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Hydrodynamic Analysis of Reservoir due to Vertical Component of Earthquake using an Analytical Solution

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Abstract**—**This paper presents an analytical solution to get a reliable estimation of the hydrodynamic pressure on gravity dams induced by vertical component earthquake when solving the fluid and dam interaction problem. Presented analytical technique is presented for calculation of earthquake-induced hydrodynamic pressure in the reservoir of gravity dams allowing for water compressibility and wave absorption at the reservoir bottom. This new analytical solution can take into account the effect of bottom material on seismic response of gravity dams. It is concluded that because the vertical component of ground motion causes significant hydrodynamic forces in the horizontal direction on a vertical upstream face, responses to the vertical component of ground motion are of special importance in analysis of concrete gravity dams subjected to earthquakes.

Keywords**—**Dam, Reservoir, Analytical solution, Vertical component, Earthquake

I. Introduction

THERE are a large number of concrete gravity dams all over the world. Some of these dams are built in seismically active areas. The analysis of dams is a complex problem due to the dam-water-foundation interaction. An important factor in the design of dams in seismic regions is the effect of hydrodynamic pressure exerted on the face of dam as a result of earthquake ground motions [1]. For a rational analysis of a dam-reservoir system, it is essential that the hydrodynamic effects and interaction between the foundation and reservoir are properly considered. The dynamic behavior of a concrete gravity dam is affected by the adjacent reservoir and the flexible strata consisting of porous sediments on which it rests. The developed hydrodynamic pressure on dam is dependent on the physical characteristic of the boundaries surrounding the reservoir including reservoir bottom. In various methods proposed by different researchers for simplification of the analytical procedures, the reservoir bottom is generally considered to be rigid. This assumption does not represent the actual behavior of the system. The hydrodynamic pressure in the reservoir is usually affected by radiation of waves from foundation. When the reservoir bottom is considered to be rigid, the pressure waves are reflected from the reservoir bed and consequently the hydrodynamic pressure is over-estimated. Due to the absorption at the reservoir bottom, the magnitude of hydrodynamic pressure due to ground motion will be reduced.

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Therefore, the hydrodynamic pressure exerted on the upstream face of dams will be less and the displacement and stress field in the dam will be affected. Thus, an accurate evaluation of hydrodynamic pressure on the dam must consider the effect of induced hydrodynamic pressure under vertical component of earthquake beside of horizontal component to achievement of reliable seismic behavior of concrete gravity dams. In this paper, hydrodynamic pressures induced by vertical component of earthquake ground motions, including bottom absorption are investigated [2].

II. Governing Equation and Boundary Conditions

Consider a gravity dam with a vertical upstream face, impounding a reservoir of constant depth and extending to infinity in the upstream direction. It is assumed that the dam and reservoir are resting on a flexible foundation which is modeled as a viscoelastic half-plane. Assuming the water in the reservoir to be inviscid, compressible, irrotational and its motion to be of small amplitude, the hydrodynamic pressure equation will be [3]:

$$\Delta P = \frac{1}{C^2}\ddot{P} \qquad \text{in} \quad \Gamma \tag{1}$$

where Δ, P, C and Γ are the Laplacian operator, hydrodynamic pressure in the reservoir, velocity of sound in water and reservoir domain, respectively. Two-dimensional form of Eq. 1 can be written as [4, 5]:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = \frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} \tag{2}$$

where x and y are the cartesian coordinates and t is the time variable. Eq. 2 together with the appropriate boundary conditions, defines completely the hydrodynamic aspects of the problem. Figure 1 shows reservoir domain and boundaries under vertical component of earthquake.

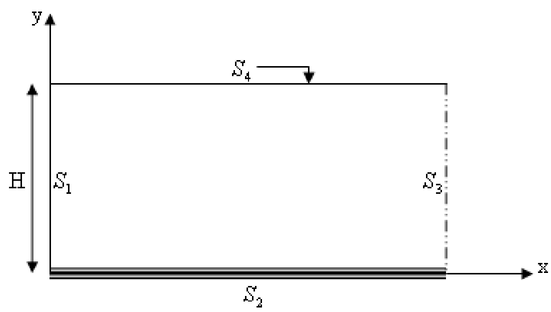

Fig. 1 Reservoir domain and boundaries

The boundary conditions to be satisfied are as follows:
(a) At the dam-reservoir interface:

$$\frac{\partial P}{\partial n} = -\rho_w a_n \quad \text{on} \quad S_1 \tag{3}$$

in which n denotes the inward normal direction to a boundary, a_n the normal component of boundary acceleration and ρ_w the mass density of water.

