International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:7, No:6, 2013

# Effect of U-Turn in Reinforced Concrete Dog-Legged Stair Slabs

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***Abstract*—**Reinforced concrete stair slabs with mid landings i.e. Dog-legged shaped are conventionally designed as per specifications of standard codes of practices which guide about the effective span according to the varying support conditions. Presently, the behavior of such slabs has been investigated using Finite Element method. A single flight stair slab with landings on both sides and supported at ends on wall, and a multi flight stair slab with landings and six different support arrangements have been analyzed. The results obtained for stresses, strains and deflections are used to describe the behavior of such stair slabs, including locations of critical moments and deflections. Values of critical moments obtained by F.E. analysis have also have been compared with that obtained from conventional analysis. Analytical results show that the moments are also critical near the kinks i.e. junction of mid-landing and inclined waist slab. This change in the behavior of dog-legged stair slab may be due to continuity of the material in transverse direction in two landings adjoining the waist slab, hence additional stiffness achieved. This change in the behavior is generally not taken care of in conventional method of design.

***Keywords*—**Dog-legged, Stair slab, F.E. Analysis, Landing, Reinforced concrete.

## I. Introduction

In low and medium rise buildings, stair slab are provided for vertical movements. These stairs are connected at floor level directly or through beam-column arrangement. Dog-legged stair slabs, commonly used for easy movements, are supported at landing levels on walls or beams provided along and/or at right angles to the direction of flight. These supports provide significant rigidity in the stair slab and hence a redistribution of moments is possible along the span of the flight. This phenomenon is normally ignored when designers consider the stair slab as simple beam supported at the ends and the span of the stair slab is taken as the distance between the end supports. Even the American code of practice [1] does not provide any recommendation regarding the restraining effect in the slab due to inherent support conditions. British code [2], however, considers the above fact and a reduction in the effective span of the stair slab are suggested which results in some saving in the design as compared to the conventional design. For stair slabs with landings supported parallel to the direction of flight, Indian code of practice for plain and reinforced concrete [3] suggests a reduced span of the stair slabs i.e. equal to the going of the stairs (the horizontal projection of the inclined portion) plus at each end either half the width of landing or 1m, whichever is smaller.

An experimental study conducted in Bangladesh [4] also claimed a considerably high ultimate load (about 133%) achieved in the stair slab designed as per specifications of the British Code [2]. A similar study by Ahmad et. al. [5] proved that the landings supporting the stair slab derive considerable restraining effect both at inner and outer edges of the flight. Consequently, a reduction in span of 30% and 20% of the going respectively in dog-legged and open-well stair slabs of selected size was suggested. Aslam [6], based his analytical study on the behavior of dog-legged stair slab, reported that the maximum sagging moment obtained as per provisions of Indian code [3] are on conservative side. Further, the study revealed that considerable hogging and twisting moments are developed at different locations in stair slab and the landings for which no care had been suggested by Indian code. Yahya [7] has studied the effect of evacuation time, in case of emergency, on the stair slab with and without intermediate landing

Since the complex behavior of stair slab is still not well understood, a comprehensive test study is therefore needed to establish the behavior of dog-legged stair slabs with different support arrangements.

## II. Method of Analysis

The conventional method of design and analysis of stair slab is mostly based on the recommendation of the codes of practices which specify the effective span according to the support conditions and the critical values of moments are calculated for given loads. However, most of these codes do not take into account the effect of kinks (i.e. the junction of waist slab and landing) and restraining effect on inner or outer edges of waist slab with the presence of landings. For example, as per the Indian code of practice (IS 456:2000), the effective span and hence the critical moments for given loads for a single fight stair slab attached with landing on one or both the sides and simply supported at the ends shall be same as that of a similar double flight stair slab.

### *A. Present Study*

Firstly, the behavior of a single flight stair slab attached with horizontal landings on both sides and supported at ends has been compared with a similar double flight stair slab attached with landings. The geometry and height of a single

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fight stair slab is kept same as one flight of double flight stair slab. The design procedure as per the conventional method i.e. according to the provision of standard codes e.g. Indian code of practice for Reinforced Concrete [3] is same, the behavior of the two stair slab may be judged by theoretical analysis.

