grid representation of the test system. The region domain consists of 13,530 nodes and 68,122 elements.



a) 3 layers



b) 5 layers



c) 7 layers

Fig. 1 Detail of the multi-layer piezoelectric transformers

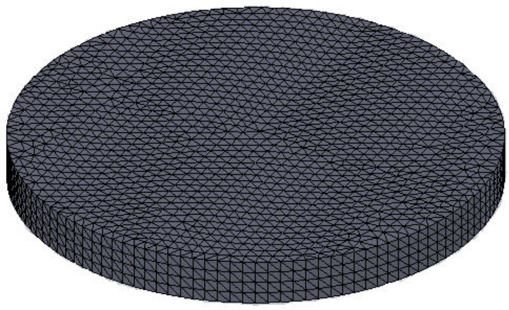


Fig. 2 Discretization of the piezoelectric transformer

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B. 3-D FEM Formulation

An equation governing each element is derived from the electro-mechanical equations directly by using Galerkin approach, which is the particular weighted residual method for which the weighting functions are the same as the shape functions. The shape function for 3-D FEM used in this research is the application of 4-node tetrahedron element (three-dimensional linear element) [7]-[9]. According to the method, the result is expressed as follows

$$A(x,y,z) = A_1N_1 + A_2N_2 + A_3N_3 + A_4N_4 \tag{7}$$

…, where N_i , i = 1, 2, 3, 4 is the element shape function and the A_i , i = 1, 2, 3, 4 is the approximation of the result at each node (1, 2, 3, 4) of the elements, which is

$$N_i = \frac{1}{6V}(a_i + b_i x + c_i y + d_i z)$$

…, where V is the volume of the tetrahedron element, which is expressed as

$$V = \frac{1}{6}\begin{vmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix}$$

and

$$\begin{aligned}
a_1 &= x_4(y_2z_3 - y_3z_2) + x_3(y_4z_2 - y_2z_4) + x_2(y_3z_4 - y_4z_3) \\
a_2 &= x_4(y_3z_1 - y_1z_3) + x_3(y_1z_4 - y_4z_1) + x_1(y_4z_3 - y_3z_4) \\
a_3 &= x_4(y_1z_2 - y_2z_1) + x_2(y_4z_1 - y_1z_4) + x_1(y_2z_4 - y_4z_2) \\
a_4 &= x_3(y_2z_1 - y_1z_2) + x_2(y_1z_3 - y_3z_1) + x_1(y_3z_2 - y_2z_3)
\end{aligned}$$

$$\begin{aligned}
b_1 &= y_4(z_3 - z_2) + y_3(z_2 - z_4) + y_2(z_4 - z_3) \\
b_2 &= y_4(z_1 - z_3) + y_1(z_3 - z_4) + y_3(z_4 - z_1) \\
b_3 &= y_4(z_2 - z_1) + y_2(z_1 - z_4) + y_1(z_4 - z_2) \\
b_4 &= y_3(z_1 - z_2) + y_1(z_2 - z_3) + y_2(z_3 - z_1)
\end{aligned}$$

$$\begin{aligned}
c_1 &= x_4(z_2 - z_3) + x_2(z_3 - z_4) + x_3(z_4 - z_2) \\
c_2 &= x_4(z_3 - z_1) + x_3(z_1 - z_4) + x_1(z_4 - z_3) \\
c_3 &= x_4(z_1 - z_2) + x_1(z_2 - z_4) + x_2(z_4 - z_1) \\
c_4 &= x_3(z_2 - z_1) + x_2(z_1 - z_3) + x_1(z_3 - z_2)
\end{aligned}$$

$$\begin{aligned}
d_1 &= x_4(y_3 - y_2) + x_3(y_2 - y_4) + x_2(y_4 - y_3) \\
d_2 &= x_4(y_1 - y_3) + x_1(y_3 - y_4) + x_3(y_4 - y_1) \\
d_3 &= x_4(y_2 - y_1) + x_2(y_1 - y_4) + x_1(y_4 - y_2) \\
d_4 &= x_3(y_1 - y_2) + x_1(y_2 - y_3) + x_2(y_3 - y_1)
\end{aligned}$$

The method of the weighted residue with Galerkin approach is then applied to the differential equation, refer to (5) and (6), where the integrations are performed over the element domain Ω.

