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Rational Chebyshev Tau method for solving natural convection of Darcian fluid about a vertical full cone embedded in porous media whit a prescribed wall temperature

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Abstract—The problem of natural convection about a cone embedded in a porous medium at local Rayleigh numbers based on the boundary layer approximation and the Darcy's law have been studied before. Similarity solutions for a full cone with the prescribed wall temperature or surface heat flux boundary conditions which is the power function of distance from the vertex of the inverted cone give us a third-order nonlinear differential equation. In this paper, an approximate method for solving higher-order ordinary differential equations is proposed. The approach is based on a rational Chebyshev Tau (RCT) method. The operational matrices of the derivative and product of rational Chebyshev (RC) functions are presented. These matrices together with the Tau method are utilized to reduce the solution of the higher-order ordinary differential equations to the solution of a system of algebraic equations. We also present the comparison of this work with others and show that the present method is applicable.

Keywords—Tau method, Semi-infinite, Nonlinear ODE, Rational Chebyshev, Porous media

I. INTRODUCTION

SPECTRAL methods have been successfully applied in the approximation of differential boundary value problems defined in unbounded domains. For problems whose solutions are sufficiently smooth, they exhibit exponential rates of convergence/spectral accuracy. We can apply different spectral methods that are used to solve problems in semi-infinite domains. The most common method is the use of polynomials that are orthogonal over unbounded domains, such as the Hermite and Laguerre spectral methods [1], [2], [3], [4], [5], [6], [7], [8]. Guo [9], [10], [11] proposed a method that proceeds by mapping the original problem in an unbounded domain to a problem in a bounded domain, and then using suitable Jacobi polynomials to approximate the resulting problems. Another approach is replacing infinite domain with $[-L, L]$ and semi-infinite interval with $[0, L]$ by choosing L, sufficiently large. This method is named domain truncation [12]. Another effective direct approach for solving such problems is based on rational approximations. Christov [13] and Boyd [14], [15] developed some spectral methods on unbounded intervals by using mutually orthogonal systems of rational functions. Boyd [15] defined a new spectral basis, named rational Chebyshev functions on the semi-infinite interval, by mapping to the Chebyshev polynomials. Guo et al. [16] introduced a new set of rational Legendre functions which are mutually orthogonal in $L^2(0, +\infty)$. They applied a spectral scheme using the rational Legendre functions for solving the Korteweg-de Vries equation on the half line. Boyd et al. [17] applied pseudospectral methods on a semi-infinite interval and compared rational Chebyshev, Laguerre and mapped Fourier sine. Parand et al. [18], [19], [20], [21], [22], [23], [24] and Tajvidi et al. [25] applied spectral method to solve nonlinear ordinary differential equations on semi-infinite intervals. Their approach was based on rational Tau and collocation method.

A porous medium means a material consisting of a solid matrix with an interconnected void. The solid matrix is either rigid or it undergoes small deformation. The interconnectedness of the pores allows the flow of one or more fluids through the material [26].

Natural convective heat transfer in porous media has received considerable attention during the past few decades. This interest can be attributed due to its wide range of applications in ceramic processing, nuclear reactor cooling system, crude oil drilling, chemical reactor design, ground water pollution and filtration processes. External natural convection in a porous medium adjacent to heated bodies was analyzed by Nield and Bejan [26], Merkin [27], [28], Minkowycz and Cheng [29], [30], [31], Pop and Cheng [32], [33], Ingham and Pop [34] and vafai [35]. In all of these analysis, it is assumed that boundary layer approximations are applicable and the coupled set of governing equations are solved by numerical methods.

In this paper, we introduce a new computational method to solve the problem of natural convection about an inverted heated cone embedded in a porous medium of infinite extent. No similarity solution exists for the truncated cone, but for the case of full cone similarity solutions exist if the prescribed wall temperature or surface heat flux is a power function of distance from the vertex of the inverted cone [26], [32], [36].

