International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:4, No:2, 2010

Unsteady Flow between Two Concentric Rotating Spheres along with Uniform Transpiration

O. Mahian, A. B. Rahimi, A. Kianifar, and A. Jabari Moghadam

Abstract**—**In this study, the numerical solution of unsteady flow between two concentric rotating spheres with suction and blowing at their boundaries is presented. The spheres are rotating about a common axis of rotation while their angular velocities are constant. The Navier-Stokes equations are solved by employing the finite difference method and implicit scheme. The resulting flow patterns are presented for various values of the flow parameters including rotational Reynolds number Re, and a blowing/suction Reynolds number Re_w. Viscous torques at the inner and the outer spheres are calculated, too. It is seen that increasing the amount of suction and blowing decrease the size of eddies generated in the annulus.

Keywords**—**Concentric spheres, numerical study, suction and blowing, unsteady flow, viscous torque.

I. INTRODUCTION

THE flow and heat transfer in an annulus between two spheres has been studied in various cases by many researchers. Such studies can be classified into two main groups. In the first group, there is neither suction nor blowing at the spherical walls. Such containers are used in engineering designs like centrifuges and fluid gyroscopes.

The first numerical study of time-dependent viscous flow between two rotating spheres has been presented by Pearson [1] in which the cases of one (or both)sphere is given an impulsive change in angular velocity starting from a state of either rest or uniform rotation. Munson and Joseph [2] have considered the case of steady motion of a viscous fluid between concentric rotating spheres using perturbation techniques for small values of Reynolds number and a Legendre polynomial expansion for larger values of Reynolds numbers. Recently a numerical study of flow and heat transfer between two rotating spheres has been done by Jabari Moghadam and Rahimi [3] in which the fluid contained between two vertically eccentric spheres maintained at different temperature and rotating about a common axis with different angular velocities when the angular velocities are arbitrary functions of time. Jabari Moghadam and Rahimi [4] have also studied the similarity solution for spheres rotating with constant angular velocity.

In the second group, the effects of transpiration on flow in an annulus between two spheres have been investigated. The study of flow in a spherical annulus along with transpiration is used in many practical applications, such as rotary machines and spherical heat exchangers and in the design of spherical fluid storage systems. In these applications transpiration is used to regulate the rate of heat transfer.

Effects of transpiration on free convection in an annulus between two stationary concentric porous spheres have been considered by Gulwadi et al. [5]. Gulwadi et al. [6] studied the laminar flow in an annulus between rotating porous spheres and with injection and suction at spherical walls. Their results are valid for small values of the rotational Reynolds number and an injection/suction Reynolds number. A review of the literature reveals that there is no study on the transient motion between two rotating spheres with uniform transpiration. In the present study, a numerical solution of unsteady momentum equations is presented for concentric spheres.

II. PROBLEM FORMULATION

The geometry of the spherical annulus considered is indicated in Fig. 1.

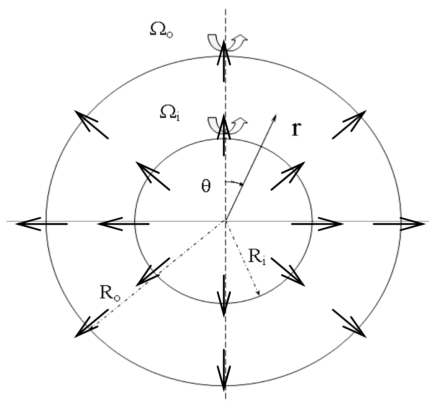


Fig. 1 Geometry of problem

A Newtonian, viscous, incompressible fluid fills the gap between the inner and outer spheres which are of radii R_i and R_o. The inner and outer spheres rotates about a common

O. Mahian is with the Young Researchers Club Isalamic Azad University Mashhad Branch, Mashhad, Iran (corresponding author to provide fax:(+98)5118763304; e-mail: omid.mahian@gmail.com).

A.B. Rahimi is Professor in Mech.Eng .Ferdowsi university of Mashhad ,Iran.(e-mail:rahimiab@yahoo.com).

A. Kianifar is associated professor in Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran (e-mail: a_kianifar@yahoo.com).