(b) At the reservoir-bed interface assuming the absorbing boundary for reservoir bottom, the condition will be:

$$\frac{\partial P}{\partial n} + q\dot{P} = -\rho a_y(x,t) \quad \text{on} \quad S_2 \tag{4}$$

where

q is the damping coefficient of the reservoir bottom

$$q = \frac{\rho_w}{C_s \rho_s} \tag{5}$$

ρ_s and C_s are mass density and sound wave velocity in sediment. The portion of the wave amplitude reflected back to the reservoir can be represented by the wave reflection coefficient α defined by

$$\alpha = \frac{1 - Cq}{1 + Cq} \tag{6}$$

where α may vary from 0 for full wave absorption to 1 for full wave reflection.

$a_y(x,t)$ is the vertical acceleration of ground motion at foundation. For a vertical bottom excitation, the vertical acceleration is zero. In this case, the bottom boundary condition leads to an eigenvalue problem.

(c) At the reservoir farfield, perfect damping is assumed:

$$\lim_{x \to \infty} P = 0 \quad \text{on} \quad S_3 \tag{7}$$

(d) At the free surface, neglecting the effects of water surface waves, the boundary condition is:

$$P = 0 \quad \text{on} \quad S_4 \tag{8}$$

When an excitation is caused by a harmonic vertical acceleration, the pressure field in the rectangular reservoir is governed by the following boundary conditions:

$$\frac{\partial P(0,y,\omega)}{\partial x} = 0 \tag{9}$$

$$\frac{\partial P(x,0,\omega)}{\partial y} + i\omega q P = -\rho a_y \tag{10}$$

$$P(x,H,\omega) = 0 \tag{11}$$

$$\lim_{x \to \infty} P(x,y,\omega) = 0 \tag{12}$$

For harmonic ground motion, the pressure in the reservoir can be expressed in the frequency domain as $P(x,y,t) = \overline{P}(y)e^{i(\omega t - kx)}$, where ω is the excitation frequency, k the wave number and $\overline{P}(y)$ the complex-valued frequency response function for hydrodynamic pressure. Substitution of this expression into equation (2) yields to the classical Helmholtz equation:

$$\frac{\partial^2 \overline{P}}{\partial y^2} + \lambda^2 \overline{P} = 0 \tag{13}$$

where

$$\lambda^2 = \frac{\omega^2}{C^2} - k^2 \tag{14}$$

The general solution of Eq. 13 is obtained as:

$$\overline{P}(y) = A\cos\lambda y + B\sin\lambda y \tag{15}$$

In the above equation, A and B are constant and are determined using boundary conditions.

At first the case with no bottom absorption is considered, in which $q = 0$. Applying the boundary condition to the general solution of equation (15) which already satisfies the free surface boundary condition yields:

$$\overline{P}(H) = A\cos\lambda H + B\sin\lambda H = 0 \tag{16}$$

and

$$B = -A\cot\lambda H \tag{17}$$

So, the general solution of equation (13) led to:

$$\overline{P}(y) = A(\cos\lambda y - \cot\lambda H \sin\lambda y) \tag{18}$$

To satisfy the dam-reservoir interface boundary condition k must be equal to zero in equation (13). So, it can be mentioned that for a vertical excitation, the hydrodynamic pressure independent of x. The solution for the hydrodynamic pressure is written as following:

$$P = \overline{P}(y)e^{i\omega t} \tag{19}$$

The reservoir-foundation interface boundary condition is applied with assumption of a vertical time-harmonic acceleration in the form of $a_y = \hat{a}_y e^{i\omega t}$ to get:

$$-A\lambda(\sin\lambda y + \cot\lambda H\cos\lambda y)e^{i\omega t} = -\rho\hat{a}_y e^{i\omega t} \text{ at } y = 0 \tag{20}$$

or

$$-A\lambda\cot\lambda H = -\rho\hat{a}_y \tag{21}$$

This yields for A:

$$A = \frac{\rho\hat{a}_y}{\lambda\cot\lambda H} \tag{22}$$

And

$$\overline{P}(y) = \frac{\rho\hat{a}_y}{\lambda\cot\lambda H}(\cos\lambda y - \cot\lambda H\sin\lambda y) \tag{23}$$