Bejan and Khair [37] used Darcy's law to study the vertical natural convective flows driven by temperature and concentration gradients. Nakayama and Hossain [38] applied the integral method to obtain the heat and mass transfer by free convection from a vertical surface with constant wall temperature and

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concentration. Yih [39] examined the coupled heat and mass transfer by free convection over a truncated cone in porous media for variable wall temperature and variable heat and mass fluxes, Also he [40] applied the uniform transpiration effect on coupled heat and mass transfer in mixed convection about inclined surfaces in porous media for the entire regime. Cheng [41] used an integral approach to study the heat and mass transfer by natural convection from truncated cones in porous media with variable wall temperature and [42] studied the Soret and Dufour effects on the boundary layer flow due to natural convection heat and mass transfer over a vertical cone in a porous medium saturated with Newtonian fluids with constant wall temperature. Natural convective mass transfer from upward-pointing vertical cones, embedded in saturated porous media, has been studied using the limiting diffusion [43]. The natural convection along an isothermal wavy cone embedded in a fluid-saturated porous medium are presented in [44], [45]. The problem of steady laminar hydromagnetic heat transfer over a vertical plate embedded in a uniform porous medium is studied in [46], [47], [48]. In [36] fluid flow and heat transfer of vertical full cone embedded in porous media have been solved by Homotopy analysis method.

Also in this work, we have applied rational Chebyshev Tau approach to find the numerical solutions of nonlinear ordinary differential equations arising from similarity solution of natural convection of Darcian fluid about a vertical full cone embedded in porous media. The operational matrices of the derivative and the product of rational Chebyshev functions are derived. These matrices together with the Tau method are then utilized to evaluate the solution to this equation. The Tau method was invented by Lanczos [49]. The method is based on expanding the required approximate solution as the elements of a complete set of orthogonal functions. In the Tau method, unlike the Galerkin approximation, the expansion functions are not required to satisfy the boundary constraint individually [50], [51]. we have made a comparison with [32], [36] and other solutions based on Runge-Kutta method. The similarity solution which was applied by [32] is based on thermal boundary conditions as wall temperature power function of distance from the vertex of the inverted cone. The transformed ordinary differential equation, with the corresponding boundary conditions, was solved by Runge-Kutta method for the systematically guessing of the missing initial conditions [36]. In Spectral methods an important question is: What sets of ”basis functions” will work? It is obvious that we would like our basis sets to have a number of properties: (i) easy computation (ii) rapid convergence and (iii) completeness, which means that any solution can be represented to arbitrarily high accuracy by taking the truncation N to be sufficiently large[12]. In this area we find that the orthogonal rational Chebyshev functions have these properties. As a sample, in this paper we show that rational Chebyshev functions with these properties can be applied to approximate third order nonlinear differential equations. Numerical results indicate the convergence and effectiveness of the present approach

II. Problem formulation

Consider an inverted cone with semi-angle γ and take axes in the manner indicated in Fig. 1(a). The stream function ψ defined as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x}. \tag{1}$$

In terms of the heated frustum $x = x_0$ is considered [26], [36].

The boundary layer equations are:

$$\begin{aligned} &\frac{1}{r}\frac{\partial^2 \psi}{\partial y^2} = \frac{g\beta K}{v}\frac{\partial T}{\partial y}, \\ &\frac{1}{r}\left(\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y}\right) = \alpha\frac{\partial^2 T}{\partial y^2}. \end{aligned} \tag{2}$$

We have approximately $r = xsin(\gamma)$ and suppose the temperature is power function of distance from the vertex of the inverted cone. Accordingly, the boundary conditions are:

$$\begin{aligned} &u = 0,\ T = T_\infty && y \to \infty, \\ &u = 0 && y = 0, x_0 \le x < \infty, \\ &T = T_w = T_\infty + A(x - x_0)^\lambda && y = 0, x_0 \le x < \infty. \end{aligned} \tag{3}$$