A. Jabari Moghadam is Assistant Professor in Department of Mechanical Engineering, Shahrood University of Technology, Iran (e-mail: amak4jm@yahoo.com).

axis with constant angular velocities Ω_i and Ω_o, respectively. The components of velocity in directions r, θ, and ϕ are v_r, v_θ, and v_ϕ, respectively. These velocity components for incompressible flow and in meridian plane satisfy the continuity equation and are related to stream function ψ and angular momentum function Ω in the following manner:

$$v_r = \frac{\psi_\theta}{r^2 \sin\theta}, v_\theta = \frac{-\psi_r}{r \sin\theta}, v_\phi = \frac{\Omega}{r \sin\theta} \tag{1}$$

The blowing/suction Reynolds number is defined as:

$$\mathrm{Re}_w = \frac{v_{r_o} r_o}{\nu} \tag{2}$$

in Which v_{r_o} (radial velocity) and r_o (radius) are reference values, which are selected as v_{R_i} and $\mathrm{R_i}$ respectively.

The blowing/suction Reynolds number $\mathrm{Re_w}$ is positive for blowing at the inner sphere and negative for suction. Since the flow is assumed to be independent of the longitude, ϕ, the non-dimensional Navier-Stokes equations and energy equation can be written in terms of the stream function and the angular velocity function as follows:

$$\frac{\partial\Omega}{\partial t} + \frac{\psi_\theta \Omega_r - \psi_r \Omega_\theta}{r^2 \sin\theta} = \frac{1}{(\mathrm{Re})} D^2 \Omega \tag{3}$$

$$\begin{aligned} &\frac{\partial}{\partial t}(D^2\psi) + \frac{2\Omega}{r^3 \sin^2\theta}[\Omega_r r \cos\theta - \Omega_\theta \sin\theta] \\ &- \frac{1}{r^2 \sin\theta}[\psi_r (D^2\psi)_\theta - \psi_\theta (D^2\psi)_r] \\ &+ \frac{2D^2\psi}{r^3 \sin^2\theta}[\psi_r r \cos\theta - \psi_\theta \sin\theta] = \frac{1}{(\mathrm{Re})} D^4\psi \end{aligned} \tag{4}$$

in which the Reynolds number (Re) is defined as:

$$\mathrm{Re} = \frac{\omega_o r_o^2}{\nu} \tag{5}$$

The following non-dimensional parameters have been used in the above equations and then the asterisks have been omitted:

$$t^* = t\omega_o,\ r^* = \frac{r}{r_o},\ \psi^* = \frac{\psi}{r_o^3 \omega_o},\ \Omega^* = \frac{\Omega}{r_o^2 \omega_o} \tag{6}$$

in which ω_o is reference value which is selected as Ω_i. The non-dimensional boundary and initial conditions for the above governing equations are:

For $t = 0$:

$$\begin{cases} \psi = 0 \\ \Omega = 0 \end{cases} \quad \text{every where}$$

For $t > 0$: (7)

$$\begin{aligned} &\theta = 0 \to \{\psi_r = 0, \psi_\theta = 0, \Omega = 0\} \\ &\theta = \pi \to \{\psi_r = 0, \psi_\theta = 0, \Omega = 0\} \\ &r = R_i / R_i \to \left\{ \psi_\theta = \frac{\mathrm{Re}_w}{\mathrm{Re}} \sin\theta, \psi_r = 0, \Omega = \frac{\Omega_i R_i^2}{\omega_o r_o^2} \sin^2\theta \right. \\ &r = R_o / R_i \to \left\{ \psi_\theta = \frac{\mathrm{Re}_w}{\mathrm{Re}} \sin\theta, \psi_r = 0, \Omega = \frac{\Omega_o R_o^2}{\omega_o r_o^2} \sin^2\theta \right. \end{aligned}$$

where:

$$D^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta} \tag{8}$$

III. Problem Solution

In this section, we firstly present the Computational Procedure and discuss then on the obtained results.

A. Computational Procedure

The two equations governing the fluid motion show that each is describing the behavior of one of the dependent variables Ω and ψ. On the other hand, these two equations are coupled only through nonlinear terms. To solve the problem, the momentum equations were discretized by the finite-difference method and implicit scheme.

In each time step (n+1), the value of the dependent variables are guessed from their values at previous time steps (n), (n-1), and (n-2) and after using them in difference equations and repeating it until the desired convergence, will lead to the corrected values at this time step. This procedure is applied for the next time step.

The flow field considered is covered with a regular mesh. To solve the system of linear difference equations, a tridiagonal method algorithm is used in both directions r and θ.