Secondly, the behavior of a multi flight dog-legged stair slab with six different types of supports (Fig. 1) has been studied to observe the effect of varying support conditions and landing width. For this, the stair slab was chosen for three floor heights 3m each, with mid height landings each at 1.5m from the floor landing and the width of waist slab and landings kept equal. Each waist slab, 1.5m wide and 2.7m horizontal projection attached with 1.5m wide landings was supposed to have a uniform thickness in waist slab and landings.

Initially, a 190mm thick waist slab was found suitable for a span of 5.7m and the same was adopted for a single flight stair slab model also, for a better comparison of theoretical results. Each of the stair slab model has been analyzed for a uniformly distributed dead and live loads over the waist slab and landings.

A portion of the stair case including the intermediate flight and adjacent landings (Fig. 2) is considered to be representing the true behavior of staircase, since it was contemplated that this particular flight would be free from any undue effect of the boundary conditions imposed at the terminating landings.

The study was carried out with the following assumptions:

(i) The thickness and width of waist slab and landing slab are equal.
(ii) The material is linearly elastic, homogeneous and isotropic.
(iii) The value of modulus of elasticity of concrete is $20.5 kN/mm^2$ and Poisson's ratio is 0.17.

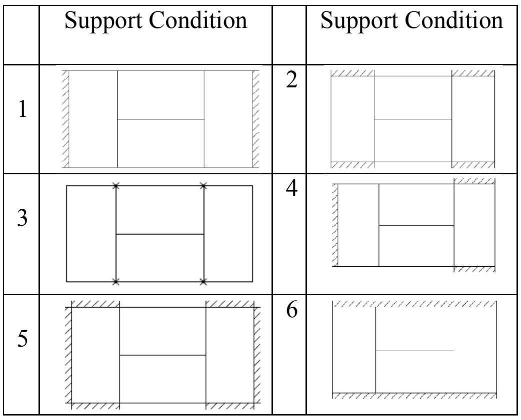



Fig. 1 Different Support Types

## III. Finite Element Analysis

In a simple finite element analysis, the composite material such as reinforced concrete is considered as homogeneous material with steel embedded at suitable location. The structure such as stair slab may be idealized as an assembly of different finite elements and its behavior is defined in terms of the characteristics of these elements. The nonlinear characteristic of reinforced concrete as continuum is defined in mathematical form and an incremental iterative procedure is adopted to incorporate the non-linear characteristics of reinforced concrete. The stiffness matrices of constituent material are updated at each load step according to the nonlinear material behavior and the failure due to cracking or crushing of reinforced concrete. The total load is applied in a number of load steps and for load step iterations are carried out to reduce unbalanced load within a prescribed limit. The failure of structure is indicated by stiffness matrices becoming non-positive definite. Thus, the behavior of stair slab analyzed as plate may be predicted in pre and post cracking stages.

### *A. Geometrical Modeling and Discretization*

In present analysis the stair slabs have been discretized by four-nodded plate bending elements of size 150mm×150mm with six degree of freedom per node. A uniform thickness of waist slab as well as landing slab has been taken as 190mm. The model consists of 200 elements each in mid landing, floor landing and the waist slab.