For the case of a full cone $(x_0 = 0, Fig.1(b))$ a similarity solution exists. In the case of prescribed wall temperature, we let:

$$\begin{aligned} &\psi = \alpha r Ra_x^{1/2} f(\eta), \\ &T - T_\infty = (T_w - T_\infty)\theta(\eta), \\ &\eta = \frac{y}{x} Ra_x^{1/2}, \end{aligned} \tag{4}$$

where the Rayleigh number is

$$Ra_x = \frac{g\beta K \cos(\gamma)(T_w - T_\infty)x}{\nu\alpha}. \tag{5}$$

The governing equations are, Ref. [26], [32], [36]:

$$\begin{aligned} &f' = \theta, \\ &\theta'' + \left(\frac{\lambda+3}{2}\right)f\theta' - \lambda f'\theta = 0, \end{aligned} \tag{6}$$

with boundary conditions,

$$f(0) = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0 \tag{7}$$

Finally, we have:

$$\begin{cases} ODE. \quad f''' + (\frac{\lambda+3}{2})ff'' - \lambda(f')^2 = 0, \\ B.C. \quad f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0. \end{cases} \tag{8}$$

It is of interest to obtain the value of the local Nusselt number which is defined as:

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)}. \tag{9}$$

Where q_w for the case of prescribed wall temperature can be computed from:

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{10}$$

From Eqs. (4), (5), (9) and (10), it follows that the local Nusselt number is given by [26], [36]:

$$Nu_x = Ra_x^{1/2}[-\theta'(0)]. \tag{11}$$

Nomenclature

A	prescribed constant
f	similarity function for stream function
g	acceleration due to gravity
K	permeability of the fluid-saturated porous medium
Nu	local Nusselt number
q_w	surface heat flux
r	local radius of the cone
Ra_x	local Raleigh number
T	temperature
T_∞	ambient temperature
T_w	wall temperature
u, v	velocity vector along x,y axis
x, y	Cartesian coordinate system
x_0	distance of start point of cone from the vertex

Greek symbols

θ	similarity function for temperature
η	independent dimensionless parameter
λ	prescribed constants
β	expansion coefficient of the fluid
α	thermal diffusivity the fluid-saturated porous medium
ν	kinematics viscosity of the fluid
ψ	stream function

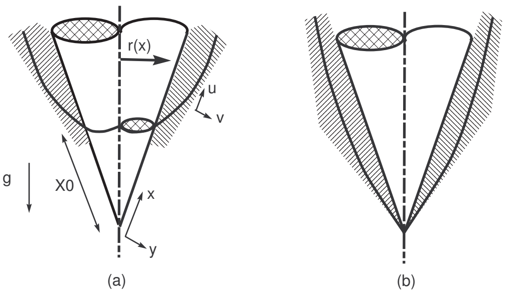

Fig. 1. (a) Coordinate system for the boundary layer on a heated frustum of a cone, (b) full cone, $x_0 = 0$

III. Properties of Rational Chebyshev functions

A. Rational Chebyshev functions

The well-known Chebyshev polynomials are orthogonal in the interval $[-1, 1]$ with respect to the weight function $w(y) = \frac{1}{\sqrt{1-y^2}}$ and can be determined with the aid of the following recurrence formula:

$$T_0(y) = 1, \quad T_1(y) = y,$$
$$T_{n+1}(y) = 2yT_n(y) - T_{n-1}(y), \quad n \geq 1.$$

The RC functions are defined in [12] as

$$R_n(x) = T_n\left(\frac{x-L}{x+L}\right).$$

Thus RC functions satisfy:

$$R_0(x) = 1, \quad R_1(x) = \frac{x-L}{x+L}, \tag{12}$$
$$R_{n+1}(x) = 2\left(\frac{x-L}{x+L}\right)R_n(x) - R_{n-1}(x), \quad n \geq 1.$$

Rational Chebyshev functions are orthogonal with respect to the weight function $w(x) = 1/(\sqrt{x}(x+L))$ in the interval $[0, +\infty)$, with the orthogonality property:

$$R_n(x) = \int_0^\infty R_n(x)R_m(x)w(x)dx = \frac{c_m\pi}{2}\delta_{nm}, \tag{13}$$

$$c_m = \begin{cases} 2, & m = 0, \\ 1, & m \neq 1, \end{cases} \tag{14}$$

and δ_{nm} is the Kronecker function.

