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# Delay-Distribution-Dependent Stability Criteria for BAM Neural Networks with Time-Varying Delays

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*Abstract*—This paper is concerned with the delay-distribution-dependent stability criteria for bidirectional associative memory (BAM) neural networks with time-varying delays. Based on the Lyapunov-Krasovskii functional and stochastic analysis approach, a delay-probability-distribution-dependent sufficient condition is derived to achieve the globally asymptotically mean square stable of the considered BAM neural networks. The criteria are formulated in terms of a set of linear matrix inequalities (LMIs), which can be checked efficiently by use of some standard numerical packages. Finally, a numerical example and its simulation is given to demonstrate the usefulness and effectiveness of the proposed results.

*Keywords*—BAM neural networks, Probabilistic time-varying delays, Stability criteria.

## I. Introduction

IN the past few decades, dynamical behavior of neural networks has been studied much in science and technology area, such as signal processing, parallel computing, optimization problems, and so on [1], [2]. This led to significant attention on stability analysis of various kind of neural network models such as Hopfield neural networks, cellular neural networks, Cohen-Grossberg neural networks, and BAM neural networks [3]–[4]. As is well known that BAM is a type of recurrent neural network which was introduced by Kosko in 1988 [5] for their potential application in pattern recognition, solving optimization problems, and automatic control engineering. Therefore, stability of BAM neural networks has attracted attention in the recent time [6]-[8].

On the other hand, time delays are commonly encountered in various physical, engineering and neural based systems which cause the poor performance and instability of dynamic systems. Many existing works investigated about neural networks with deterministic time-delay, but at the same time, latest literatures are concerned with neural networks with stochastic time-delay. For example, if some values of the delay are very large but the probabilities of its occurrence are very small. In this case, we do not get less conservative results when only the information of variation range of the time delay is considered. Thus, the stability analysis of dynamic neural networks with random time-delay deserves to receive much attention and has been studied in recent years [9]-[12]. The authors in [9] addressed the problem of delay-distribution-dependent state estimation for discrete-time stochastic neural networks with random delay. Similarly, other authors have also proposed the delay-distribution-dependent stability of stochastic discrete-time neural networks with randomly mixed time-varying delays in [10]. To the best of our knowledge, none have worked on the issue of stability criteria for BAM neural networks with time-delays in the leakage term and probabilistic time-varying delays, till now.

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Motivated by the above discussion, the main objective of this paper is to propose the delay-distribution-deepened stability criteria for BAM neural networks with probabilistic time-varying delay functions by using a combination of Lyapunov-Krasovskii functional, stochastic stability theory, Jenson's inequality and free-weighting matrices. A delay-dependent criteria is expressed in terms of LMIs. Finally a numerical example is given to show the effectiveness and significance of the proposed criterion.

**Notations:** $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the $n$-dimensional Euclidean space and the set of all $n \times n$ real matrices respectively. The superscript $T$ denotes the transposition and the notation $X \geq Y$ (similarly, $X > Y$), where $X$ and $Y$ are symmetric matrices, means that $X-Y$ is positive semi-definite (similarly, positive definite). $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. $Pr\{\alpha\}$ means the occurrence probability of the event $\alpha$. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ , respectively, mean the expectation of the stochastic variable $x$ and the expectation of the stochastic variable $x$ conditional on the stochastic variable $y$. $diag\{\cdots\}$ stands for a block diagonal matrix. The notation $*$ always denotes the symmetric block in one symmetric matrix. $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the minimum and maximum eigenvalues of a given matrix.

## II. Problem Description and Preliminaries

The delayed BAM neural networks can be described as follows:

$$\begin{cases} \dot{u}_i(t) = -a_iu_i(t) + \sum_{j=1}^m b_{ij}^1 \tilde{f}_j(v_j(t)) \\ \qquad + \sum_{j=1}^m b_{ij}^2 \tilde{f}_j(v_j(t-\tau(t))) + I_i, \ i = 1,\cdots,n, \\ \dot{v}_j(t) = -c_jv_j(t) + \sum_{i=1}^n d_{ij}^1 \tilde{g}_i(u_i(t)) \\ \qquad + \sum_{i=1}^n d_{ij}^2 \tilde{g}_i(u_i(t-\sigma(t))) + J_j, \ j = 1,\cdots,m, \end{cases} \tag{1}$$

or be rewritten in the following vector-matrix form:

$$\begin{cases} \dot{u}(t) = -Au(t) + B_1\tilde{f}(v(t)) + B_2\tilde{f}(v(t-\tau(t))) + I, \\ \dot{v}(t) = -Cv(t) + D_1\tilde{g}(u(t)) + D_2\tilde{g}(u(t-\sigma(t))) + J, \end{cases} \tag{2}$$

where $u(t) = [u_1(t), u_2(t), \cdots, u_n(t)]^T \in \mathbb{R}^n, v(t) = [v_1(t), v_2(t), \cdots, v_n(t)]^T \in \mathbb{R}^m$ are neuron state vectors,