$$\int_\Omega \nabla \cdot N_i(c^E Bu)d\Omega + \int_\Omega N_i(\rho\omega^2 u)d\Omega + \int_\Omega \nabla \cdot N_i(e^{\mathrm{T}}\nabla\Phi)d\Omega = 0$$

$$\int_\Omega \nabla \cdot N_i(eBu)d\Omega - \int_\Omega \nabla \cdot N_i(\varepsilon^S\nabla\Phi)d\Omega = 0$$

Finally, a set of linear equations in the compact matrix form is obtained

$$\begin{bmatrix} [K_{uu}] - \omega^2[M] & [K_{u\Phi}] \\ [K_{u\Phi}]^{\mathrm{T}} & [K_{\Phi\Phi}] \end{bmatrix}_{16\times16} \begin{bmatrix} u \\ \Phi \end{bmatrix}_{16\times1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{16\times1} \tag{8}$$

…, where the matrices $[M]$, $[K_{uu}]$, $[K_{u\Phi}]$ and $[K_{\Phi\Phi}]$ are the mass, stiffness, piezoelectric coupling, and dielectric matrices, respectively.

$$[M]_{12\times12} = \frac{-\rho V}{20}\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & \cdot & 1 & 1 \\ 1 & 1 & \cdot & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

$$[K_{uu}]_{12\times12} = \frac{1}{36V} B_u^{\mathrm{T}}(c^E)B_u$$

$$[K_{u\Phi}]_{12\times4} = \frac{1}{36V} B_u^{\mathrm{T}}(e^{\mathrm{T}})B_\Phi$$

$$[K_{\Phi\Phi}]_{4\times4} = \frac{1}{36V} B_\Phi^{\mathrm{T}}(-\varepsilon^S)B_\Phi$$

…, where

$$B_u = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \end{bmatrix}$$

$$B_\Phi = \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix}$$

For one element containing 4 nodes, the expression of the FEM approximation is a 16×16 matrix. With the account of all elements in the system of *n* nodes, the system equation is sizable as the *4n×4n* matrix.

IV. Simulation Results

The boundary conditions applied here are that to set zero potential at the ground electrode and 50 V at the input electrode. This simulation uses the system frequency of 50 Hz. The piezoelectric material properties shown by [2]:

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & 11.7 & 0 \\ 0 & 0 & 0 & 11.7 & 0 & 0 \\ -5.4 & -5.4 & 13.5 & 0 & 0 & 0 \end{bmatrix} (\mathrm{C/m^2})$$

$$c^E = \begin{bmatrix} 14.9 & 10.1 & 9.8 & 0 & 0 & 0 \\ 0 & 14.9 & 9.8 & 0 & 0 & 0 \\ 0 & 0 & 14.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.4 \end{bmatrix} \times 10^{10} (\mathrm{N/m^2})$$

$$\varepsilon^S = \begin{bmatrix} 8.0 & 0 & 0 \\ 0 & 8.0 & 0 \\ 0 & 0 & 7.2 \end{bmatrix} \times 10^{-9} (\mathrm{F/m^2})$$

$$\rho = 7600 \mathrm{kg/m^3}$$

The FEM-based simulation conducted in this paper is coded with MATLAB programming for calculation of electric potentials and mechanical displacements of multi-layer piezoelectric transformer when considering 3, 5, and 7 layers. Each of which was tested with a range of 90-125 kHz of input sources. Fig. 3 displayed electric potentials in forms of voltage gain in association with its operating frequency for all different three numbers of layers. As a result, 3-layer case gave the highest voltage gain at 35.12, 116 kHz (natural frequency). For 5-layer case, the highest voltage gain was 168.15 at 119 kHz. The final case, 7-layer, the highest voltage gain was 211.16 at 92 kHz. This revealed that when the number of layers is increased, the voltage gain is also increased.

