Simulation of Population Dynamics of *Aedes aegypti* using Climate Dependent Model

Nuraini Yusoff, Harun Budin, and Salemah Ismail

***Abstract*—**A climate dependent model is proposed to simulate the population of Aedes aegypti mosquito. In developing the model, average temperature of Shah Alam, Malaysia was used to determine the development rate of each stage of the life cycle of mosquito. Rainfall dependent function was proposed to simulate the hatching rate of the eggs under several assumptions. The proposed transition matrix was obtained and used to simulate the population of eggs, larvae, pupae and adults mosquito. It was found that the peak of mosquito abundance comes during a relatively dry period following a heavy rainfall. In addition, lag time between the peaks of mosquito abundance and dengue fever cases in Shah Alam was estimated.

***Keywords*—**simulation, *Aedes aegypti*, Lefkovitch matrix, rainfall dependent model, Shah Alam

I. Introduction

AEDES AEGYPTI mosquito has been identified to be one of the vectors that transmit the dengue virus from one host to another. The dengue virus will be in the blood of an infected host for about seven days. An uninfected mosquito that feeds on this individual during this period will acquire the virus. Once infected, the virus carried by the mosquito will be transmitted to another host during another blood meal. This transmission cycle will continue until there is no more infected mosquito.

It follows that countries whose climates are conducive to the development and survival of mosquito are among the 100 or more countries that are fighting with the spread of dengue fever. These countries are located in tropical (between the latitudes of 23.5^{o}N and 23.5^{o}S) and sub-tropical regions (between latitudes 23.5^o and 40^o in northern and southern hemispheres). The tropical countries are warm and humid all year round with two distinct seasons, the wet and the dry season with mean temperature of above 18^{o}C. Being located adjacent to the tropical regions, the subtropics will have similar climate for at most eight months in a year. When the wet season coincides with the high temperature, we will show later that this is a suitable environment for mosquito breeding and hence, will be followed by the rise of the dengue fever incidences.

There had been several studies that relate the climate and the increase of Aedes population. Refences [1]- [3] elaborated in their papers on how the increase in temperature will increase the development rate of each stage of the mosquito. This phenomena reduces the time they are in each stage, hence reduces their time from egg to adult. In addition to high temperature, the amount of rainfall is found to trigger the aggressiveness of the breeding of Aedes mosquito [4],[5]. In all of these papers, models were developed to support the theories proposed and validated by the actual data obtained from experiments conducted. There are many studies that relate the increase of dengue fever incidences to climate and have been reported such as by [6]-[9]. Since the weather conditions are some what periodic (similar conditions at about the same time each year), this leads to the establishement of the periodicity and seasonality of dengue fever occurences, for example as reported by [10],[11].

The climatic factors will not be meaningful if there is no breeding sites for the mosquito. The three stages in the life cycle of mosquito (egg, larva, pupa) are under water. Indoor, they will breed in containers such as vases, ant traps, unused toilet bowls etc., while in the outdoors they will breed in small discarded containers (bottles, food packaging), large discarded containers (tires, damaged appliances), utensils or tools (trash cans, pails) and cavities in structures (holes in concrete floors, roof gutter) [12]. A study was conducted in Thailand which incorporated the temporal patterns (dry and wet) and the spatial patterns (tires, earthenware jars and outdoor tanks). It was reported that in almost all occasions studied (14 out of 16), there is a strong correlation between pupal and adult population with time and space [13].

The purpose of this study is to see whether there is any relation between climate and the dengue fever incidences. In this paper, a stage structured model will be proposed based on the concept of matrix population model [14], and will be adapted using research done by [15] and [5]. The model is constructed based on the climate and conditions of the city of Shah Alam, Malaysia. This model will take into account the temperature and rainfall. The population of each stage of the Aedes mosquito will be simulated and related to the actual data of Aedes House Index and dengue fever incidences reported in the city of Shah Alam.

In the next section, the related theories and steps involved in the construction of the model will be presented. Results obtained from the simulation will be compared to the actual

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data of larval population and In the section of Result and Discussion, the simulated values will be compared to the actual data of larval population and will use this model to simulate the population of Aedes aegypti. The relation of these results and the number of dengue fever cases will be discussed. Summary of the findings and recommendations will be presented in the final section of Conclusion.