So, the hydrodynamic pressure distribution in the reservoir will be in the following form:

$$P(x,y,t) = \frac{\rho\hat{a}_y}{\lambda\cot\lambda H}(\cos\lambda y - \cot\lambda H\sin\lambda y)e^{i\omega t} \tag{24}$$

The peaks arise at the eigenfrequencies of the undamped system $\frac{\omega H}{C} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$.

It is interesting to investigate the limit as $\omega \to 0$. Then also $\lambda \to 0$, $\lambda\cot\lambda H \to \frac{1}{H}$ and $\cot\lambda H\sin y \to \frac{y}{H}$. In this case, the pressure has the hydrostatic pressure distribution:

$$\overline{P}(y) \to \rho \hat{a}_y H(1-\frac{y}{H}) \quad (25)$$

And

$$P(y) \to \rho a_y (H-y) \quad (26)$$

If bottom absorption is included, the boundary condition at the reservoir bottom becomes as equation (10). This boundary condition is applied to the general solution for P which already satisfies the free surface boundary condition to get

$$-A\lambda(\sin\lambda y + \cot\lambda H \cos\lambda y)e^{i\omega t} = \quad \text{at} \quad y=0 \quad (27)$$
$$-\rho\hat{a}_y e^{i\omega t} - Ai\omega q(\cos\lambda y - \cot\lambda H \sin\lambda y)e^{i\omega t}$$

Or

$$-A\lambda\cot\lambda H = -\rho\hat{a}_y - Ai\omega q \quad (28)$$

Which determines A as

$$A = \frac{\rho\hat{a}_y}{\lambda\cot\lambda H - i\omega q} \quad (29)$$

The boundary condition at the dam is automatically satisfied because $k = 0$. Then the $\overline{P}(y)$ is determined as:

$$\overline{P}(y) = \frac{\rho\hat{a}_y}{\lambda\cot\lambda H - i\omega q}(\cos\lambda y - \cot\lambda H \sin\lambda y) \quad (30)$$

And

$$P(y) = \frac{\rho\hat{a}_y}{\lambda\cot\lambda H - i\omega q}(\cos\lambda y - \cot\lambda H \sin\lambda y)e^{i\omega t} \quad (31)$$

As for the bottom with no absorption, at the limit as $\omega \to 0$, the pressure has the hydrostatic pressure distribution and the peaks arise at the eigenfrequencies of the undamped system $\frac{\omega H}{C} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$.

III. Model Analysis

To assess the effectiveness and ability of the proposed analytical solution, the dam-reservoir model is considered in this section. To have a good understanding of the effect of this parameter and check the feasibility of the present analytical solution, several situations were studied where the wave reflection coefficient and excitation frequency are varied over a wide range. Case study is the model of a rigid dam with reservoir which is subjected to harmonic excitation in the vertical direction. The full depth of the reservoir was considered to be $100\,m$. The mass density of water was assumed to be 1000 kg/m^3 and the acoustic velocity of water $1440\,m/s$. The amplitude of the external vertical acceleration was assumed to be equal to $0.3g$. The model is analyzed for two cases of neglecting and considering bottom absorption.

Figures 2 to 4 show the results for the case of no bottom absorption.

Figure 3 illustrates the hydrodynamic pressure distribution along the height for the case of no botton absorption. In addition the normalized hydrodynamic pressure $P/\rho gH$ was selected as variable for the cases of $\omega < \Omega$ and $\omega > \Omega$ to compare the results, where Ω is the natural frequency of the reservoir given by $\Omega = \pi C/2H$ and C is the acoustic velocity.