B. Function approximation

A function $f(x)$ defined over the interval $[0, +\infty)$ may be expanded as

$$f(x) = \sum_{i=0}^{\infty} a_i R_i(x), \tag{15}$$

where

$$a_i = \frac{2}{c_i\pi}\int_0^\infty R_i(x)f(x)w(x)dx. \tag{16}$$

If $f(x)$ in Eq. (15) is truncated up to the Nth terms, then it can be written as

$$f_N(x) = \sum_{i=0}^{N-1} a_i R_i(x) = A^T R(x), \tag{17}$$

with

$$A = [a_0, a_1, ..., a_{N-1}]^T, \tag{18}$$
$$R(x) = [R_0(x), R_1(x), ..., R_{N-1}(x)]^T. \tag{19}$$

C. Operational Matrix of Derivative

The derivative of the vector $R(x)$ defined in Eq. (19) can be expressed as

$$R'(x) = \frac{dR(x)}{dx} \simeq DR(x), \tag{20}$$

where D is the $N \times N$ operational matrix for the derivative. Differentiating Eq. (12) we get:

$$R'_{n+1}(x) = \frac{4L}{(x+L)^2}R_n(x) + 2\left(\frac{x-L}{x+L}\right)R'_n(x) - R'_{n-1}(x), \quad n \geq 1.$$

By using

$$\frac{2L}{(x+L)^2} = \frac{1}{L}\left[\frac{3}{4}R_0(x) - R_1(x) + \frac{1}{4}R_2(x)\right] = R'_1(x), \tag{21}$$

the elements d_{ij} of the matrix D can be obtained from

$$\begin{cases} R'_{n+1}(x) = 2(R_1(x)R_n(x))' - R'_{n-1}(x), & n \geq 1, \\ R'_1(x) = \frac{1}{L}\left[\frac{3}{4}R_0(x) - R_1(x) + \frac{1}{4}R_2(x)\right], \\ R'_0(x) = 0. \end{cases} \tag{22}$$

The general form of the matrix D is a lower-Heisenberg matrix. The matrix D can be expressed as $D = \frac{1}{L}(D_1 + D_2)$, where D_1 is a tridiagonal matrix which is obtained from

$$D_1 = diag\left(\frac{7}{4}i, -i, \frac{i}{4}\right), \quad i = 0, ..., N-1, \tag{23}$$

and the d_{ij} elements of matrix D_2 are obtained from $d_{10} = -1$ and

$$d_{ij} = \begin{cases} 2, & j \geq i-1, \\ kic_j, & j < i-1, \end{cases} \tag{24}$$

where $k = (-1)^{i+j+1}, c_0 = 1$ and $c_j = 2$, for $j > 1$.
for $N = 7$ we have

$$D = \frac{1}{L} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3/4 & -1 & 1/4 & 0 & 0 & 0 & 0 \\ -2 & 7/2 & -2 & 1/2 & 0 & 0 & 0 \\ 3 & -6 & 21/4 & -3 & 3/4 & 0 & 0 \\ -4 & 8 & -8 & 7 & -4 & 1 & 0 \\ 5 & -10 & 10 & -10 & 35/4 & -5 & 5/4 \\ -6 & 12 & -12 & 12 & -12 & 21/2 & -6 \end{pmatrix}$$

D. The product operational matrix

The following property of the product of two rational Chebyshev function vectors will also be used. Since

$$R_m(x)R_n(x) = \frac{1}{2}[R_{m+n}(x) + R_{|n-m|}(x)], \tag{25}$$

if we truncate the terms higher than R_{N-1}, then we have

$$R(x)R^T(x)A \simeq \tilde{A}R(x), \tag{26}$$

where $\tilde{A}$ is an $N \times N$ product operational matrix for the vector A and the elements $\tilde{a}_{ij}$ of the matrix $\tilde{A}$ can be obtained from

$$\tilde{a}_{ij} = \begin{cases} a_j, & i = 0, \\ \frac{1}{2}a_i, & j = 0, i \neq 0, \\ a_{i-j} + \frac{1}{2}a_{i+j}, & i = j,\ i,j \neq 0, \\ \frac{1}{2}(a_{|i-j|} + a_{i+j}), & i \neq j,\ i,j \neq 0, i+j \leq N, \\ \frac{1}{2}(a_{|i-j|}), & i \neq j,\ i,j \neq 0, i+j > N, \end{cases}$$