II. Materials and Methods

This section of the paper will elaborate the related theories required in constructing a stage-structured model of the population dynamics of *Aedes aegypti*. As mentioned in the previous section, the stage-structured model used in this paper is the matrix population model. The proposed model will take into account the temperature and rainfall for the city of Shah Alam, Malaysia. The data sets were obtained from the Malaysian Meteorology Department for the weather station in Subang, Malaysia, the closest weather station to Shah Alam.

In our previous study, it is found that there are only small fluctuations in daily temperature in Malaysia and thus the development of each stage of the mosquito varies only in the third decimal place. Hence, for this study, parameters will be computed based on the average temperature 27.2^{o}C. However, parameters that are rain dependent will be computed daily, based on the actual daily rainfall.

A. Temperature Dependent Model

Matrix model was initiated by P.H. Leslie in 1945 [14]. This model makes analysis and projection of female population according to age-classes possible. In this model, the female population is divided into *n* age classes with the class interval *h*, and their birth and death patterns are required. The projection interval has to be done for every *h* years (or whichever time unit used) and the transition matrix is known as the Leslie matrix.

However, there are species in our biological systems whose life-cycles are not well-defined by age, but rather, by stage or size. For example, insect's life-cycle is divided into the stages of egg, larva, pupa and adult, while size is more suitable in the classification of plants. Moreover, the duration of each stage is different. The transition matrix constructed from such life cycle is known as the Lefkovitch matrix [15]. Let *A* be the transition matrix and $X^{(k)}$ is the population at any time *k*. A projection of population at time *k* will be a matrix multiplication between matrix *A* and population at time *k*-1, as given in (1).

$$X^{(k)} = AX^{(k-1)}, \qquad (1)$$

An example of a five-stage Lefkovitch matrix is given in (2).

$$A = \begin{bmatrix} P_1 & F_2 & F_3 & F_4 & F_5 \\ G_1 & P_2 & 0 & 0 & 0 \\ 0 & G_2 & P_3 & 0 & 0 \\ 0 & 0 & G_3 & P_4 & 0 \\ 0 & 0 & 0 & G_4 & P_5 \end{bmatrix}. \qquad (2)$$

Element P_i denotes the probability of surviving and staying in stage *i*, G_i is the probability of surviving and growing from stage *i* to stage (*i* + 1), and F_i represents the fertility of stage *i*. One big advantage of the Leftkovitch model is that it allows different duration in each stage and projection can be done by days, weeks or whatever unit used.

If s_i is the survival rate for stage *i* and d_i is the duration in stage *i*, then we can compute P_i and G_i as follows:

$$G_i = \frac{s_i^{d_i}(1-s_i)}{1-s_i^{d_i}}. \qquad (3)$$

Since the total population of stage *i* that survive equals to the proportion that remains in stage *i* plus the proportion that has grown to stage (*i* + 1), or, $s_i = P_i + G_i$, then

$$P_i = \frac{s_i\left(1-s_i^{d_i-1}\right)}{1-s_i^{d_i}}. \qquad (4)$$

For a detailed derivation of (3) and (4), readers may refer to [15]. Next, we will discuss on how these information can be obtained based on the development rates of each stage.

Classification of the life cycle of Aedes that is used in this study is egg (*E*), larva (*L*), pupa (*P*), adult 1 (*A*1) and adult 2 (*A*2). An adult mosquito is classified as *A*1 before the first egg laying. Fig. 1 gives the life cycle of an Aedes mosquito from egg to adult 2. The parameters P_i, G_i and F_i are the entries of the matrix given by (2).

