# Finding Equilibrium in Transport Networks by Simulation and Investigation of Behaviors 

Gábor Szücs, Gyula Sallai


#### Abstract

The goal of this paper is to find Wardrop equilibrium in transport networks at case of uncertainty situations, where the uncertainty comes from lack of information. We use simulation tool to find the equilibrium, which gives only approximate solution, but this is sufficient for large networks as well. In order to take the uncertainty into account we have developed an interval-based procedure for finding the paths with minimal cost using the Dempster-Shafer theory. Furthermore we have investigated the users' behaviors using game theory approach, because their path choices influence the costs of the other users' paths.


Keywords-Dempster-Shafer theory, S-O and U-O transportation network, uncertainty of information, Wardrop equilibrium.

## I. INTRODUCTION

T'HIS paper is the extended version of the previous our work [25]. This paper is about the path optimization for transport networks, which is a current topic in the transportation literature. The paper is based on a previous work [18] of finding a solution for path planning in a road network, where the costs of roads are uncertain. Our concept is based on the Dempster-Shafer theory, which helps to model the uncertainty: the lack of the information. In [18] we have presented a path search algorithm for any individual driver taking this kind of uncertainty into account. New question arises in regard to this best path search: if every driver traveled on the base of his/her best path, they may influence to each other; so will be this situation the best for all participants?

Wardrop [17] has investigated this question and he has recognized alternative possible behaviors of users of transport networks, and stated two principles, which are commonly named after him:

- First principle: The journey times of all paths actually used are equal. These are equal or less than those which would be experienced by a single vehicle on any unused path.
- Second principle: The average journey time is minimal.
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The first principle corresponds to the behavioral principle in which travelers seek to (unilaterally) determine their minimal costs of travel whereas the second principle corresponds to the behavioral principle in which the total cost in the network is minimal. Many researches focus on this kind of equilibrium, e.g. on realizing Wardrop equilibria with realtime traffic information [10], or on analysis of Wardrop equilibrium [12].
The equilibrium depends on many variables and parameters of the network structure. The Braess Paradox [16] is a wellknown phenomenon for this: adding a new road to a transport network reduces the total travel time in general, but in special cases the total travel time may be increasing. In fact, some road users may be better off, but they contribute to an increase in travel time for other users. This situation happens because drivers do not face the true social cost of an action. Many works [8] have analyzed this phenomenon, some works [14] have been proposed to at least minimize the effects of the paradox (by giving path recommendation to drivers).
Two kinds of optimized system can be distinguished: useroptimized (U-O) and system-optimized (S-O) transportation networks. In the former the users act unilaterally, in selecting their paths; and in the latter the users select paths according to what is optimal from a societal point of view, in that the total cost in the system is minimized. The former problem (the U-O network problem, also commonly referred to in the transportation literature as the traffic assignment problem [3], [4], [6], [7] or the traffic network equilibrium problem [9]) coincides with Wardrop's first principle, and the latter with Wardrop's second principle. In this paper we focus on both of them (U-O and the S-O transportation networks).

## II. Finding Equilibrium

## A. Formalization of the Problem

Consider a general network $\mathrm{G}=(\mathrm{N}, \mathrm{E})$, where N denotes the set of nodes, and E the set of directed links (edges). Let $e$ denote an edge of the network connecting a pair of nodes, and let $p$ denote a path consisting of a sequence of edges connecting an origin/destination (O/D) pair. In transport networks the nodes represent the intersections (origins and destinations may be only nodes as well), and edges correspond to roads. Thus a path is a sequence of edges which comprise a path from an origin to a destination.
We follow the formalization described in [1]. Let $\mathrm{P}_{\mathrm{rs}}$ denote the set of paths connecting the origin $r$ to destination $s$ pair of
nodes. Let S represent the set of origin/destination pairs of nodes, and Let P denote the set of all paths in the network assuming that S is given. Let $x_{p}$ represent the flow on path $p$ and let $f_{e}$ denote the flow on edge $e$. The following conservation of flow equation must be held:
$f_{e}=\sum_{p \in P} x_{p} \cdot \delta_{e p}, \quad \forall e \in E$
where $\delta_{e p}$ is equal to 1 , if edge $e$ is contained in path $p$, and 0 , otherwise. Expression (1) states that the flow on an edge $e$ is equal to the sum of all the path flows on paths $p$ that contain (traverse) edge $e$. Let $d_{r s}$ denote the demand associated with $\mathrm{O} / \mathrm{D}$ pair $r s$, which should be the sum of the flows on different paths:

$$
\begin{equation*}
\sum_{p \in P_{r s}} x_{p}=d_{r s}, \quad \forall r s \in S \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
x_{p} \geq 0, \quad \forall p \in P \tag{3}
\end{equation*}
$$