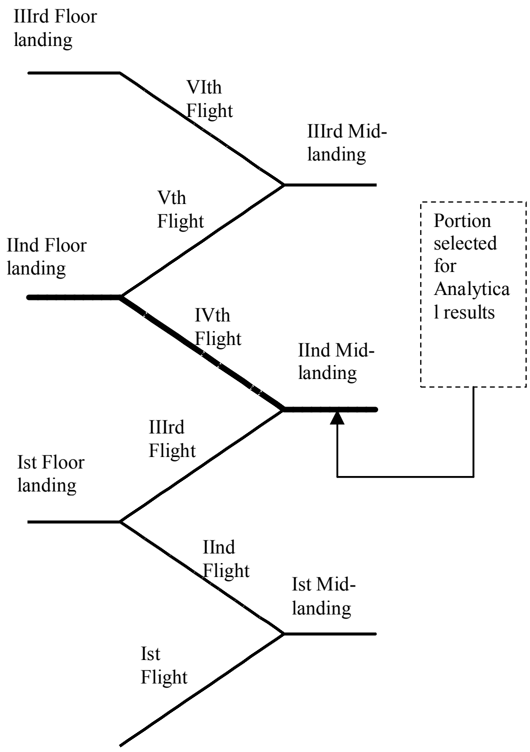



Fig. 2 Side view of dog-legged staircase

In present analysis the stair slabs have been discretized by four-nodded plate bending elements of size 150mm×150mm with six degree of freedom per node. A uniform thickness of waist slab as well as landing slab has been taken as 190mm. The model consists of 200 elements each in mid landing, floor landing and the waist slab (Fig.3).

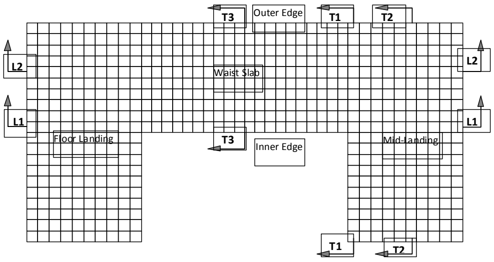



Fig. 3 F.E. Mesh of Chosen portion of Multi-flight stair slab

*B. Support Conditions*

Wall supports at the ends of single flight and double flight stair slabs were modeled as hinge supports, thus the moments along the supports were released whereas the reactions in the three directions were considered as restraints. In multi flight slab models the first waist slab started at ground level has been modeled as fixed supported at its lower edge while landings supported on walls or beams assumed as hinged. A fixed support assumes both translational and rotational restraints in all the three directions (i.e. $F_X$, $F_Y$, $F_Z$, $M_X$, $M_Y$, $M_Z$ are restraints) whereas a hinged support has restraints against all translational movement and none against rotational movement. In other words, a hinged support assumes $F_X$, $F_Y$, $F_Z$ as restraints and $M_X$, $M_Y$, $M_Z$ as releases.

The following six varying support conditions were chosen for three storied dog legged stair as described in Fig.1.

- SUPPORT TYPE-1: Landing slab simply supported at the extreme edges.
- SUPPORT TYPE-2: Landing slab supported on the two edges parallel to the direction of span.
- SUPPORT TYPE-3: Landing slab cantilevered, with beam simply supported at ends.
- SUPPORT TYPE-4: Mid-landing is supported on the two edges parallel to the direction of span and floor landing is simply supported at the extreme edge.
- SUPPORT TYPE-5: Landing slab simply supported along the three external edges.
- SUPPORT TYPE-6: Landing slab and waist slab is simply supported at its outer edges

## IV. Analytical Results

*A. Double Flight Stair Slab*

Behavior of double flight stairs has been investigated for various supporting arrangements. For convenience, the analytical results have been presented for a selected portion of staircase consisting of II mid-landing, IV waist slab and II floor landing which represents the general behavior (Fig. 2).

For each support conditions a landing width of 1.5m has been selected and the results compared for bending moment and deflection along few critical sections as discussed below.

Fig.4 shows a general distribution of moments in x-direction. Variation of bending moment $M_x$ along sections L1-L1 for different support types is shown Fig. 5 (a) wherein it is clear that at kinks, maximum hogging moment is developed in all types of stair slabs except in type-1. A similar variation of bending moment $M_x$ as obtained at section L2-L2 is shown in Fig. 5(b). Transverse bending moment $M_y$ (parallel to steps), as shown in Fig. 5 (c) and (d), is found to be negligibly small in the inclined waist slab for all support types at landing. This moment is of considerable magnitude, especially in a strip close to the kink, where the inclined waist slab meets the landings.