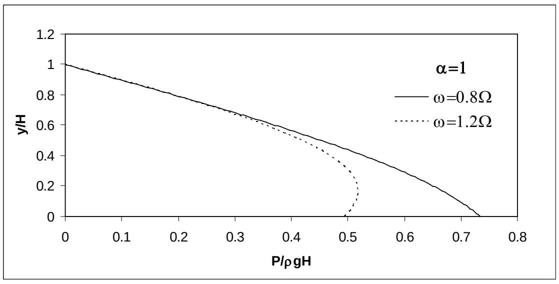

Fig. 2 Hydrodynamic pressure distribution on dam height for different values of ω

It is interesting to illustrate the hydrodynamic pressure distribution along the height for the limit as $\omega \to 0$. Figure 3 shows the case in which the excitation frequency is equal to zero.

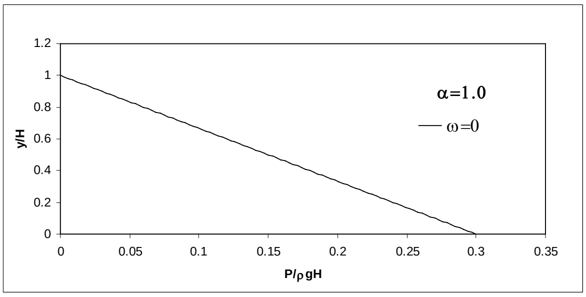

Fig. 3 Hydrodynamic pressure distribution on dam height for the case $\omega \to 0$

According to figure 3, it can be concluded that at the limit as $\omega \to 0$, the pressure has the hydrostatic pressure distribution and shows a linear relation with the height.

The reservoir of example is analyzed to illustrate the influence of the frequency ratio ω/Ω and reflection coefficient α equal to unit on the variation of maximum hydrodynamic pressure exerted on dam. Figure 4 shows the effect of ω/Ω on maximum hydrodynamic pressure for the case of bottom with no absorption.

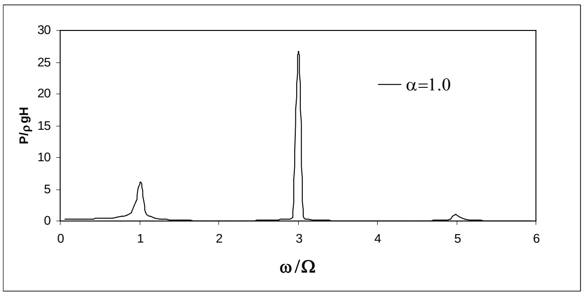

Fig. 4 Maximum hydrodynamic pressure variation due to ω/Ω for $\alpha = 1.0$

To show the effect of bottom absorption on the hydrodynamic response of reservoir, the model was analyzed considering the absorption at the bottom and the results are obtained for the condition of partial damping due to wave absorption in the alluvial deposit at the bottom of the reservoir. The results shown in figures 5 and 6 have been obtained from proposed analytical solution for different condition of bottom absorption and excitation frequency.

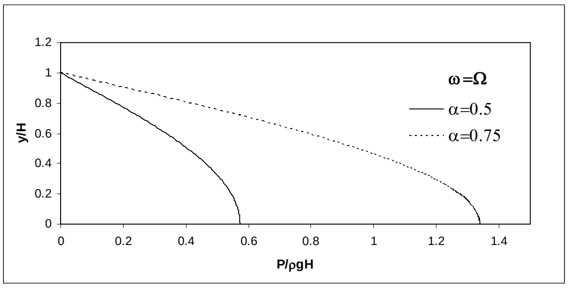

Fig. 5 Hydrodynamic pressure distribution on dam height for different values of α

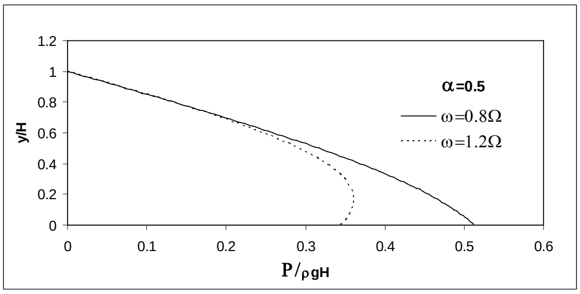

Fig. 6 Hydrodynamic pressure distribution on dam height for different values of ω

Similar to case of no bottom absorption, the pressure has the hydrostatic pressure distribution at the limit as $\omega \to 0$ for all value of reflection coefficient. Figure 7 show the result for this case.