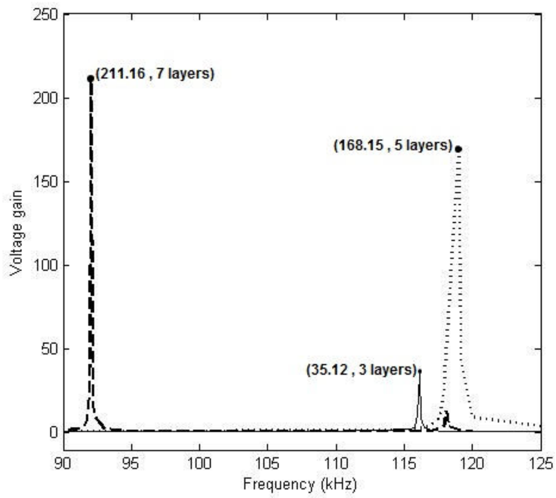

Fig. 3 Frequency responses for three different numbers of layers

For which 3-D FEM result, it can be graphically presented in the filled polygon of potentials and displacements dispersed thoroughly the volume of multi-layer piezoelectric transformer. Fig. 4-6 illustrate the result of potential distribution in form of voltage gain of 3-D FEM at natural frequency where the voltage gain of the piezoelectric transformer is high when considering 3, 5, and 7 layers, respectively. When the number of layers increases, the voltage gain is also increased. Also, at the natural frequency excitation of the 3, 5, and 7 layers, the displacement distribution of those were shown in Fig. 7-9, respectively. The 3-layer case gave the highest displacement at 1.18×10^{-6} m. The 5-layer case gave the highest displacement at 6.68×10^{-6} m. Whereas the 7-layer case gave the highest displacement at 16.46×10^{-6} m. As can be seen, when the number of layers increases, the highest displacement is also increased in such a way that the voltage gain increases. This characteristic is vital and can be described by the mathematical model of electro-mechanical coupling.

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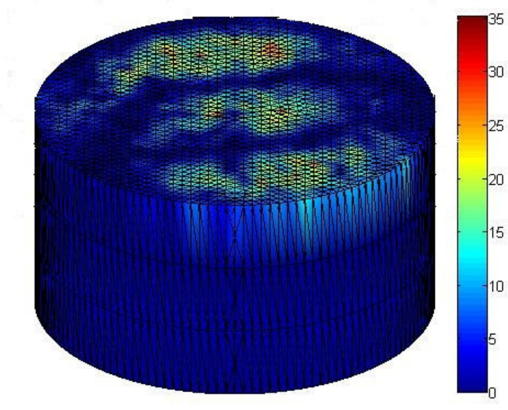

Fig. 4 Voltage gain at natural frequency for 3-layer piezoelectric transformer

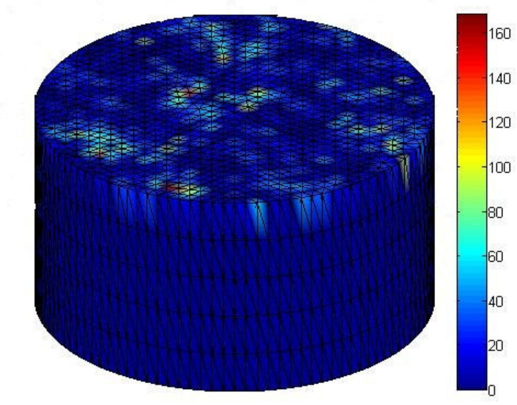

Fig. 5 Voltage gain at natural frequency for 5-layer piezoelectric transformer

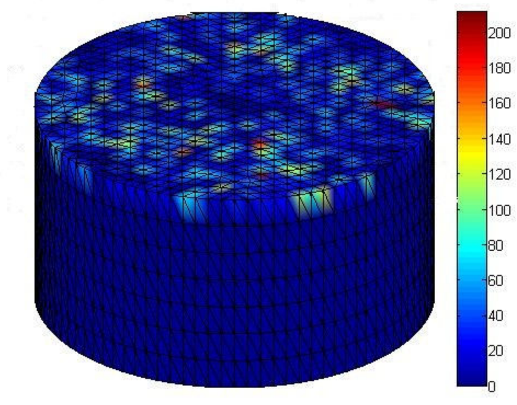

Fig. 6 Voltage gain at natural frequency for 7-layer piezoelectric transformer

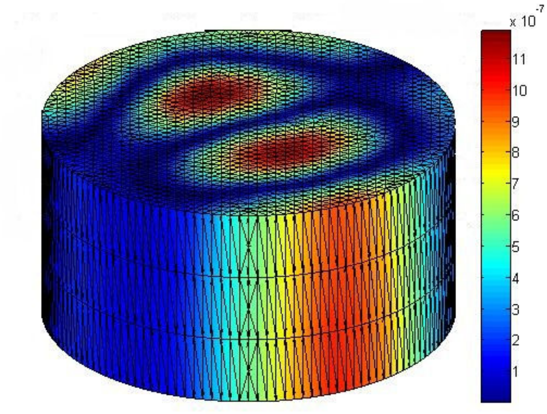

Fig. 7 Displacements distribution (m) at natural frequency for 3-layer piezoelectric transformer