Direct substitution of previous values of dependent variables by new calculated values can cause calculation un-stability in general. To overcome this problem, a weighting procedure is used in which the optimum weighting factor depends on Reynolds number. The mesh size used in numerical solution for equator of the circle is a uniform 40x20, 60x30, 80x40 and 100x50(θ-direction x r-direction, respectively) with the ratio of R_{out}/R_{in}=2, which all of them show that the problem is independent of mesh size, but on the one hand by noting to calculations time and on the other hand since a finer mesh size is better we choose the 80x40 mesh size. To verify the validity of the numerical procedure used in this work, comparison with Ref [6] have been done which show very good agreement.

B. Results and Discussions

The contours of flow field in the meridian plane for blowing $(\mathrm{Re}_w = 0.5)$ at the inner boundary for, $\mathrm{Re} = 20$, $\Omega_{oi} = \dfrac{\Omega_o}{\Omega_i} = 0$ (outer sphere is stationary) and $\lambda = \dfrac{R_i}{R_o} = 0.5$ in t $=50^{\sec}$ is shown in Fig. 2. From Fig. 2, it is seen that, because of relatively low value of blowing Reynolds number $(\mathrm{Re}_w = 0.5)$, two stagnation points exist on the streamlines at the poles ($\theta = 0^\circ$ and $\theta = 180^\circ$) in which the value of ν_r (radial velocity component) is zero. In this condition, the eddies created by the centrifugal effect generated by the rotation of the inner sphere are confined within regions near the poles, as the size of the eddies decreases with increase in value of Re_w. To verify the results, comparisons with Ref [6] have been done which show very good agreements.

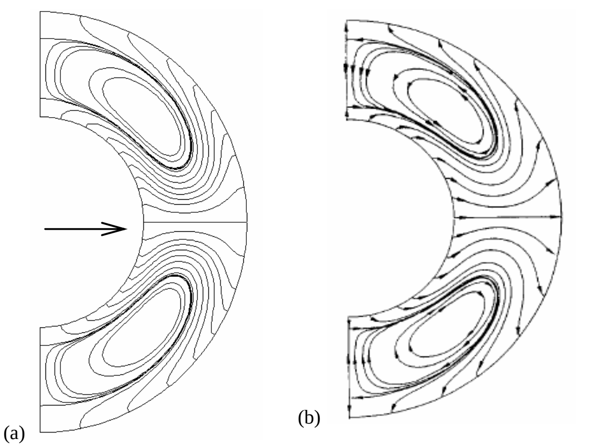


Fig. 2 Contour of flow field for $\mathrm{Re} = 20$, $\mathrm{Re}_w = 0.5$, $\Omega_{oi} = 0$, (a) Presented work, (b) Ref [6]

In Fig. 3 the streamlines for $\mathrm{Re} = 20$, $\mathrm{Re}_w = 2$ and $\Omega_{oi} = 0$ in t $=50^{\sec}$ is shown. It is observed that, with increasing the blowing the eddies in the annulus are removed. Figure (4) show the streamlines for the case suction for $\mathrm{Re} = 20$ and $\Omega_{oi} = 0$ in t $=50^{\sec}$. In the case (a), $\mathrm{Re}_w = -0.5$ the eddies are formed in the equator. As can be seen in the case (b), $\mathrm{Re}_w = -1$ with increasing the suction the eddies are removed.

