

# Human Induced Dynamic Loading on Stairs

L. Gaile, I. Radinsh

**Abstract**—Based on experimental data using accelerometry technology there was developed an analytical model that approximates human induced ground reaction forces in vertical, longitudinal and lateral directions ascending and descending the stairs. Proposed dynamic loading factors and corresponding phase shifts for the first five harmonics of continuous walking force history in case of stair ascend and descend. Into account is taken imperfectness of individual footfall forcing functions, differences between continuous walking force histories among individuals. There is proposed mean synthetic continuous walking force history that can be used in numerical simulations of human movement on the stairs.

**Keywords**—Footfall force, Ground reaction forces, Serviceability limit state, Stair vibration, Walking force history

## I. INTRODUCTION

CONTEMPORARY structural design often governs serviceability criteria. Light weight slender structures such as pedestrian bridges, slender floors are subject to vibrations caused by human walking activities. Thus to assess the dynamic response and performance of the structure during the early stage of the designing process becomes more important.

Human walking induces dynamic and time varying forces which have components in vertical, lateral and longitudinal directions. Therefore there is a need for realistic models of human footfall forces. Numerous loading models are proposed and usually based on the assumption of perfect periodicity of human walking which is not entirely true. Lately there are proposed models of human footfall forces that attempt to take into account imperfect individual forcing functions [1].

Researchers mostly concentrate on obtaining realistic models for the force vertical direction. Relatively recent problems of lateral vibrations experienced by some of pedestrian lightweight bridges increased researcher's interest in horizontal component of the footfall forcing function.

In the case of the sightseeing towers with low natural frequency of the structure, human and structure interaction could play role in the tower design. The longitudinal footfall force component could initiate tower vibration in lateral and torsional directions [2]. The example of such situation is a lattice steel core sightseeing tower located in city Jurmala, Latvia. The total height of the tower is 36.48 m and since the

tower is opened for public, there have been complaints about the tower's excessive vibration.

## II. BACKGROUND

### A. Literature Review

Recent literature reviews [3], [4] show an increased interest in modeling of human walking forces. Mostly it is connected with a vibration of footbridges under human induced lateral excitations. A lot of research is done on human ground reaction forces (GRF) in biomechanics field. In this field there is an interest in GRF values for distinct points and their chronological occurrence on the single foot step force time's history. Those points, so called force parameters allow comparison between different people in human gait analysis [3].

Only limited amount of studies are performed to obtain GRF for the stair ascending and descending cases. Most of GRF are experimentally obtained for individual foot step by using force plates mounted in laboratory stairs. The most relevant and recent research work on this subject is done by S.C. Kerr, N.W.M. Bishop [5] and M. Kasperski [6]. S.C. Kerr presented more than 500 individual footstep measurements of 25 subjects ascended and descended stairs with inclination  $22^{\circ}$  -  $28^{\circ}$  and walking on a flat surface. Comparing results of measurements he concluded that footstep forces significantly differ weather walking on a flat surface or on stairs. [5].

There have been reports that a single step force record suggested to be unreliable [7] because of potential inability of the single step force record to present continuous walking force. To get more than one single step force record M. Kasperski [6] obtained load time histories of 105 persons. His test stair had four active steps and four passive steps which served as lead-in or lead-out section. Four active steps were mounted on one multi component force plate through wooden frame. He concluded that about 50% of the tested persons showed relative difference larger than 4% between both legs. However M. Kasperski experiment is questionable how the wooden structure's above force plate dynamic properties influence precision of obtained load time histories.

Other researchers evaluate human loading on staircases by measuring actual stair structure dynamic response either to a single person or a group loading. By measuring dynamic response of a steel fire escape staircase due to different activities of individuals, small groups and uncontrolled crowds, A. Bougard [8] obtained ratio  $P_{dyn}/P_{stat}$  which is applied dynamic load to applied static load (DAF). B. Davis and M. Murray [9] tested a slender monumental stair to obtain response to human walking. For analytical comparison of results finite element technique where used. To predict a peak steady - state acceleration response there was applied a

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footstep forcing function reported by S.C. Kerr, N.W.M. Bishop [5]. B. Davis and M. Murray reported a big difference between predicted and measured accelerations due to walking (Table I).