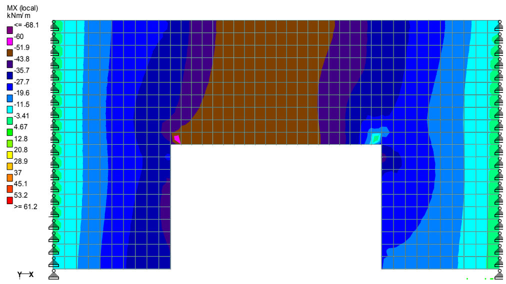
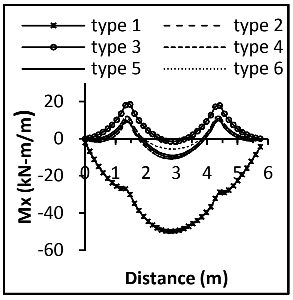
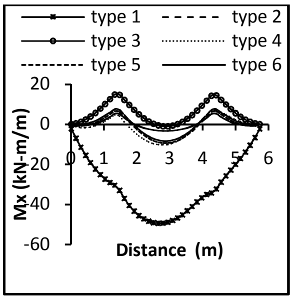


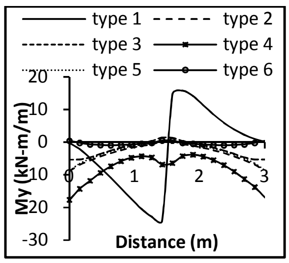
Fig. 4 Bending Moment $M_x$ in Dog-legged Stair Slab



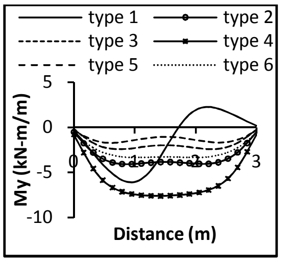

(a) $M_x$ at section L1-L1




(b) $M_x$ at section L2-L2




(c) $M_y$ at section T1-T1




(d) $M_y$ at section T1-T1

Fig. 5 $M_x$ and $M_y$ at different sections for Dog-legged stair slab

Vertical deflection at mid span for different support conditions is shown in Fig. 6. It may be observed here that there is little variation in deflection along the section and that the maximum deflection occurs for support type-1.

From the above analytical results it may be realized that the behaviour of stair slab with support type-1 very closely resembles that of simply supported beam. This implies that the stair slab does not require any special treatment because of its folded nature. Also, that waist slabs supported on landings running at right angle to the direction of the flight (i.e. support type 2-6), derive significant restraining effect from such supporting arrangements. Therefore, additional hogging moments are seen in longitudinal direction near the junction of waist slab and landings.

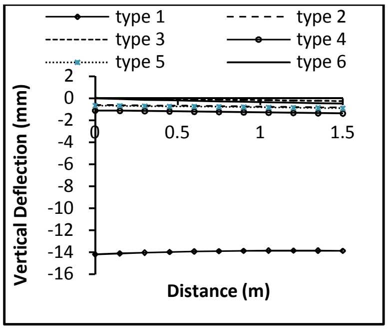



Fig. 5 Vertical Deflection at section T3-T3 for double flight stair slabs

### *B. Single Flight Stair Slab*

The behavior of single flight stair slab was also studied by analyzing 1.5m wide stair flight with landing on both sides and supported at the ends on simple supports. The slab model was descritized into rectangular elements and subjected to uniformly distributed load over complete span. Contours of moments $M_x$ as obtained for maximum load are shown in Fig. 6, wherein maximum stresses are near mid span and minimum near the supports. Variation of deflection along the flight is compared with obtained for a similar sized double flight stair slab in Fig. 7.

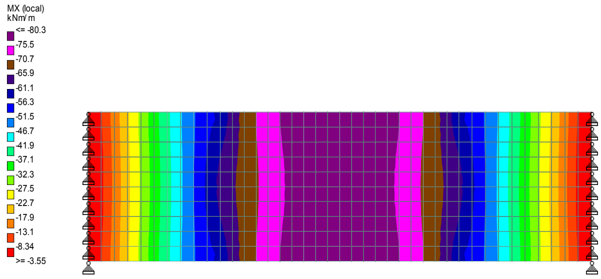



Fig. 6 Bending Moment $M_x$ in Single Flight Stair Slab

### *C. Flexural Analysis as per Indian Code of Practice (IS 456:2000) vs. F.E. Method*

The flexural behavior of multi flight stair slab as described above and analyzed using Finite Element technique could be also determined by calculating effective span in different arrangements as per the specifications of Indian Code of also determined by calculating effective span in different arrangements as per the specifications of Indian Code of Practice (3). Based on the calculated effective span and applied loads the moment $M_x$ in longitudinal direction could be determined for different support arrangements. For most of these support arrangements (types 1,2,4,5 & 6), the stair slab with landings supported on walls is considered as simply supported and hence the longitudinal moment is found to be hogging all along the span. This method does not give any idea of the hogging developed anywhere on stair slab. The critical values of the longitudinal moment are given in Table 2, along with the corresponding values obtained by F.E. analysis.