Let ${ }_{l} c_{e}$ denote the edge cost associated with traversing edge $e$ for a user. Assume that the edge cost function is given by a separable function, furthermore this function is assumed to be continuous and an increasing function of the edge flow $f_{e}$ in order to model the effect of the edge flow on the cost.

$$
\begin{equation*}
{ }_{1} c_{e}={ }_{1} c_{e}\left(f_{e}\right) \quad \forall e \in E \tag{4}
\end{equation*}
$$

Let $c_{e}$ denote the total cost of edge $e$ for all users traversing this edge:
$c_{e}={ }_{1} c_{e}\left(f_{e}\right) \cdot f_{e} \forall e \in E$

The total cost of the whole network is the sum of the all edge costs:

$$
\begin{equation*}
C_{S O}=\sum_{e \in E} c_{e}\left(f_{e}\right) \tag{6}
\end{equation*}
$$

The system-optimization problem can be expressed by finding the minimum of $\mathrm{C}_{\mathrm{SO}}$.

The user-optimization problem is very similar, the constraint equations are identical in both of them, but the aim is different. The goal of the U-O problem is to minimize the following:

$$
\begin{equation*}
C_{U O}=\sum_{e \in E} \int_{0}^{f_{e}}{ }_{1} c_{e}(f) d f \tag{7}
\end{equation*}
$$

subject to (1)-(3).

## B. Problem with Uncertainty

Some researches in this topic of the transportation literature deal with uncertainty by fuzzy logic [11] in path choice models, but not one of them handles the lack of information by Dempster-Shafer (DS) theory.
In DS theory the set $\Omega=\left\{\mathrm{H}_{1}, \ldots, \mathrm{H}_{\mathrm{n}}\right\}$ of all the possible states of the system, $\mathrm{H}_{1}, \ldots \mathrm{H}_{\mathrm{n}}$ still mutually exclusive. We denote by $\mathrm{G}(\Omega)$ the powerset $2^{\Omega}$, and by $A$ an element of $G(\Omega)$.
$G(\Omega)=2^{\Omega}=\left\{\{ \},\left\{H_{1}\right\},\left\{H_{2}\right\},\left\{H_{3}\right\}, \ldots,\left\{H_{1}, H_{2}\right\}, \ldots, \Omega\right\}$
DS theory defines functions ( $m$ ) called basic belief assignment (BBA) on the $G(\Omega)$.
$m: \quad 2^{\Omega} \rightarrow[0,1]$
$A \rightarrow m(A)$

Thus it enables to work with non-mutually exclusive pieces of evidence, represented by powerset $G(\Omega)$. A basic belief assignment ( $m$ ) function has to satisfy:

$$
\begin{align*}
& m(\phi)=0  \tag{11}\\
& \sum_{A \in G(\Omega)} m(A)=1 \tag{12}
\end{align*}
$$

Using DS theory we can define a lower and an upper limit for $\operatorname{prob}(A)$, the real probability of the evidence. The belief function $\operatorname{Bel}(A)$ for a set $A$ is defined as the sum of all the BBA of subsets of $A$, saying that a portion of belief assigned to $B$ must be assigned to other hypothesis that it implies. The DS theory also defines the plausibility $P l(A)$ as the sum of all the BBA of sets $B$ that intersects the set of $A$. Hereby:

$$
\begin{align*}
& \operatorname{Bel}(A)=\sum_{B \mid B \subseteq A} m(B)  \tag{13}\\
& P l(A)=\sum_{B \mid B \cap A \neq 0} m(B) \tag{14}
\end{align*}
$$

Then the measures are related to each other as follows:
$\operatorname{Bel}(A) \leq \operatorname{prob}(A) \leq \operatorname{Pl}(A)$
Based on DS theory the cost of an edge can be expressed by an interval (we have called it cost-interval). The details of calculation of the cost-interval are described in [18]. The costinterval of an edge depends on the flow and on other influencing factors, like weather, actual lane numbers. Let us define $k$ different influencing factors: $\mathrm{g}_{1}, \mathrm{~g}_{2}, \ldots, \mathrm{~g}_{\mathrm{k}}$. The actual
values of these factors for an edge $e$ are: $\mathrm{g}_{\mathrm{c} 1}, \mathrm{~g}_{\mathrm{c} 2}, \ldots, \mathrm{~g}_{\mathrm{ck}}$, briefly $\bar{g}_{e}$.