and the matrix $\tilde{A}$ is in the form

$$\tilde{A} = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & \cdots & a_{N-1} \\ \frac{1}{2}a_1 & a_0 + \frac{1}{2}a_2 & \frac{1}{2}(a_1 + a_3) & \frac{1}{2}(a_2 + a_4) & \cdots & \frac{1}{2}a_{N-2} \\ \frac{1}{2}a_2 & \frac{1}{2}(a_1 + a_3) & a_0 + \frac{1}{2}a_4 & \frac{1}{2}(a_1 + a_5) & \cdots & \frac{1}{2}a_{N-3} \\ \frac{1}{2}a_3 & \frac{1}{2}(a_2 + a_4) & \frac{1}{2}(a_1 + a_5) & a_0 + \frac{1}{2}a_6 & \cdots & \frac{1}{2}a_{N-4} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ \frac{1}{2}a_{N-1} & \frac{1}{2}a_{N-2} & \frac{1}{2}a_{N-3} & \frac{1}{2}a_{N-4} & \cdots & a_0 \end{pmatrix}$$

.

IV. SOLVING THE PROBLEM

To solve Eq. (8) with initial conditions, we set

$$f(x) \simeq f_N(x), \tag{27}$$

$$f_N(x) = \sum_{i=0}^{N-1} a_i R_i(x) = A^T R(x), \tag{28}$$

$$f_N^{(j)}(x) = \sum_{i=0}^{N-1} a_i R_i^{(j)}(x) \simeq A^T D^j R(x) \quad j = 1, 2, 3. \tag{29}$$

where $R_i(x)$ defined in Eq. (12) in a way that $L = 1$ and D^j is the jth power of the matrix D given in Eq. (20).

TABLE I
A COMPARISON OF METHODS IN [32], [36] AND THE PRESENT METHOD WITH THE VALUES FOR $f''(0)$

		RCT method(N=26 and L=1)	Other methods	
λ	Runge-Kutta[36]	RCT	HAM[36]	Ref. [32]
0	-0.76854	-0.76600	-0.77363	-0.769
$1/4$	-0.88498	-0.88830	-0.88890	$-$
$1/3$	-0.92101	-0.92100	-0.92433	-0.921
$1/2$	-0.98956	-0.98581	-0.99382	-0.992
$3/4$	-1.08518	-1.08598	-1.08840	$-$
1	-1.17372	-1.17040	-1.17686	$-$

Using Eqs. (28) and (29) we get

$$f_N'^2(x) \simeq A^T D R(x) R^T(x) D^T A = A^T D R(x) R^T(x) E \simeq A^T D \tilde{E} R(x), \tag{30}$$

$$f_N(x) f_N''(x) \simeq A^T R(x) R^T(x) (D^2)^T A = A^T R(x) R^T(x) F \simeq A^T \tilde{F} R(x), \tag{31}$$

Where $E = D^T A$ and $F = (D^2)^T A$ and the matrices $\tilde{A}$, $\tilde{E}$ and $\tilde{F}$ can be calculated similarly to Eq. (26). Using Eqs. (28-31) the residual function $(Res(x))$ for Eq. (8) can be written as

$$Res(x) = [A^T D^3 + (\frac{\lambda + 3}{2}) A^T \tilde{F} - \lambda A^T D \tilde{E}] R(x). \tag{32}$$

As in typical Tau method [51], [19], [25] we generate $N-2$ algebraic equations by applying

$$\langle Res(x), R_k(x) \rangle = \int_0^{+\infty} Res(x) R_k(x) w(x) dx = 0, \quad k = 0, 1, ... N-3. \tag{33}$$

using Eqs. (28-29), for the initial conditions we get

$$f_N(0) = A^T R(0) = 0, \qquad f_N'(0) = A^T D R(0) = 1. \tag{34}$$

Take into account

$$\lim_{x \to \infty} R'(x) = 0,$$

and Eq. (29) the boundary condition $f'(\infty) = 0$ already is satisfied.