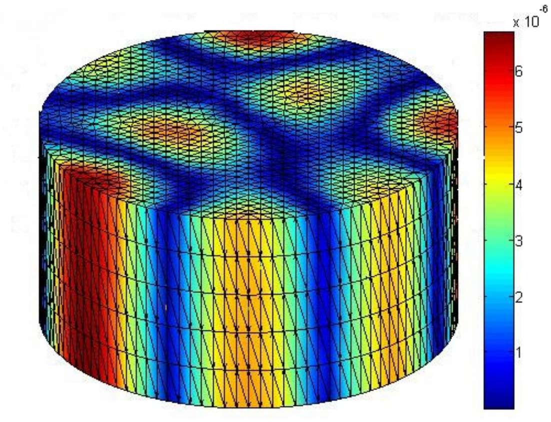

Fig. 8 Displacements distribution (m) at natural frequency for 5-layer piezoelectric transformer

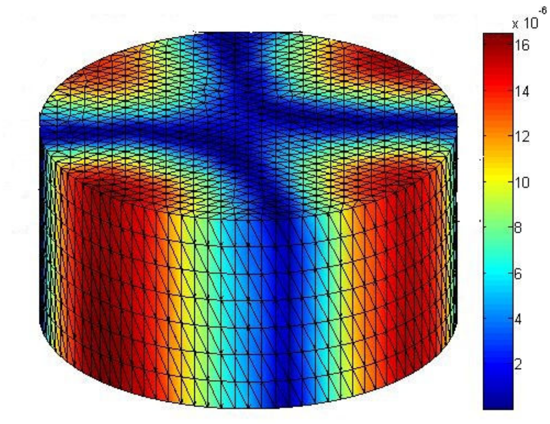

Fig. 9 Displacements distribution (m) at natural frequency for 7-layer piezoelectric transformer

V. Conclusion

This paper presents FEM based simulation for visualization of potential and displacement distribution when considering the multi-layer piezoelectric transformer of 3, 5, and 7 layers. The test multi-layer piezoelectric transformers are circular ring type made form PZT. The computer simulation is performed by using 3-D finite element method (3-D FEM) instructed in MATLAB programming codes. As a result,when the number of layers increases, the electric potential and the mechanical displacement of multi-layer piezoelectric transformers are also increased due to the appearance of electro-mechanical coupling in piezoelectric transformers.

References

[1] N.A. Demerdash and D.H. Gillott, "A new approach for determination of eddy current and flux penetration in nonlinear ferromagnetic materials," *IEEE Transactions on Magnetics*, Vol.74, pp. 682-685, 1974. K.F. Kwok, P. Dong, K.W.E. Cheng, K.W. Kwok, Y.L. Ho, X.X. Wang and H. Chan, "General Study on Piezoelectric Transformer," *IEEE Transactions on Ultrasonics*, Vol. 55, pp.216-220, 2002.

[2] R. Lerch, "Simulation of Piezoelectric Devices by Two and Three Dimension Finite Element," *IEEE Transactions on Ultrasonics*, Vol. 37, No. 2, pp.233-247, 1990.

[3] J.S. Wang and D.F. Ostergaard, "A Finite Element-Electric Circuit Coupled Simulation Method for Piezoelectric Transducer," *IEEE Transactions on Ultrasonics*, Vol. 59, pp.1105-1108, 1999.

[4] C. Christopoulos, *The Transmission-Line Modeling Method: TLM*, IEEE Press, USA, 1995.

[5] P. Pao-la-or, T. Kulworawanichpong, S. Sujitjorn and S. Peaiyoung, "Distributions of Flux and Electromagnetic Force in Induction Motors: A Finite Element Approach," *WSEAS Transactions on Systems*, Vol. 5, No. 3, pp.617-624, 2006.

[6] P. Pao-la-or, A. Isaramongkolrak and T. Kulworawanichpong, "Finite Element Analysis of Magnetic Field Distribution for 500-kV Power Transmission Systems," *Engineering Letters*, Vol. 18, No. 1, pp.1-9, 2010.

[7] R.W. Lewis, P. Nithiarasu and K.N. Seetharamu, *Fundamentals of the Finite Element Method for Heat and Fluid Flow*, John Wiley & Sons, USA, 2004.

[8] M.A. Bhatti, *Advanced Topics in Finite Element Analysis of Structures*, John Wiley & Sons, USA, 2006.

[9] P.I. Kattan, *MATLAB Guide to Finite Elements (2nd edition)*, Springer Berlin Heidelberg, USA, 2007.