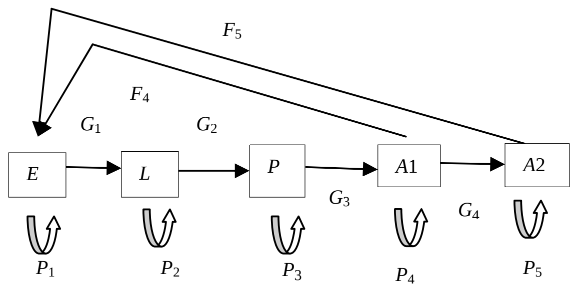

Fig. 1 Life cycle of Aedes mosquito

For *Aedes aegypti*, the values of F_4 and F_5 represent the number of (female) eggs laid by the female each female adult in the stage *A*1 and *A*2, respectively. This is computed by taking the average number of eggs laid per oviposition, divided by the duration the mosquito stay in the respective stages. In this study, we use the number 63 eggs/oviposition with ratio of male:female is taken to be 1:1, and it does not depend on temperature [2]. There will be one oviposition in stage $A1$(*ovr*1) and up to five ovipositions in stage $A2$(*ovr*2) in the Malaysian temperature . Values of F_2 and F_3 are zero since both larva and pupa do not lay egg. For this study, G_1 represents the development from stage E to L and will be referred to as egg-hatching or *elr* while G_2 and G_3 are from larva to pupa (*lpr*) and pupa to adult (*par*), respectively. In both (3) and (4), the values of s_i and d_i are needed in order to compute the entries of the matrix. As mentioned earlier, s_i is the survival rate of each stage. This parameter is temperature dependent and its computation was discussed in detail in [16]. The values of s_i used in this study is given in Table 1, extracted from the same paper.

TABLE I
SURVIVAL RATES AT TEMPERATURE 27.2°C

s_i	Stage	Survival rate
s_1	Egg	0.9890
s_2	Larva	0.9898
s_3	Pupa	0.9898
s_4	Adult 1	0.9100
s_5	Adult 2	0.9100

In order to compute d_i, the duration of each stage, development rate at the average temperature, $R_D(27.2^oC)$, of each stage was computed by [16]. Let CD_t be the cumulative developmental rate such that

$$CD_t = \sum_{t=0}^{n} R_D(T_t) \quad . \tag{5}$$

This term accumulates the developmental rate for t days, based on the temperature of the day. The developments from egg to larva, larva to pupa and pupa to adult are considered complete when $CD_t > 0.95$ while *ovr*1 occurs on the day when $CD_t > 1.0$. Subsequent cycles (*ovr*2) occur on the day when CD_t increases an additional 0.58 [1]. Detailed calculation was discussed in [16] and the values obtained for the duration of each stage can be extracted from Table II.

TABLE II
AGES AND NUMBER OF EGGS LAID PER DAY FOR EACH STAGE

Stage	Ages (days)	Eggs laid
Eggs	< 4	0
Larva	4 - 8	0
Pupa	9 - 10	0
Adult1	11 - 14	8
Adult2	15 - 24	16

It is important to note that the number of eggs laid given in Table 2 is the female eggs only. In the next subsection, the rainfall is taken into account in constructing the matrix.

B. Rainfall Dependent Model

The above model is extended to take into account the daily rainfall. In the *Aedes* mosquito life cycle, water is required in the development of egg, larva and pupa. However, flooding is a necessity for egg hatching. Breeding sites indoors can be monitored by house owners, by making sure that no water is left uncovered for a long period. However, some mosquitoes breed outdoors, inside any containers left scattered in the field, by the road sides and abandoned premises. This proposed model is to help in the simulation of *Aedes* population if rainfall contributes to the availability of breeding sites.

For this paper, only the egg hatching process (G_1) is considered while all other parameters are kept constant at the temperature specified earlier. Female mosquito lays eggs on the water edge and flooding is necessary for hatching [5]. Hence, egg hatching is taken to be a function with respect to time t. It can be represented as follows:

$$G_1(t) = s_1 c_1 \left(\frac{wd(t-1)}{wd_{max}} \right)^{c_2} . \tag{6}$$

In (6), $wd(t)$ is the water depth at time t and wd_{max} is the maximum height of the breeding site. Parameters c_1 and c_2 are greater than zero. The choice for the values of c_1 and c_2 are based on the following conditions:

1) When $wd(t-1) = 0$, $G_1(t) = 0$, no hatching occurs.
2) When $wd(t-1) = wd_{max}$, the hatching of the surviving eggs equal to the development of egg at the specified temperature [16].
3) Value of c_2 is chosen such that the hatching increases more rapidly for higher value of *wd*, thus will create a quadratic-like function.