$A = diag\{a_1, \cdots, a_n,\} > 0$, $C = diag\{c_1, \cdots, c_n,\} > 0$ are diagonal matrices with positive entries $a_i > 0$ and $c_i > 0$, $B_1$ and $D_1$ are the connection weight matrices, $B_2$ and $D_2$ are the delayed connection weight matrices, $\tilde{f}(v(t)) = [\tilde{f}_1(v_1(t)), \cdots, \tilde{f}_m(v_m(t))]^T$, $\tilde{g}(u(t)) = [\tilde{g}_1(u_1(t), \cdots, \tilde{g}_n(u_n(t)]^T$ denote neuron activation functions, $I = [I_1, I_2, \cdots, I_n]^T$ and
$J = [J_1, J_2, \cdots, J_m]^T$ are external inputs, and $\tau(t)$ and $\sigma(t)$ are time-varying delays and satisfy

$$0 \le \tau(t) \le \tau, \quad 0 \le \sigma(t) \le \sigma,$$

where $\tau$ and $\sigma$ are positive constants.

The initial condition of the system (2) are assumed to be

$$\begin{aligned} u(s) &= \phi(s) \quad s \in [-\tau, 0], \\ v(s) &= \psi(s) \quad s \in [-\sigma, 0]. \end{aligned}$$

**Assumption 2.1** : The neuron activation functions $\tilde{f}_i(\cdot)$ and $\tilde{g}_j(\cdot)$ in (1) satisfy

$$l_j^- \le \frac{\tilde{f}_j(a) - \tilde{f}_j(b)}{a-b} \le l_j^+, \tag{3}$$

$$k_i^- \le \frac{\tilde{g}_i(a) - \tilde{g}_i(b)}{a-b} \le k_i^+, \tag{4}$$

for all $a, b \in \mathbb{R}$, $a \neq b$, $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$. The constants $l_i^-, l_i^+, k_j^-, k_j^+$ in Assumption 2.1 are allowed to be positive, negative or zero.

**Remark 2.2** Assumption 2.1 was proposed in [13]. The constants $l_i^-, l_i^+, k_j^-, k_j^+$ in Assumption 2.1 are allowed to be positive, negative or zero. Then, Assumption 2.1 is less restrictive than the descriptions on both the Lipschitz-type activation functions and the sigmoid activation functions.

Now, let $u^* = [u_1^*, \cdots, u_n^*]^T$, $v^* = [v_1^*, \cdots, v_n^*]^T$ be the equilibrium points of (2), and define the following variables:

$$x(t) = u(t) - u^*, \ y(t) = v(t) - v^*.$$

Then, the neural networks (2) is transformed to

$$\begin{cases} \dot{x}(t) = -Ax(t) + B_1 f(y(t)) + B_2 f(y(t - \tau(t))), \\ \dot{y}(t) = -Cy(t) + D_1 g(x(t)) + D_2 g(x(t - \sigma(t))), \end{cases} \tag{5}$$

where $f(y(t)) = [f_1(y_1(t)), \cdots, f_m(y_m(t))]^T$, $g(x(t)) = [g_1(x_1(t)), \cdots, g_n(x_n(t))]^T$, $f_j(y_j(t)) = \tilde{f}_j(y_j(t) + v^*) - \tilde{f}_j(v^*)$, $g_i(x_i(t)) = \tilde{g}_i(x_i(t) + u^*) - \tilde{g}_i(u^*)$.