TABLE I  
MEASURED AND PREDICTED ACCELERATIONS DUE TO WALKING [5]

Description	Measured peak acceleration (%g)	Predicted peak acceleration (%g)
Ascending, 2 <sup>nd</sup> harmonic	4.1	1.9
Ascending, 3 <sup>rd</sup> harmonic	1.1	1.3
Descending, 2 <sup>nd</sup> harmonic	4.7	4.2
Descending, 3 <sup>rd</sup> harmonic	1.7	1.9

Bin Zhou [10] also obtained slender indoor spiral steel stair accelerations experimentally and numerically by measuring human walking and running activities. For numerical analysis he used only the first two harmonic dynamic components of walking force to load the stairs. The predicted stair response of numerical analysis in some reference points were even 27% less than experimentally measured ones.

*B. Actuality of the Problem*

Analysing the previous research there is a clear necessity to improve existing load model of pedestrian induced forces to obtain better agreement between numerically calculated and experimentally measured structure response to the human activities. It should be taken into account that each force record of the step in the person’s time load history is different during ascend or descend activities. In addition the continuous walking force histories for different persons differ as well.

It is suggested that it is not enough to apply to the model of the structure only a few harmonics of human walking. The phase shift of those separate harmonics is very important parameter to obtain more realistic load model of human walking. For dynamic analyses of the slender vertical structures like sightseeing towers where one of the key element is a stair, vertical component of human induced forces usually is not the critical one (Fig. 1).

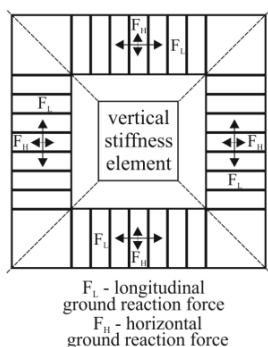


Fig. 1 Critical human induced force directions for sightseeing towers with a vertical stiffness element at the centre of the tower

For the numerical analysis there are required continuous

walking histories of longitudinal and lateral force components which authors cannot find in the literature.

*C. Theoretical Background*

The force plate technologies are not suitable to obtain the continuous walking force histories on the actual stairs. Theoretically these can be obtained by using kinematics of the motion of human centre of gravity (COG) [11]. The vertical walking force function we obtain from a simple dynamic equilibrium based on the Newton’s Second law (1).

$$F(t) = Mg + Ma(t), \tag{1}$$

where  $M$  is body mass of the person,  $g$  – gravitational constant,  $a$  – acceleration of the COG.

To obtain accurate and reliable COG’s acceleration function  $a(t)$  authors propose to utilize accelerometry technology. Lately there is a huge improvement of accelerometer measurement accuracy, as well as reduction of the size and cost of apparatus.

If individual would generate identical and perfectly repeatable footfalls, the convenient way to represent vertical walking force in the time domain as a sum of Fourier harmonic components [3]:

$$F_y(t) = G + \sum_{i=1}^n G\lambda_y^i \sin(2\pi f_i t + \varphi_y^i). \tag{2}$$

For longitudinal or lateral walking force:

$$F(t) = \sum_{i=1}^n G\lambda^i \sin(2\pi f_i t + \varphi^i), \tag{3}$$

where  $G$  is a static weight of the subject body ( $M$ ),  $i$  – order number of the harmonic,  $n$  – the total number of contributing harmonics,  $\lambda^i$  – the Fourier coefficient of the  $i^{th}$  harmonic often referred as dynamic loading factor (DLF),  $f_i$  –  $i^{th}$  harmonic frequency (Hz),  $\varphi^i$  – the phase shift of the  $i^{th}$  harmonics.