The cost-interval of an edge for a user:

$$
\begin{equation*}
{ }_{1} c_{e, \text { Min }} \leq{ }_{1} \widetilde{c}_{e}\left(f_{e}, \bar{g}_{e}\right) \leq{ }_{1} c_{e, M a x} \quad \forall e \in E \tag{16}
\end{equation*}
$$

where the left expression is the minimal, the right one is the maximal value of the interval.

The total cost-interval of an edge for all user:

$$
\begin{equation*}
\widetilde{C}_{e}\left(f_{e}, \bar{g}_{e}\right)={ }_{1} \widetilde{c}_{e}\left(f_{e}, \bar{g}_{e}\right) \cdot f_{e} \forall e \in E \tag{17}
\end{equation*}
$$

The total cost-interval of all edges for the whole network at the system-optimization and at the user-optimization problem respectively:
$\widetilde{C}_{S O}=\sum_{e \in E} \widetilde{C}_{e}\left(f_{e}, \bar{g}_{e}\right)$
$\widetilde{C}_{U O}=\sum_{e \in E} \int_{0}^{f_{e}}{ }_{1} \widetilde{c}_{e}(f) d f$
The left size of (18) and (19) are intervals; at both problems the aim is to minimize these costs. In order to minimize we need a definition of interval comparison, this will be discussed in section III.B. (after that the uncertainty is represented by intervals). In this paper a simulation method is presented for solving the problem described above.

## III. Solution with Simulation

## A. Structure of the solution procedure

The aim of the drivers is to minimize their own travel times, for solving the problem described above so-called userequilibrium program solution algorithms (e.g. the FrankWolfe algorithm) are well known. These theoretical algorithms are appropriate for small and average models, but not efficient for large networks. Some works [3] deals with changing state in time, e.g. the time expansion of the spatial network is solved in [2] by MILP (Mixed Integer Linear Programming), but this leads to a very large graph (the number of the nodes is the number of the original nodes multiplied by the number of the time steps).

Other works [5], [13], [15] use simulation tools to find the equilibrium, which give only approximate solution (but this tends to theoretical solution by increasing simulation run length), but capable for large networks.

Our solution based on work in [19] using simulation with an iterative procedure, which converges to the conditions mentioned above. The solution consists of two main
components: a method to determine a new set of timedependent path flows given the experienced path travel times on the previous iteration, and a method to determine the actual travel times that result from a given set of path flow rates. The latter problem is referred to as the "network-loading problem", and can be solved using any path-based dynamic traffic model (e.g. INTEGRATION, CORSIM, AIMSUN2, VISSIM, PARAMICS, MITSIM). The iterative procedure furthermore requires a set of initial path flows, which are determined by assigning all vehicles to the shortest paths, based on free-flow conditions. The general structure of the procedure is shown schematically in Fig. 1.
The inputs of the simulation model can be divided into two parts: common and unique. The common part contains the network structure with nodes, edges, cost functions, and the BBA functions for network parameters. The unique part of the input is unique for the given problem situation, so this consists of the user demands and the actual facts.
The influencing factors of the edges using uncertain probabilities are described by probability intervals. Based on these intervals the cost-intervals of each edge can be calculated, so the "network with parameter" is given. The further part of the procedure takes the uncertain values of costs into account such a way.


Fig. 1 Structure of the solution procedure

## B. Details about the solution of uncertainty

The block "Demand ordering" of the procedure in Fig. 1 determinates the order of the demands in the further execution, this order may influence the speed of the
convergence
The part "Determination paths and path flow" of the procedure in Fig. 1 determinates the paths in several iterations. In the first iteration the shortest (least cost) paths are based on free flow. At calculation for first demand the network is empty, then the edge travel times are updated and new shortest paths are computed for the second demand. The further calculations are similar, so this is repeated for all demands. At the end of the first iteration the network-loading is executed: this will be used for calculation the input flow for the next iteration.