According to the Assumption 2.1, one can obtain that

$$l_j^- \le \frac{f_j(a)}{a} \le l_j^+, \quad f_j(0) = 0, \ j = 1, \cdots, m, \tag{6}$$

$$k_i^- \le \frac{g_i(a)}{a} \le k_i^+, \ g_i(0) = 0, \ i = 1, \cdots, n. \tag{7}$$

**Assumption 2.3** Considering the information of probability distribution of the time-delays $\tau(t), \sigma(t)$ are defined

$$\text{Prob}\Big\{\tau(t) \in [0, \tau_1]\Big\} = \alpha_0, \text{Prob}\Big\{\tau(t) \in (\tau_1, \tau_2]\Big\} = 1 - \alpha_0,$$

$$\text{Prob}\Big\{\sigma(t) \in [0, \sigma_1]\Big\} = \beta_0, \text{Prob}\Big\{\sigma(t) \in (\sigma_1, \sigma_2]\Big\} = 1 - \beta_0,$$

where $0 \le \alpha_0 \le 1$, $0 \le \beta_0 \le 1$ are constants.

Therefore, the stochastic variables $\alpha(t), \beta(t)$ can be defined as

$$\alpha(t) = \begin{cases} 1, & \tau(t) \in [0, \tau_1], \\ 0, & \tau(t) \in (\tau_1, \tau_2] \end{cases} \qquad \beta(t) = \begin{cases} 1, & \sigma(t) \in [0, \sigma_1], \\ 0, & \sigma(t) \in (\sigma_1, \sigma_2], \end{cases} \tag{8}$$

with its probabilities $\text{Prob}\{\alpha(t) = 1\} = \alpha_0$, $\text{Prob}\{\beta(t) = 1\} = \beta_0$, $\text{Prob}\{\alpha(t) = 0\} = 1 - \alpha_0$, $\text{Prob}\{\beta(t) = 0\} = 1 - \beta_0$.

From Assumption 2.3, it is easy to see that $\mathbb{E}\{\alpha(t) - \alpha_0\} = 0$, $\mathbb{E}\{(\alpha(t) - \alpha_0)^2\} = \alpha_0(1 - \alpha_0)$, $\mathbb{E}\{\beta(t) - \beta_0\} = 0$, and $\mathbb{E}\{(\beta(t) - \beta_0)^2\} = \beta_0(1 - \beta_0)$. Now, we introduce time-varying delays $\tau_1(t)$, $\tau_2(t)$, $\sigma_1(t)$ and $\sigma_2(t)$ such that

$$\tau(t) = \begin{cases} \tau_1(t), & \tau(t) \in [0, \tau_1], \\ \tau_2(t), & \tau(t) \in (\tau_1, \tau_2] \end{cases}$$

$$\sigma(t) = \begin{cases} \sigma_1(t), & \sigma(t) \in [0, \sigma_1], \\ \sigma_2(t), & \sigma(t) \in (\sigma_1, \sigma_2]. \end{cases} \tag{9}$$

By using the new functions $\tau_1(t)$, $\tau_2(t)$, $\sigma_1(t)$ $\sigma_2(t)$ and stochastic variables $\alpha(t)$, $\beta(t)$, the system (5) can be written as

$$\begin{cases} \dot{x}(t) = -Ax(t) + B_1 f(y(t)) + \alpha(t) B_2 f(y(t - \tau_1(t))) \\ \quad + (1 - \alpha(t)) B_2 f(y(t - \tau_2(t))), \\ \dot{y}(t) = -Cy(t) + D_1 g(x(t)) + \beta(t) D_2 g(x(t - \sigma_1(t))) \\ \quad + (1 - \beta(t)) D_2 g(x(t - \sigma_2(t))), \end{cases} \tag{10}$$

which is equivalent to

$$\begin{cases} \dot{x}(t) = -Ax(t) + B_1 f(y(t)) + \alpha_0 B_2 f(y(t - \tau_1(t))) \\ +(1 - \alpha_0) B_2 f(y(t - \tau_2(t))) + (\alpha(t) - \alpha_0)\Big[B_2 f(y(t - \tau_1(t))) \\ -B_2 f(y(t - \tau_2(t)))\Big], \\ \dot{y}(t) = -Cy(t) + D_1 g(x(t)) + \beta_0 D_2 g(x(t - \sigma_1(t))) \\ +(1 - \beta_0) D_2 g(x(t - \sigma_2(t))) + (\beta(t) - \beta_0)\Big[D_2 g(x(t - \sigma_1(t))) \\ -D_2 g(x(t - \sigma_2(t)))\Big]. \end{cases}$$