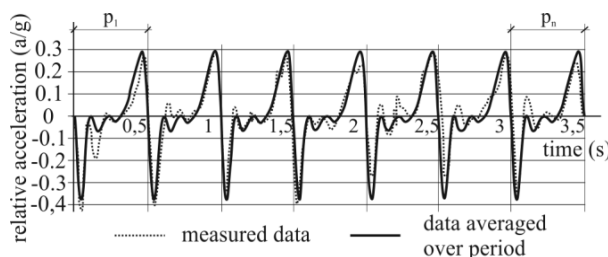


Fig. 2 Relative acceleration longitudinal component of an individual during the stair ascent

By taking into account that during walking the footfalls are not perfectly identical it is proposed to divide continuous walking time history into multiple periods  $p_n$  (Fig.2) and

obtain equivalent period  $p_{eq}$ :

$$p_{eq} = \sum_{i=1}^n p_n / n, \quad (4)$$

where  $n$  is a number of periods looked at during continuous stair ascend or descend.

Further equivalent period  $p_{eq}$  may be multiplied to obtain a perfect periodicity of the subject's continuous walking time history  $c_n$  where differences in individual steps are taken onto account (Fig.6 - 11).

Each test of the subject's continuous walking time history  $c_n$  further is separated into harmonics by using Fast Fourier Transform (FFT) technique. Each subject's DLF and phase values were determined and used to obtain subject's walking force analytical expression.

The test shows that character of walking time history for different person's appear to be similar (Fig.6 - 11). Particular harmonic's DLF and phase values show significant scatter which is in agreement with literature [5], [1]. Therefore authors suggest averaging between individual's continuous walking time histories (4):

$$c_{eq} = \sum_{i=1}^n c_n / n, \quad (4)$$

where  $n$  is a number of subject's continuous walking time histories during continuous ascend and descend the stair.

Further the perfectly periodic function  $c_{eq}$  can be again divided into harmonics using Fast Fourier Transform (FFT) technique. The obtained analytical expression of pedestrian load model is convenient to apply for numerical simulations of structures dynamic response to human induced loadings ascending or descending stairs.

### III. EXPERIMENT

#### A. Methods and Materials

The acceleration of the person's centre of gravity (COG) in vertical, lateral and longitudinal directions during stair ascent and descent were measured to obtain individual continuous walking force time histories. There were 11 men and 7 women with a normal walking pattern that took part in the experiment.

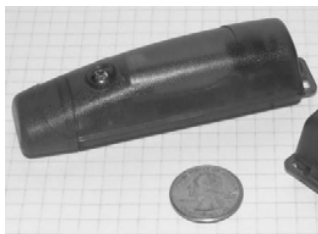


Fig. 3 Accelerometer X6-1A

During the experiment there were more than 216 continuous walking acceleration time histories recorded which resulted in more than 2160 individual footfall traces.

For analysis altogether there were used 60 continuous walking acceleration time histories which contained 540 individual footfall traces.

Two 3-axis light weight (55g) USB accelerometers (Model X6-1A (Fig.3)) were used to record the accelerations.

Accelerometers were fixed to the foam plastic light weight boards that were tightly attached with straps to a subjects COG horizontal axis in front and back of the body (Fig. 4). The measurement sample rate is 160 Hz.

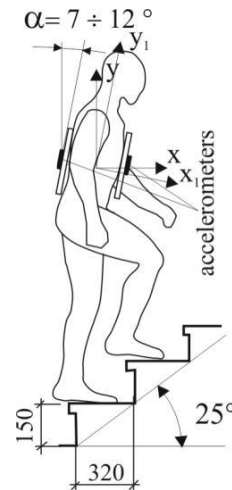


Fig. 4 Illustration of the experimental setup

Experiment took place in Riga Technical University's staircase. Inclination of the test stair is  $25^0$  which is a common inclination. Two flights of the stair were used to perform the test, see Fig. 5.



Fig. 5 Stairs used to perform the test

Measurements of the second flight during stair ascend and measurements of the first flight during stair descent were taken for the data processing.

#### B. Test Procedure

The board with attached accelerometers was tightly strapped on the test subject. The equipment is very light and does not vibrate independently from individual's body.

Individuals admitted that equipment did not restrict their natural movement on the stair and chose their own convenient constant walking rate and path during the two flight stair ascent and descent.