Starting at the second iteration, and up to a prespecified maximum number of iterations, the edge travel times after each loading are used to determine a new set of shortest paths (and path flows) that are added to the current set of paths (and path flows). At each iteration $n$ the volume assigned as input flow to each path in the set is $x_{p} / n$, where $x_{p}$ is the calculated flow on path $p$ in the previous iteration.

At the feedback we can use ordering optimization. Without this the order of the demands is random, but the iteration of the whole process may be faster by using optimization. The ordering optimization method tries to estimate the importance of the unique demands based on some features (e.g. how many other users would like to travel from the same origin to the same destination). This estimated importance can be used for ordering.

The convergence criteria can be chosen from the known ones in the literature. This may be based on the iteration counter limit, the computational time limit, the changing lower limit.

The finding the shortest path for a given demand at a given iteration is based on [18], where the uncertainty is taken into account, and intervals to model the uncertainty are calculated. An interval (cost-interval) is given by two boundaries: $X_{\text {Min }}$ $X_{M a x}$ :

$$
\begin{equation*}
X_{M i n} \leq \widetilde{X} \leq X_{M a x} \tag{20}
\end{equation*}
$$

The addition rules of some intervals are the following:

$$
\begin{equation*}
Y_{M i n}=\sum_{i} X_{i, M i n}, \quad Y_{M a x}=\sum_{i} X_{i, M a x} \tag{21}
\end{equation*}
$$

In some cases the result of the comparison of intervals is unambiguous; it can not been decided which interval is less, because the intervals are overlapped such way that one of the intervals contains the other interval (e.g. $X_{M i n}<Y_{M i n}$ and $X_{M a x}$ $>Y_{\text {Max }}$ ). A rule from some predefined rules can be applied in order to solve this unambiguous situation according to the user decision. Three basic rules can be predefined based on the risk attitude of the user:

- Pessimistic: If $\mathrm{X}_{\text {Max }}<\mathrm{Y}_{\text {Max }}$, then interval $X$ is considered as less.
- Optimistic: If . $\mathrm{X}_{\text {Min }}<\mathrm{Y}_{\text {Min }}$, then interval $X$ is considered as less.
- Centralistic: If $\left\{\mathrm{X}_{\text {Min }}+\mathrm{X}_{\text {Max }}\right\} / 2<\left\{\mathrm{Y}_{\text {Min }}+\mathrm{Y}_{\text {Max }}\right\} / 2$, then interval $X$ is considered as less.


## C. Heuristics

The convergence of the iteration may be slow, we give a heuristic idea to reach the convergence criteria faster: Not all the possible paths are computed for a demand, but some of the best (smallest cost) ones based on the following rule: The paths with smallest cost are computed at the given iteration and situation, then its cost is stored as minimal cost $\left(c_{\text {min }}\right)$. The further (second, third, etc.) paths are computed until the cost of the path is less, than the $r \cdot c_{\text {min }}$, where $r(1 \leq r)$ is the predefined constant for acceptance. Thus the consequence of the reduced search space will be the faster running. The $r$ is similar to parameter of the local beam search, where the search can be influenced by a focus parameter.

## D. Competition or Cooperation among the Users

We can investigate the users' behaviors in the transport system: they travel on the same transport network, and their path choices influence the costs of the other users' paths (conflict can occur among them). There are three general strategies of conflict resolution in interpersonal relationships among the persons:

- Avoidance behaviors: People employ no or indirect communication with denial, equivocation, changing the subject, noncommittal remarks, unfocused or rephrasing the question, joking.
- Competitive behaviors: Persons involve negative communication with confrontative remarks, personal criticism, rejection, hostile questioning, sarcasm, denial of responsibility.
- Cooperative behaviors: People involve open and positive communication with describing the problem, analytical remarks, open disclosure, soliciting criticism, great empathy, ability to concessions, accepting responsibility.
Avoidance behaviors can not be implemented in the games, so the rest (competition or cooperation) lead to dilemma. This could be the key problem of any strategy maker in the games. There is a problem which is famous for this question, the prisoner's dilemma [20][21]. This basic prisoner's dilemma can be extended to several ( N ) people, this can be called N person prisoner's dilemma [22]. The simplest version of this game, when each person should select between two alternatives: C and $\mathrm{D}, \mathrm{C}$ represents the intention of the people to cooperation with others and D represents the uncooperative behavior, which leads to defection. Each player who selected C causes each of the other persons to receive $\$ 1$. Each player who selected D gets $\$ 1$, but this has no effect on the payoff for others. If everyone selects C , each gets $\mathrm{N}-1$; in case of everyone selects D , each gets 1 . Maximal gain is N , when everyone except one player selects $C$ and this player select $D$.