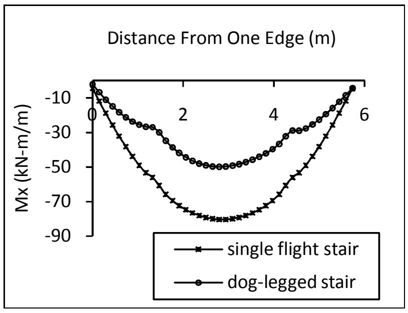



Fig. 7 Comparison of Bending Moment $M_x$ along mid section

It may be observed here that the moment calculated as per conventional method is always more than that obtained by F.E. analysis, except in Type-3 where the critical moment is hogging due to cantilevered landings on both the sides. The difference varies from 21% to 92% corresponding to different support arrangements and the width of landings.

## V. Comparison of Results

### *A. Effect of Landing Width*

A parametric study was conducted on double flight stair slab with landings for each type of support condition by varying width of landing and waist slab. The maximum value of the moment or deflection developed anywhere in the slab hereafter referred as 'critical'. Such critical values of longitudinal moment ($M_x$) along the span, at mid span section, obtained with F.E. analysis have been compared with the corresponding values as per conventional analysis, for different support conditions and landing width varying as 1m, 1.5m, 2m and 2.5m. A glance study on the results as given in Table I, suggests that the complex interaction of the stair slab with the supporting landing slab as well as the adjoining flights influence the behavior of the stair slab. It is also observed that:

1) The longitudinal sagging moment is least affected with variation of landing width, for type-1 it increases linearly with increasing landing width, whereas for Type-3 this moment is decreased with increasing landing width from 1m to 1.5m.
2) The critical longitudinal hogging moment is generally increasing with increasing landing width for all support conditions. In Type-1 there is no critical longitudinal hogging moment.
3) The critical transverse sagging moment is almost increasing linearly with increasing landing width for all support conditions.
4) Except for Type-1, the critical hogging moments in transverse direction are only marginal as compared to the longitudinal moments. Therefore the effect of landing width on the critical hogging moment is also marginal.
5) Vertical Deflection: The maximum vertical deflection for Type-1 increased linearly with increasing landing width, whereas for Type-5 there is no effect of landing width on maximum vertical deflection. For Type 2, 4 & 6 the maximum deflection remained constant up to landing width of 2.0m where after it decreased marginally.

The values of critical moments and deflection in support type-1 are found maximum as compared to other support types. Also, increase in the width of landing and flight increases the critical moments and deflection and it also changes the location of critical moments.

In type-3 (with cantilevered landing) theoretical moments (both sagging and hogging) obtained by Finite Element analysis are found to be more than the moments obtained by the conventional method by 12 to 15%. Therefore, extra safety has to be considered in case of cantilevered landings.

In case of landing and flight width 1.0m, critical sagging moment obtained by Finite Element analysis are found to be less than the moments obtained by the conventional method by 20%, 70%, 74%, 70% and 85% for support type 1, 2, 4, 5 and 6 respectively.

In case of landing and flight width 1.5m, critical sagging moment obtained by Finite Element analysis are found to be less than the moments obtained by the conventional method by 23%, 75%, 80%, 75% and 85% for support type 1, 2, 4, 5 and 6 respectively.

In case of landing and flight width 2.0m, critical sagging moment obtained by Finite Element analysis are found to be less than the moments obtained by the conventional method by 23%, 80%, 85%, 75% and 88% for support type 1, 2, 4, 5 and 6 respectively.