## III. Main Results

For representation convenience, we introduce the following notations: $L_1 = \text{diag}\{l_1^- l_1^+, \ l_2^- l_2^+, \cdots, l_m^- l_m^+,\}$, $L_2 = \text{diag}\Big\{\frac{l_1^- + l_1^+}{2}, \frac{l_2^- + l_2^+}{2}, \cdots, \frac{l_m^- + l_m^+}{2}\Big\}$, $L^- = \text{diag}\{l_1^-, l_2^-, \cdots, l_m^-\}$, $L^+ = \text{diagdiag}\{l_1^+, l_2^+, \cdots, l_m^+\}$, $K_1 = \text{diag}\{k_1^- k_1^+, k_2^- k_2^+, \cdots, k_n^- k_n^+,\}$, $K_2 = \text{diag}\Big\{\frac{k_1^- + k_1^+}{2}, \frac{k_2^- + k_2^+}{2}, \cdots, \frac{k_n^- + k_n^+}{2}\Big\}$, $K^- = \text{diag}\{k_1^-, k_2^-, \cdots, k_n^-\}$, $K^+ = \text{diagdiag}\{k_1^+, k_2^+, \cdots, k_n^+\}$.

Now, a new delay-dependent stability analysis of delayed system (11) is given in the following theorem.

**Theorem 3.1** The system (11) is globally asymptotically stable in the mean square if there exist positive definite symmetric matrices $P_1 > 0, P_2 > 0$, $Q_l > 0, R_l > 0$ $(l = 1, \cdots, 4)$ positive diagonal matrices $\Lambda_1, \Lambda_2, \Delta_1, \Delta_2, W_r$ $(r = 1, \cdots, 6)$ and real matrices $S_a (a = 1, \cdots, 4)$ of appropriate dimensions, such that the following LMIs hold:

$$\begin{bmatrix} \Xi & \Gamma_1\mathcal{R}_1 & \Gamma_2\mathcal{R}_1 & \Gamma_3\mathcal{R}_2 & \Gamma_4\mathcal{R}_2 \\ * & -\mathcal{R}_1 & 0 & 0 & 0 \\ * & * & -\mathcal{R}_1 & 0 & 0 \\ * & * & * & -\mathcal{R}_2 & 0 \\ * & * & * & * & -\mathcal{R}_2 \end{bmatrix} < 0,$$

$$\begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_2 & S_2 \\ * & R_2 \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} R_3 & S_3 \\ * & R_3 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_4 & S_4 \\ * & R_4 \end{bmatrix} \geq 0, \qquad (11)$$