In general in the game theory, the summation of pay-off functions of two persons is not zero. A finite (nonzero-sum) two-person game is usually referred to as a bimatrix game, since it is completely determined by the pay-off matrices of the players. The game is given by the pair of (A, B) pay-off matrices, and we can define the mixed extension of the game, where the average pay-off $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ (belonging to player 1 and player 2 respectively) are the following:
$E_{1}=p^{T} \cdot A \cdot q \quad E_{2}=p^{T} \cdot B \cdot q$
where the p and q are the distribution vectors of the two players. The pair of $\left(\mathrm{p}^{0}, \mathrm{q}^{0}\right)$ strategies is the Nash equilibrium point, if:

$$
\begin{array}{ll}
E_{1}\left(p^{0}, q^{0}\right) \geq E_{1}\left(p, q^{0}\right) & \forall p \\
E_{2}\left(p^{0}, q^{0}\right) \geq E_{2}\left(p^{0}, q\right) & \forall q \tag{24}
\end{array}
$$

In the transport system the number of the users is not two, but very large (let us denote by $n$ ). The strategy n-tupple of the all users is the Nash equilibrium point, if:

$$
\begin{array}{ll}
\mathrm{E}_{1}\left(\mathrm{p}_{1}^{0}, \mathrm{p}_{2}^{0}, \ldots, \mathrm{p}_{\mathrm{n}}^{0}\right) \geq \mathrm{E}_{1}\left(\mathrm{p}_{1}, \mathrm{p}_{2}^{0}, \ldots, \mathrm{p}_{\mathrm{n}}^{0}\right) & \forall \mathrm{p}_{1} \\
\mathrm{E}_{2}\left(\mathrm{p}_{1}^{0}, \mathrm{p}_{2}^{0}, \ldots, \mathrm{p}_{\mathrm{n}}^{0}\right) \geq \mathrm{E}_{2}\left(\mathrm{p}_{1}^{0}, \mathrm{p}_{2}, \ldots, \mathrm{p}_{\mathrm{n}}^{0}\right) & \forall \mathrm{p}_{2} \tag{26}
\end{array}
$$

end so forth, but this can be formalized in more general ( $i$ is an integer from 1 to $n$ ):
$\mathrm{E}_{\mathrm{i}}\left(\mathrm{p}_{1}^{0}, \mathrm{p}_{2}^{0}, \ldots, \mathrm{p}_{\mathrm{n}}^{0}\right) \geq \mathrm{E}_{\mathrm{i}}\left(\mathrm{p}_{1}^{0}, \mathrm{p}_{\mathrm{i}}, \ldots, \mathrm{p}_{\mathrm{n}}^{0}\right) \quad \forall \mathrm{p}_{\mathrm{i}} \quad \forall \mathrm{i}$
In general the computation of Nash equilibrium of noncooperative games is hard [24]. In the transport system the users' pay-off (which should be maximized) is the opposite of the cost of the chosen path (which should be minimized). In order to compute the Nash equilibrium we should know all the users' strategies (with hundreds of players), which leads a very complex mathematical problem. In the future work the fast solution of this problem is planned.

## IV. Conclusion

The aim of this paper was to find Wardrop equilibrium in transport networks at case of uncertainty situations, where the uncertainty comes from lack of information. Two kinds of problems are investigated: user-optimized (U-O) and systemoptimized (S-O) transportation networks. In the former the users act unilaterally, they select their paths from own point of view; and in the latter the users select paths according to what is optimal for the whole community of users (minimized total cost). We have used simulation tool to find the equilibrium, which gives only approximate solution, but this is sufficient for large networks as well. In order to take the uncertainty into account we have developed an interval-based procedure for finding the paths with minimal cost using the DempsterShafer theory. A game on the transport network (like game on
graphs [23]) has been investigated, in which the users' behaviors can be cooperative or competitive. In the future work an extension of our concept is planned with examination of transient phenomena. This is important in real situations, where the state is changed after the calculated Wardrop equilibrium.

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