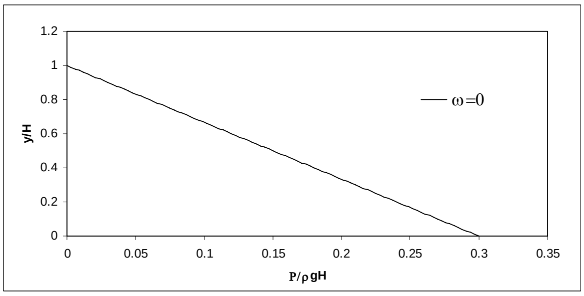

Fig. 7 Hydrodynamic pressure distribution on dam height for the case $\omega \to 0$

Finally, the example of this case is analyzed to illustrate the influence of the frequency ratio ω/Ω and reflection coefficient α on the variation of maximum hydrodynamic pressure exerted on the upstream face near the bottom of dam. Figures 8 and 9 show the effect of ω/Ω and α on maximum hydrodynamic pressure for different cases.

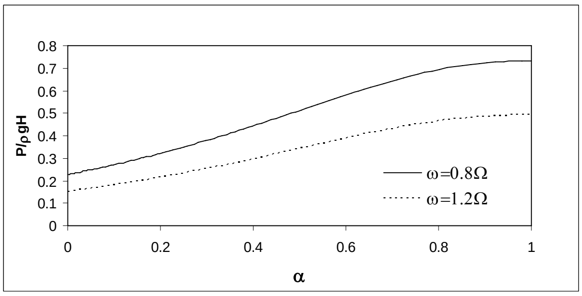

Fig. 8 Maximum hydrodynamic pressure variation due to α for different values of ω

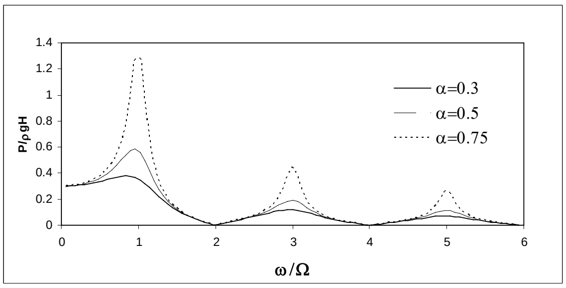

Fig. 9 Maximum hydrodynamic pressure variation due to ω/Ω for different values of α

It is obvious from figures 4 and 9 that the response becomes so much complicated when excitation frequencies are close to natural frequency of reservoir, where resonant is observed. With increasing wave absorption at the reservoir bottom and decreasing α, the fundamental resonant peak due to vertical ground motion decreases in amplitude. Figure 8 represents similarity in behavior of maximum pressure variation due to α for both case of excitation frequencies (less and more than the natural frequency of the reservoir). In addition the magnitude of hydrodynamic pressure exceeds when $\omega < \Omega$ compared with the case of $\omega > \Omega$.

IV. Conclusion

The paper has presented a new analytical technique to estimate the earthquake-induced hydrodynamic pressure on gravity dams allowing for water compressibility and wave absorption at the reservoir bottom. This new analytical solution can take into account the effect of bottom material on seismic response of gravity dams under vertical loading. Obtained results show the considerable values of induced hydrodynamic pressure due to vertical component of earthquake which can affect on dam. This effect has been neglected in most previous researches. Obtained results show the effect of bottom absorption and excitation frequency on hydrodynamic response of reservoir under vertical loading. Consider to results it can be concluded that the fundamental resonant peak due to vertical ground motion decreases in amplitude with increasing wave absorption and the magnitude of hydrodynamic pressure exceeds when the excitation frequency less than the natural frequency of the reservoir compared with the case in which the excitation frequency more than the natural frequency. In addition the method can be easily incorporated in dynamic analysis of dam.

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