Now, the transition matrix for this model will be as given in (7).

$$A(t) = \begin{bmatrix} P_1 & 0 & 0 & F_4 & F_5 \\ G_1(t) & P_2 & 0 & 0 & 0 \\ 0 & G_2 & P_3 & 0 & 0 \\ 0 & 0 & G_3 & P_4 & 0 \\ 0 & 0 & 0 & G_4 & P_5 \end{bmatrix} . \tag{7}$$

In the next subsection, steps, conditions and assumptions for the simulation process will be discussed.

C. Simulation based on daily rainfall

In determining the best values of parameters in the function $G_1(t)$, verification is done by comparing the modeled values with the actual data obtained from the Department of Town Services, Shah Alam City Council (more popularly known as *MBSA*). The department provided us with data of reported dengue fever cases in Shah Alam and the data of percentage of premises found to be positive with aedes larvae recorded weekly. In Singapore, the latter data was referred to as the *Aedes* house index [17]. Daily average temperature and rainfall were obtained from the Meteorology Department, Malaysia as recorded at the Subang meteorology station. This is the closest station located near Shah Alam.

In addition to the conditions given in the previous subsection, it also assumed that, when $wd(t-1) > wd_{max}$, all the remaining eggs will wash out, hence there will be no eggs to hatch at time *t*, or $P_1 = 0$ and $G_1(t) = 0$. However, the remaining larva will not be washed out as the Aedes aegypti larvae were able to resist the flushing effect of rain [18].

In simulating the population of each stage of the mosquito, (8) is generated for all *t*, time in week, for years 2008 until 2010. Recall that the constants F_4 and F_5 were the number of female eggs. Hence, this simulation gives the population of female mosquito for each stage.

$$P(t) = A(t)P(t-1). \tag{8}$$

In (8),

$$P(t) = [E(t) \quad L(t) \quad P(t) \quad A1(t) \quad A2(t)]^T, \tag{9}$$

population of each stage at time *t*.

Firstly, simulation was done to determine the best value for c_2. The modeled result for larval population was compared to the *Aedes* house index, looking for the presence of similar patterns.

Next, with the chosen c_2 value, modeled population of the adult female mosquito was able to be determined. For discussion, the result will be compared to the dengue fever cases to see the presence of lag time.

In all simulations, various initial numbers of eggs were used in order to obtain reasonable number to be tabulated. It was discovered that different initial value has no significant in the result obtained and this confirmed the same result as written in [2].

III. Results and Discussions

In this section, results will be presented and discussed. This section will be divided into several subsections to present and discuss different aspect of the simulation.

A. Identification of constant c_2

As mentioned earlier, the best value of c_2 need to be identified, as compared to the *Aedes* house index (AHI). Several values of c_2 were used and Fig. 2 shows the relation between water depth and hatching rate for different c_2.

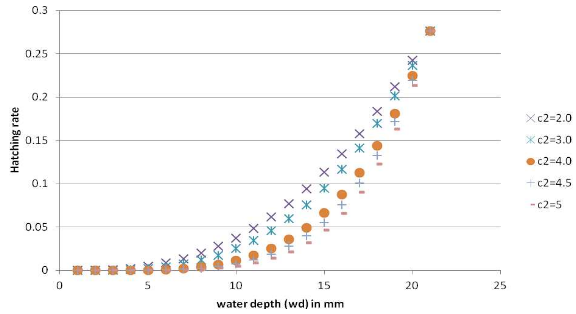

Fig. 2 Hatching rate with respect to water depth for different c_2

In this study, wd_{max} used is 20 mm, taken to be the average depth of containers considered as the breeding sites of Aedes mosquito in the area of Shah Alam. Out of these values, $c_2 = 5$ gives the best modeled population of larva, as compared to the AHI data. Comparison was done by representing the data obtained as a sine function given in the form of