TABLE I
COMPARISON OF CRITICAL LONGITUDINAL MOMENT (SAGGING) BETWEEN FINITE ELEMENT ANALYSIS AND CONVENTIONAL ANALYSIS

| *Support Type* | *Landing Width (m)* | *Effective Span as per Indian Code (m)* | *Bending moment $M_x$ (Sagging -ve)* | | *Difference with respect to Conventional Method* |
|---|---|---|---|---|---|
| | | | F.E. Analysis (kN-m/m) | Conventional Analysis (kN-m/m) | |
| | 1.0 | 4.7 | -40.960 | -52.0 | -21% |
| 1 | 1.5 | 5.7 | -57.574 | -75.1 | -23% |
| | 2.0 | 6.7 | -78.243 | -102.2 | -23% |
| | 2.5 | 7.7 | -100.726 | -133.3 | -24% |
| | 1.0 | 3.7 | -9.054 | -31.9 | -71% |
| 2 | 1.5 | 4.2 | -8.878 | -39.7 | -77% |
| | 2.0 | 4.7 | -8.014 | -48.1 | -83% |
| | 2.5 | 4.7 | -6.521 | -48.1 | -86% |
| | 1.0 | 2.007 | -11.201 | -9.8 | +14% |
| 3 | 1.5 | --- | -1.461 | ----- | --- |
| | 1.0 | 4.2 | -10.734 | -41.4 | -74% |
| 4 | 1.5 | 4.95 | -10.799 | -56.1 | -80% |
| | 2.0 | 5.7 | -10.314 | -72.7 | -85% |
| | 2.5 | 6.2 | -9.165 | -85.2 | -89% |
| | 1.0 | 3.7 | -9.269 | -31.9 | -70% |
| 5 | 1.5 | 4.2 | -9.897 | -39.7 | -75% |
| | 2.0 | 4.7 | -10.142 | -48.1 | -78% |
| | 2.5 | 4.7 | -10.032 | -48.1 | -79% |
| | 1.0 | 3.7 | -3.712 | -31.9 | -88% |
| 6 | 1.5 | 4.2 | -5.521 | -39.7 | -86% |
| | 2.0 | 4.7 | -5.226 | -48.1 | -89% |
| | 2.5 | 4.7 | -3.528 | -48.1 | -92% |

In case of landing and flight width 2.5m, critical sagging moment obtained by Finite Element analysis are found to be less than the moments obtained by the conventional method by 23%, 85%, 85%, 78% and 90% for support type 1, 2, 4, 5 and 6 respectively.

*B. Effect of Support Conditions*

1) For support type-1, critical sagging moment obtained by Finite Element analysis is 20-25% less than the moments obtained by the conventional method.
2) For support type-2 , the critical sagging moment obtained by Finite Element analysis is about 70-85% less than the

moments obtained by the conventional method.

3) For support type-3 and landing width of 1.0m, critical sagging moment obtained by Finite Element analysis is 15% more than the moments obtained by the conventional method.
4) For support type-4, critical sagging moment obtained by Finite Element analysis is 75-90% less than the moments obtained by the conventional method.
5) For support type-5, critical sagging moment obtained by Finite Element analysis is 70-80% less than the moment obtained by the conventional method
6) For support type-6, critical sagging moment obtained by Finite Element analysis is reduced to 85-90% of the moments obtained by the conventional method. There is an advantage of providing wall supports on the outer edge of the waist slab, as conventionally present in stair halls, both in the longitudinal and transverse moments. Comparing the results of Type-2 and Type-6 it is observed that both the longitudinal and transverse sagging moments are reduced by more than 50%. Also the maximum deflection is reduced due to side supported waist slab by 10-50%, the higher values correspond to lower width of landing.
7) Support types 2, 4, 5 & 6 and landings supported on outer edges, running at right angle to the direction of the flight derive significant restraining effect from such supporting arrangements, the effective span reduces significantly as compared to support type-1.

## VI. Conclusion

The behavior of dog-legged stair slab depends both on the type of supports and width of flight and landings. The recommendations of standard codes of practice available are insufficient to describe the true behavior of such slab with varying support conditions and geometry.

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