where $\Xi_{16\times16}$with entries : $\Xi_{1,1} = -P_1A - A^TP_1 + Q_1 + Q_2 - R_1 + 2K^{-T}\Lambda_1A - 2K^{+T}\Lambda_2A - K_1W_4$, $\Xi_{1,3} = S_1$, $\Xi_{1,7} = R_1 - S_1$, $\Xi_{1,11} = -A^T\Lambda_1^T + K_2W_4$, $\Xi_{1,12} = P_1B_1 - K^{-T}\Lambda_1B_1 + K^{+T}\Lambda_2B_1$, $\Xi_{1,15} = \alpha_0[P_1B_2 - K^{-T}\Lambda_1B_2 + K^{+T}\Lambda_2B_2]$, $\Xi_{1,16} = (1-\alpha_0)[P_1B_2 - K^{-T}\Lambda_1B_2 + K^{+T}\Lambda_2B_2]$, $\Xi_{2,2} = -P_2C - C^TP_2 + Q_3 + Q_4 - R_3 + 2L^{-T}\Delta_1C - 2L^{+T}\Delta_2C - L_1W_1$, $\Xi_{2,5} = S_3$, $\Xi_{2,9} = R_3 - S_3$, $\Xi_{2,11} = P_2D_1 - L^{-T}\Delta_1D_1 + L^{+T}\Delta_2D_1$, $\Xi_{2,12} = -C^T\Delta_1^T + C^T\Delta_2^T + L_2W_1$, $\Xi_{2,13} = \beta_0(P_2D_2 - L^{-T}\Delta_1D_2 + L^{+T}\Delta_2D_2)$, $\Xi_{2,14} = (1-\beta_0)[P_2D_2 - L^{-T}\Delta_1D_2 + L^{+T}\Delta_2D_2]$, $\Xi_{3,3} = -Q_1 - R_1 - R_2$, $\Xi_{3,4} = S_2$, $\Xi_{3,7} = R_1 - S_1^T$, $\Xi_{3,8} = R_2 - S_2$, $\Xi_{4,4} = -Q_2 - R_2$, $\Xi_{4,8} = R_2 - S_2^T$, $\Xi_{5,5} = -Q_3 - R_3 - R_4$, $\Xi_{5,6} = S_4$, $\Xi_{5,9} = R_3 - S_3^T$, $\Xi_{5,10} = R_4 - S_4$, $\Xi_{6,6} = -Q_4 - R_4$, $\Xi_{6,10} = R_4 - S_4^T$, $Xi_{7,7} = -2R_1 + S_1 + S_1^T - K_1W_5$, $\Xi_{7,13} = K_2W_5$, $\Xi_{8,8} = -2R_2 + S_2 + S_2^T - K_1W_6$, $\Xi_{8,14} = K_2W_6$, $Xi_{9,9} = -2R_3 + S_3 + S_3^T - L_1W_2$, $\Xi_{9,15} = L_2W_2$, $\Xi_{10,10} = -2R_4 + S_4 + S_4^T - L_1W_3$, $\Xi_{10,16} = L_2W_3$, $\Xi_{11,11} = -W_4$, $\Xi_{11,12} = \Lambda_1B_1 + D_1^T\Delta_1^T - D_1^T\Delta_2^T$, $\Xi_{11,15} = \alpha_0\Lambda_1B_2$, $\Xi_{11,16} = (1-\alpha_0)\Lambda_1B_2$, $\Xi_{12,12} = -W_1$, $\Xi_{12,13} = \beta_0(\Delta_1D_2 - \Delta_2D_2)$, $\Xi_{12,14} = (1-\beta_0)(\Delta_1D_2 - \Delta_2D_2)$, $\Xi_{13,13} = -W_5$, $\Xi_{14,14} = -W_6$, $\Xi_{15,15} = -W_2$, $\Xi_{16,16} = -W_3$, $\Gamma_1 = \Big[-A \quad \underbrace{0...0}_{10} \quad B_1 \quad 0 \quad 0 \quad \alpha_0B_2 \quad (1-\alpha_0)B_2\Big]^T$,

$\Gamma_2 = \Big[\underbrace{0...0}_{14} \quad \sqrt{\alpha_0(1-\alpha_0)}B_2 \quad -\sqrt{\alpha_0(1-\alpha_0)}B_2\Big]^T$,

$\Gamma_3 = \Big[0 \quad -C \quad \underbrace{0...0}_{8} \quad D_1 \quad 0 \quad \beta_0D_2 \quad (1-\beta_0)D_2\Big]^T$,

$\Gamma_4 = \Big[\underbrace{0...0}_{12} \quad \sqrt{\beta_0(1-\beta_0)}D_2 \quad -\sqrt{\beta_0(1-\beta_0)}D_2 \quad 0 \quad 0\Big]^T$,

$\mathcal{R}_1 = \sigma_1^2R_1 + (\sigma_2-\sigma_1)^2R_2$, $\mathcal{R}_2 = \tau_1^2R_3 + (\tau_2-\tau_1)^2R_4$.

**Proof.** Consider the following Lyapunov-Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \qquad (12)$$

where

$$\begin{aligned} V_1(t) &= x^T(t)P_1x(t) + y^T(t)P_2y(t), \\ V_2(t) &= \int_{t-\sigma_1}^t x^T(s)Q_1x(s)ds + \int_{t-\sigma_2}^t x^T(s)Q_2x(s)ds \\ &\quad + \int_{t-\tau_1}^t y^T(s)Q_3y(s)ds + \int_{t-\tau_2}^t y^T(s)Q_4y(s)ds, \\ V_3(t) &= \sigma_1\int_{-\sigma_1}^0\int_{t+\theta}^t \dot{x}^T(s)R_1\dot{x}(s)dsd\theta \\ &\quad + (\sigma_2-\sigma_1)\int_{-\sigma_2}^{-\sigma_1}\int_{t+\theta}^t \dot{x}^T(s)R_2\dot{x}(s)dsd\theta \\ &\quad + \tau_1\int_{-\tau_1}^0\int_{t+\theta}^t \dot{y}^T(s)R_3\dot{y}(s)dsd\theta \\ &\quad + (\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^t \dot{y}^T(s)R_4\dot{y}(s)dsd\theta \\ V_4(t) &= 2\sum_{i=1}^n\Big(\lambda_{1i}\int_0^{x_i(t)}(g_i(s) - k_i^-s)ds \\ &\quad + \lambda_{2i}\int_0^{x_i(t)}(k_i^+s - g_i(s))ds\Big) \\ &\quad + 2\sum_{j=1}^m\Big(\delta_{1j}\int_0^{y_j(t)}(f_j(s) - l_j^-s)ds \\ &\quad + \delta_{2j}\int_0^{y_j(t)}(l_j^+s - f_j(s))ds\Big). \end{aligned}$$