$$\alpha(t) = a + \sum_{k=1}^{n} b_k \sin(2k\omega t + \phi_1). \tag{10}$$

In (10), $\omega = \pi / 52$, taking 52 weeks in a year as one period. The *k* value classifies the sine function to be of *k*th harmonic.However, it was discovered there were some interventions done by the authorities in 2009, in order to control the high number of dengue fever cases in Shah Alam in the first 2 months of the year. It is felt that data for year 2008 is more representative of actual scenario of Shah Alam in order to test this model. Moreover, before year 2008, there is no complete data for AHI in MBSA's database. Hence, the following results presented will be compared to the respective data for the year 2008. When the simulated larval population was compared to the actual AHI, the result can be represented by the graph in Fig. 3.

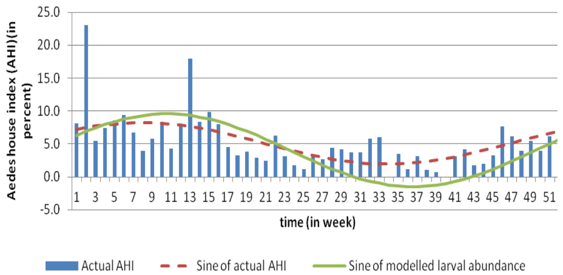

Fig. 3 Comparison of actual AHI with modeled larval population for year 2008

From the graph, it was shown that the modeled value is a close match to the actual AHI, better than for the year 2009, as given in Fig. 4.

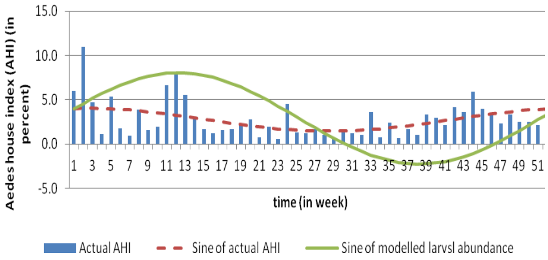

Fig. 4 Comparison of actual AHI with modeled larval population for year 2009

Hence, the simulation result for the female adult population of Aedes aegypti of this model will be taken as a good representation of the actual abundance. The result will be presented and discussed in the following subsection.

B. Simulation of Aedes aegypti population

In the above simulation, the result produced gives weekly population of each stage of mosquito based on the rainfall dependent model. From there, the population for $A1$ and $A2$ were totaled and tabulated. As mentioned earlier, the result presented does not represent the actual population but it gives the trend of the abundance of mosquito.

In presenting the modeled population of Aedes mosquito, the tabulated values are plotted together with the representation of the value as a sine function as given in (10).It is important to know that the numbers on the vertical axis do not represent the exact population, but a relative number for the initial condition used. Four different representations were computed for each $k = 1$, $k = 2$, $k = 3$ and $k = 4$. The resulted graph is given in Fig. 5.

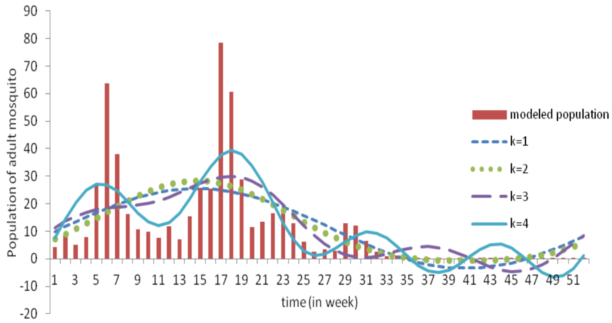

Fig. 5 Modeled population of *Aedes* mosquito for year 2008

The graph shows that, when higher value of k is used, a more accurate representation (10) gives. Although the population seems negligible in the last one third of the year, the graph given for k = 4 shows there was a slight increase and decrease of the population in that period.