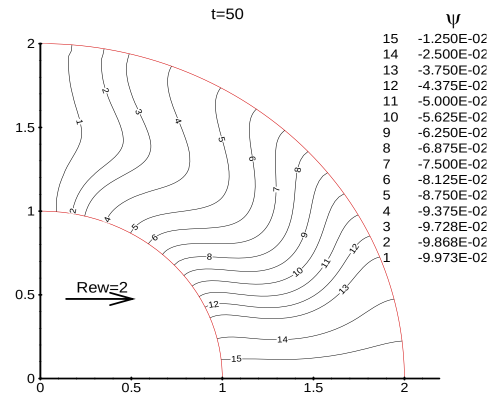


Fig. 3 Contour of flow field for $\mathrm{Re} = 20$, $\mathrm{Re}_w = 2$

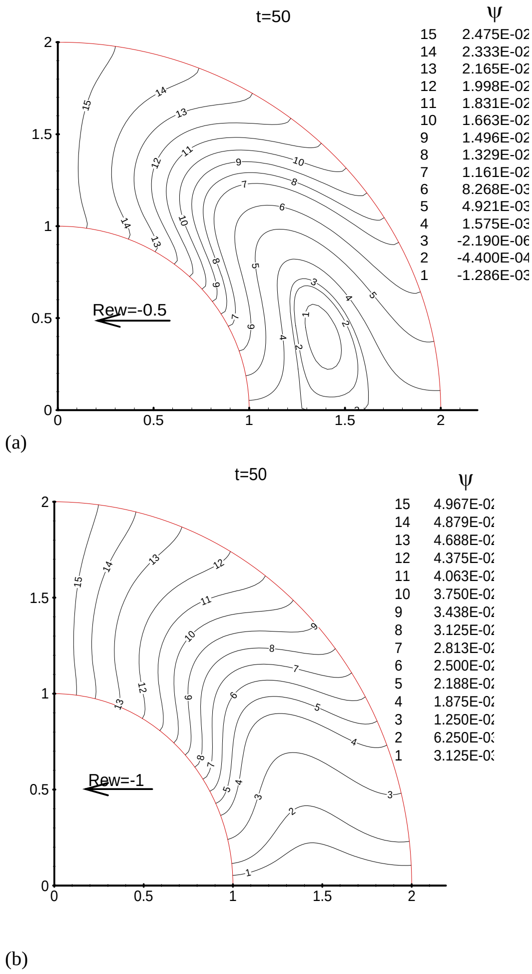


Fig. 4 Contour of flow field for $\mathrm{Re} = 20$, $\Omega_{oi} = 0$
(a) $\mathrm{Re}_w = -0.5$, (b) $\mathrm{Re}_w = -1$

In the above cases, the outer sphere was stationary. Now, we aim to examine the effect of angular velocity of outer sphere on flow field. Fig. 5 show the flow field for $\mathrm{Re}=50$, $\mathrm{Re}_w=3$ for two cases, when the outer sphere is stationary as $\Omega_{oi}=0$ (case a) and or other case (case b) when the outer sphere rotates with same angular velocity as inner sphere so that $\Omega_{oi}=1$ $(\Omega_i=1, \Omega_o=1)$. It is observed from this figure that the change in angular velocity of outer sphere can removes the eddies.

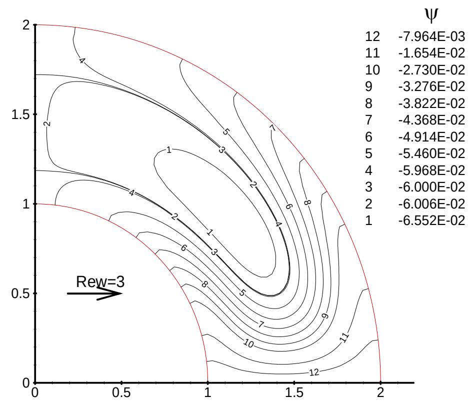
The dimensionless viscous torque T_μ at any radius r can be defined in general as:

$$T_\mu = \frac{3}{4} r^3 \int_0^\pi \tau_{r\phi} \sin^2\theta \, d\theta \qquad (9)$$

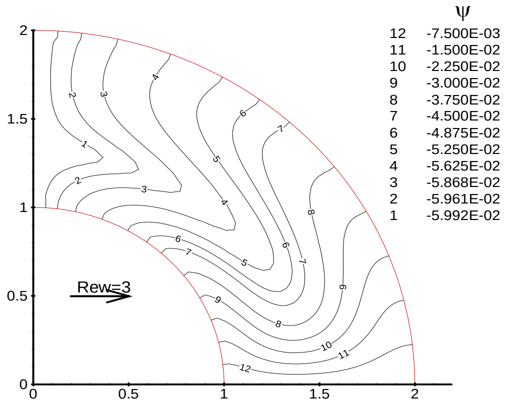
where the dimensionless shear stress $\tau_{r\phi}$ is:

$$\tau_{r\phi} = -r \frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) \qquad (10)$$

Using the above definitions, the viscous torques at the inner sphere $T_{\mu,i}$ and at the outer sphere $T_{\mu,o}$ can be calculated. Figures (6) and (7) show the variations of viscous torques with time at the inner and outer spheres for $\mathrm{Re}=20$, $\Omega_{oi}=0$ and three different suction / blowing Reynolds number. Note that in the case $\mathrm{Re}_w=0$ the viscous torque at the inner and outer spheres are nearly equal, as expected.