Define the infinitesimal operator $\mathbb{L}$ of $V(t)$ as in [11], we have

$$\mathbb{L}V(t) \leq \mathbb{L}V_1(t) + \mathbb{L}V_2(t) + \mathbb{L}V_3(t) + \mathbb{L}V_4(t), \quad (13)$$

where $\mathbb{L}V_1(t) = 2x^T(t)P_1\dot{x}(t) + 2y^T(t)P_2\dot{y}(t)$, $\mathbb{L}V_2(t) = x^T(t)Q_1x(t) - x^T(t-\sigma_1)Q_1x(t-\sigma_1) + x^T(t)Q_2x(t) - x^T(t-\sigma_2)Q_2x(t-\sigma_2) + y^T(t)Q_3y(t) + y^T(t)Q_4y(t) - y^T(t-\tau_1)Q_3y(t-\tau_1) - y^T(t-\tau_2)Q_4y(t-\tau_2)$, $\mathbb{L}V_3(t) = \dot{x}^T(t)\{\sigma_1^2R_1 + (\sigma_2-\sigma_1)^2R_2\}\dot{x}(t) - \sigma_1\int_{t-\sigma_1}^t \dot{x}^T(s)R_1\dot{x}(s)ds - (\sigma_2-\sigma_1)\int_{t-\sigma_2}^{t-\sigma_1}\dot{x}^T(s)R_2\dot{x}(s)ds + \dot{y}^T(t)\{\tau_1^2R_3 + (\tau_2-\tau_1)^2R_4\}\dot{y}(t) - \tau_1\int_{t-\tau_1}^t \dot{y}^T(s)R_3\dot{y}(s)ds - (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{y}^T(s)R_4\dot{y}(s)ds$, $\mathbb{L}V_4(t) = 2[g(x(t)) - K^-x(t)]^T\Lambda_1\dot{x}(t) + 2[K^+x(t) - g(x(t))]^T\Lambda_2\dot{x}(t) + 2[f(y(t)) - L^-y(t)]^T\Delta_1\dot{y}(t) + 2[L^+y(t) - f(y(t))]^T\Delta_2\dot{y}(t)$.
The rest of proof is omitted.

**Remark 3.2** Recently, the problem of stability analysis of BAM neural networks with time-varying delays have been reported in [6]-[8]. In this paper, we have provided a delay-dependent condition to ensure the global asymptotic stability of BAM neural networks with probabilistic time-varying delays. The global stability criterion is derived by using suitable Lyapunov-Krasovskii functional, Jensen's inequality and some inequality techniques. It is worth noting that in contrast to existing literature, the derived stability criteria in this paper is dependent on the upper bound of the probabilistic time-varying delays.

## IV. Numerical Example

In this section, a numerical example is provided along with simulation results to illustrate the potential benefits and effectiveness of the developed method for BAM neural networks.

Consider a third-order delayed BAM neural network (11) with the following parameters

$$A = diag\{3.1, 3.5, 3.8\}, B_1 = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 0.6 & 1.3 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1.2 & 0 & 1.2 \\ -1.2 & 0 & 0.7 \\ 0.5 & -0.5 & 0 \end{bmatrix},$$

$$C = diag\{2.5, 2, 2.9\}, D_1 = \begin{bmatrix} 0.5 & -1 & 0 \\ -0.7 & 1 & 0.8 \\ 1 & 0 & -0.4 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

and the activation functions are taken as follows:

$$f(y(t)) = \tanh y(t), \; g(x(t)) = \tanh x(t).$$

Further, it satisfies Assumption 2.1 with $k_b^- = l_b^- = 0$ and $k_b^+ = l_b^+ = 1$ $(b = 1, 2, 3)$. Thus, we can get the following parameters: $K_1 = L_1 = diag\{0, 0, 0\}$, $K_2 = L_2 = diag\{0.5, 0.5, 0.5\}$, $K^- = L^- = diag\{0, 0, 0\}$, $K^+ = L^+ = diag\{1, 1, 1\}$. Let $\tau_1 = \sigma_1 = 1, \tau_2 = \sigma_2 = 1.49, \alpha_0 = \beta_0 = 0.25$ and solving the LMIs in Theorem 3.1 in aid of MATLAB LMI Control Tool box, it can be easily confirmed that the LMIs are feasible. Here note that we are given only few feasible matrices of LMIs (12)-(13) for page limitation. The Fig. 1 show the state trajectories of the dynamical system converge to the zero equilibrium point with different initial conditions. Therefore, the BAM neural networks (11) is robustly globally asymptotically stable in the mean square sense.