Eq. (33-34) generates a set of N nonlinear algebraic equations, Consequently, the unknown coefficients a_i of the vector A in Eq. (28) can be calculated.

Table I shows good agreement between RCT method and Homotopy analysis and Runge-Kutta method [36] for $f''(0)$ or $\theta'(0)$ with various λ and $N = 26$.

The results for $f'(\eta)$ have been shown in Table II with two selected $\lambda = 0$ and $\lambda = 1/2$ and comparison have been made between the Runge-Kutta's solution [36] and the presented numerical solution. Absolute errors show that RCT give us approximate solution with a high degree of accuracy with a small N.

The resulting graph of Eq. (8) for $N = 26$ and $L = 1$ is shown in Fig. 2.

V. CONCLUSION

In this study, we have applied rational Chebyshev Tau method to solve three order nonlinear differential equations

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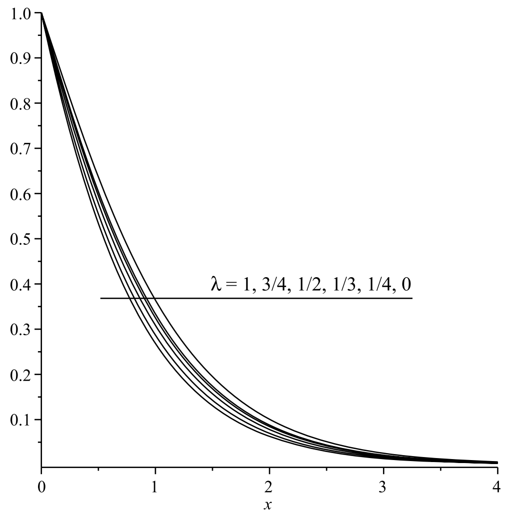

Fig. 2. RCT approximation of $f'(\eta)$ for different values $\lambda = 0,\ 1/4,\ 1/3,\ 1/2,\ 3/4$ and 1

TABLE II
COMPARISON BETWEEN RCT SOLUTION AND RUNGE-KUTTA SOLUTION FOR $f'(0)$ WITH $\lambda = 0$ AND $\lambda = 1/2$ WITH $N = 26$ AND $L = 1$

	$\lambda = 0, f'(\eta)$			$\lambda = 1/2, f'(\eta)$		
η	RCT Solution	Runge-Kutta Solution	Absolute Error	RCT Solution	Runge-Kutta Solution	Absolute Error
0	1.0000	1.0000	0.0000	0.9999	1.0000	0.0000
0.2	0.8477	0.8478	0.0000	0.8130	0.8130	0.0000
0.4	0.7036	0.7036	0.0000	0.6500	0.6500	0.0000
0.6	0.5732	0.5733	0.0001	0.5125	0.5124	0.0001
0.8	0.4598	0.4599	0.0001	0.3994	0.3994	0.0001
1	0.3641	0.3643	0.0001	0.3084	0.3084	0.0001
1.2	0.2853	0.2855	0.0001	0.2364	0.2364	0.0000
1.4	0.2218	0.2218	0.0000	0.1802	0.1802	0.0000
1.6	0.1713	0.1713	0.0001	0.1367	0.1367	0.0002
1.8	0.1316	0.1315	0.0003	0.1034	0.1034	0.0003
2	0.1007	0.1007	0.0003	0.0779	0.0780	0.0004

arising from similarity solution of natural convection of Darcian fluid about a vertical full cone embedded in porous media in whole domain, and compared the results with [36] and [32]. Local similarity solutions are obtained for a full cone with the prescribed wall temperature being a power function of distance from the vertex of the inverted cone. The obtained approximate solution by RCT provides us with calculating Nusselt number. The operational matrices of the derivative and product of RC functions together with the Tau method have been obtained to solve the higher-order ordinary differential equations numerically. The stability and convergence of the Tau approximations [50] are merits that make the approach very useful. Through the comparisons among the numerical solutions of Cheng et al. [32], Sohouli et al. [36] and the current work, it has been shown that the present work has provided acceptable approach for this type equations.

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