(a)



(b)

Fig. 5 Effect of angular velocity of the outer sphere on flow field: $\mathrm{Re}=50$, $\mathrm{Re}_w=3$,(a) $\Omega_{oi}=0$ (b) $\Omega_{oi}=1$

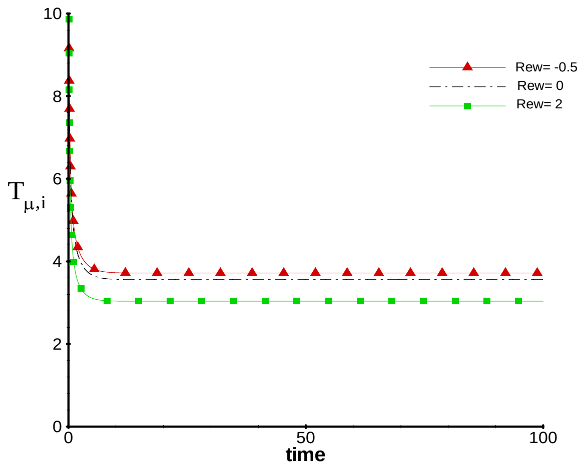


Fig. 6 Variations of viscous torque at the inner sphere with time for $\mathrm{Re}=20$, $\Omega_{oi}=0$

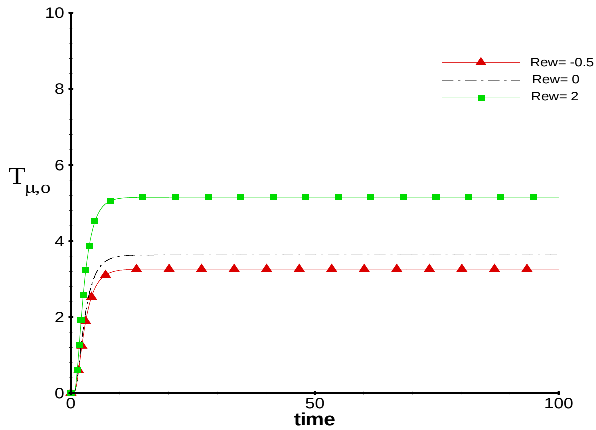


Fig. 7 Variations of viscous torque at the outer sphere with time for $\mathrm{Re}=20$, $\Omega_{oi}=0$

IV. Conclusion

A numerical study of unsteady flow between two concentric rotating spheres with uniform transpiration has been done. The obtained results showed that with increasing the value of transpiration (suction and blowing) the eddies created in the annulus by centrifugal effects can be removed. Also it is observed that with change in the angular velocity of a sphere can remove the eddies, too.

When the outer sphere is stationary, suction decreases the thickness of the boundary layer of inner sphere and corresponding to this change then the coefficient of friction and therefore viscous torques on this sphere is increased. The effect of suction on outer sphere is that the boundary layer thickness is increased and therefore the coefficient of friction and viscous torques are decreased.

Acknowledgment

This study has been supported by the research grant of Shahrood University of Technology.

References

[1] C. Pearson, “A numerical study of the time-dependent viscous flow between two rotating spheres,” *Journal of Fluid Mechanics*, vol.28, part.2, 1967, pp.323-336
[2] B.R Munson and D.D Joseph, “Viscous compressible flow between concentric rotating spheres,” *Journal of Fluid Mechanics* vol.49, Part1, 1971, pp.289-303.
[3] A. Jabari Moghadam and A.B. Rahimi, “A numerical study of flow and heat transfer between two rotating spheres with time dependent angular velocities,” *Journal of Heat transfer*, vol.130, 2008.
[4] A. Jabari Moghadam and A.B. Rahimi, “Similarity solution in study of flow and heat transfer between two rotating spheres with constant angular velocities,” *SIENTIA IRANICA*, vol.16, No.4, 2009, pp.354-362.
[5] S.D. Gulwadi and A.F Elkouh, “Effects of transpiration on free convection in an annulus between concentric porous spheres,” *Journal of Engineering Mathematics*, vol.28, 1994, pp: 483-499.
[6] S.D. Gulwadi, A.F. Elkouh and T.C. Jan, “Laminar flow between two concentric rotating porous spheres,” *ACTA MECHANICA*, vol.97, 1993, pp: 215-228.