## V. Conclusion

In this paper, we have dealt with the delay-distribution dependent stability criteria for BAM neural networks with time-varying delays. By establishing a stochastic variable with Bernoulli distribution, the information of probabilistic time-varying delay is transformed into the deterministic time-varying delay with stochastic parameters. Based on a appropriate Lyapunov-Krasovskii functional and some inequality techniques, a delay-dependent stability criteria for BAM neural networks have been derived. Finally, a numerical example has been given to show the effectiveness and superiority of the proposed results.

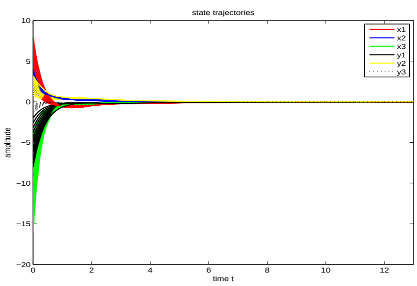


Fig. 1. state trajectories of system (11) with different initial conditions.

## References

[1] S. Haykin, *Neural Networks: A comprehensive Foundation*. NJ: Prentice Hall 1998.

[2] A. Cichocki, R. Unbehauen, Neural networks for optimization and signal processing. Wiley, Chichester (1993)

[3] C. Ji, H.G. Zhang, Y. Wei, "LMI Approach for Global Robust Stability of Cohen-Grossberg Neural Networks with Multiple Delays," *Neurocomputing*, vol.71, pp.475-485, 2008.

[4] T.Y. Lou, B.T. Cui, "Novel Global Stability Criteria for High-Order Hopfield-Type Neural Networks with Time-Varying Delays," *J. Math. Anal. Appl.*, vol.330, pp.144-158, 2007.

[5] B. Kosko, "Bidirectional Associative Memories," *IEEE Trans. Syst. Man, Cybern. B Cybern.*, vol.18, pp.49-60, 1998.

[6] L. Sheng, H. Yang, "Novel Global Robust Exponential Stability Criterion for Uncertain BAM Neural Networks with Time-Varying Delays," *Chaos, Solitons, Fractals*, vol.40, pp.2102-2113, 2009.

[7] S. Arik, "Global Asymptotic Stability Analysis of Bidirectional Associative Memory Neural Networks with Time Delays," *IEEE Trans. Neural Netw.*, vol.16, pp.580-586, 2005.

[8] B. Liu, P. Shi, "Delay-Range-Dependent Stability for Fuzzy BAM Neural Networks with Time-Varying Delays," *Phy. Let. A*, vol.373, pp.1830-1838, 2009.

[9] H. Bao, J. Cao, "Delay-Distribution-Dependent State Estimation for Discrete-Time Stochastic Neural Networks with Random Delay," *Neural Net.*, vol.24, pp.19-28, 2011.

[10] Y. Zhang, D. Yue, E. Tian, "Robust Delay-Distribution-Dependent Stability of Discrete-Time Stochastic Neural Networks with Time-Varying Delay," *Neurocomputing*, vol.72, pp.1265-1273, 2009.

[11] R. Yang, H. Gao, J. Lam, P. Shi, "New Stability Criteria for Neural Networks with Distributed and Probabilistic Delays," *Circ. Syst. Signal Process*, vol.28, pp.505-522, 2009.

[12] Y. Tang, J.A. Fang, M. Xia, D. Yu, "Delay-Distribution-Dependent Stability of Stochastic Discrete-Time Neural Networks with Randomly Mixed Time-Varying Delays," *Neurocomputing*, vol.72, pp.3830-3838, 2009.

[13] Y. Liu, Z. Wang, X. Liu, "Global Exponential Stability of Generalized Recurrent Neural Networks with Discrete and Distributed Delays," *Neural Netw.*, vol.19, pp.667-675, 2006.