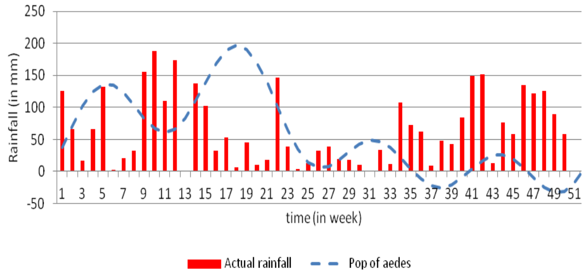

Fig. 6 Modeled population of *Aedes* mosquito and actual rainfall for year 2008

Next analysis is the relation of rainfall with the abundance of Aedes mosquito. The graph that represents both items is given in Fig. 6. Numbers on the vertical axis represent only the actual rainfall of Shah Alam, Malaysia. As for the population of Aedes, the researchers are only interested in the pattern of mosquito abundance, with respect to rainfall. This graph is for the fourth harmonic sine representation of the population of mosquito.

It was observed that population of mosquito is at its lowest during heavy rainfall. In the drier period following this heavy rainfall, the population appears to reach its peak. Since fourth harmonics is used, there are relatively four peaks and troughs, and all seem to follow this pattern. Recall that, this is so for our assumption that eggs get to be flushed out during heavy rain but the larva remain intact. In addition, we also assumed that the breeding site is of 20 mm deep.

This observation helped to indicate the need of residents and local municipal authorities to make sure that all possible breeding sites should be empty of water after heavy rainfall. Although the eggs are flushed out, the larvae might remain in the container.

C. Comparison to the dengue fever cases

The last observation is regarding the relation between the abundance of mosquito with the reported dengue fever cases in Shah Alam. It is of our interest to find the lag time between the peaks of these two occurrences. It was reported that, in Brazil, the lag time was found to 100 days, or slightly more than three months.

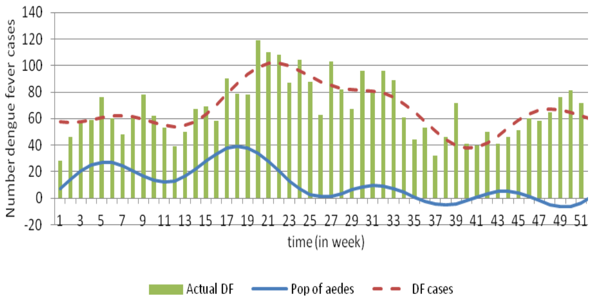

Fig. 7 Modeled population of *Aedes* mosquito and actual dengue fever cases for year 2008

Fig. 7 shows the graph of modeled Aedes population, in relation to the actual dengue fever cases in Shah Alam. Similar to previous graph in Fig. 6, the fourth harmonic representation for sine function is used. The *dashed* line is the fourth harmonic of sine of the dengue fever cases. This is required to help in the determination of the lag time, or delay between the abundance of mosquito and dengue fever cases.

It was observed that the average lag time for all the four peaks in Fig. 7 is four weeks, or one month. This agrees with the average 8 - 10 days of the viremia period in the Aedes mosquito after biting an infected host and the same duration for host after being bitten by an infected vector. These viremia periods plus the time of successful bites will be about 30 days.

IV. CONCLUSION

In this paper we have presented the development of our proposed climate dependent model to be used to simulate the population of *Aedes aegypti* mosquito. In computing the entries for our transition matrix, the average temperature of Shah Alam, Malaysia was used in obtaining the development rates of each stage of the life-cycle of Aedes and hence, the duration they stay in each stage. Actual rainfall for year 2008 was used to simulate the population and verification was done by comparing to the Aedes house index (AHI) of Shah Alam for that same year.

The model was simulated based on several assumptions. It was found that the peak of abundance in mosquito population comes about 4 weeks before the peak of dengue fever cases. In addition, the peak of mosquito abundance comes during a relatively dry period following a heavy rainfall. As part of the strategy to control the spread of dengue fever, residents and municipal authorities will need to increase the killing of larva during this period. Fogging activities should be intensified to kill adult mosquitoes from spreading the virus to other hosts.

For further studies, it is recommended that more research to be done to look more closely at the direct relation between rainfall and population of mosquito. This will be helpful to give a better prediction of the population of mosquito, with respect to rainfall so that an alert indicator can be developed.

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