Additionally, during some of the tests there was a laser streamer mounted on the front of the board that lased on the staircase wall while the video camera recorded the angle  $\alpha$  from Fig. 4. When individuals are walking with a constant speed and path in a straight line then a positive and negative accelerations are equal. Therefore the angles  $\alpha$  for the rest of the tests were calculated by transforming recorded measurements from local axis into global directions. Body's upper part forward inclination during stair ascends were found to be from  $7^0$  to  $12^0$ .

#### IV. RESEARCH RESULTS

Authors present parameters – DLF and Phase for the analytical function that approximates continuous human walking force time history in vertical (Table II), lateral (Table IV) and longitudinal (Table III) directions for the first five harmonics ascending and descending the stairs.

Approximated function is mean of individual continuous walking force time histories. These histories consist of an average period of individual footfall during the continuous movement.

TABLE II  
OBTAINED PARAMETERS FOR APPROXIMATION FUNCTION OF CONTENTIOUS HUMAN WALKING VERTICAL FORCE TIME HISTORY

Description	Dynamic loading factor (DLF)	Phase
Ascending, 1 <sup>nd</sup> harmonic	0,37	9,66
Ascending, 2 <sup>nd</sup> harmonic	0,21	2,15
Ascending, 3 <sup>rd</sup> harmonic	0,1	-142
Ascending, 4 <sup>th</sup> harmonic	0,03	84,5
Ascending, 5 <sup>th</sup> harmonic	0,01	18,5
Descending, 1 <sup>nd</sup> harmonic	0,6	20
Descending, 2 <sup>nd</sup> harmonic	0,13	-60,3
Descending, 3 <sup>rd</sup> harmonic	0,05	-84,5
Descending, 4 <sup>th</sup> harmonic	0,03	-125
Descending, 5 <sup>th</sup> harmonic	0,02	93,4

In Fig. 6 – 11 are presented mean functions for vertical, longitudinal and lateral directions.

These results are valid for  $25^0$  inclined stairs and can be applied for a slight angle variation. In other researches it is confirmed that influence of the staircase inclination on walking forces are relatively small [3].

Before the test the individuals were asked to choose their own comfortable pacing rate during the stair ascend and descend. The observed mean step frequency during stair ascend was 2.0 Hz, but during descend 2.15Hz. It is in agreement with other researchers' works where the mean

walking pace is defined 2.0 Hz [5].

To perform averaging between individual analytical functions of walking force time histories the frequencies where normalized to mean value as it is mentioned above.

TABLE III  
OBTAINED PARAMETERS FOR APPROXIMATION FUNCTION OF CONTENTIOUS HUMAN WALKING LONGITUDINAL FORCE TIME HISTORY

Description	Dynamic loading factor (DLF)	Phase
Ascending, 1 <sup>nd</sup> harmonic	0,12	-166
Ascending, 2 <sup>nd</sup> harmonic	0,11	-169
Ascending, 3 <sup>rd</sup> harmonic	0,05	173
Ascending, 4 <sup>th</sup> harmonic	0,03	169
Ascending, 5 <sup>th</sup> harmonic	0,01	146
Descending, 1 <sup>nd</sup> harmonic	0,07	-11,7
Descending, 2 <sup>nd</sup> harmonic	0,1	5,5
Descending, 3 <sup>rd</sup> harmonic	0,06	31,4
Descending, 4 <sup>th</sup> harmonic	0,02	31
Descending, 5 <sup>th</sup> harmonic	0	76

TABLE IV  
OBTAINED PARAMETERS FOR APPROXIMATION FUNCTION OF CONTENTIOUS HUMAN WALKING LATERAL FORCE TIME HISTORY

Description	Dynamic loading factor (DLF)	Phase
Ascending, 1 <sup>nd</sup> harmonic	0,1	28
Ascending, 2 <sup>nd</sup> harmonic	0,01	-133
Ascending, 3 <sup>rd</sup> harmonic	0,11	18,9
Ascending, 4 <sup>th</sup> harmonic	0,01	-123
Ascending, 5 <sup>th</sup> harmonic	0,08	4,95
Ascending, 6 <sup>th</sup> harmonic	0,02	-129
Descending, 1 <sup>nd</sup> harmonic	0,08	-6,02
Descending, 2 <sup>nd</sup> harmonic	0,03	10
Descending, 3 <sup>rd</sup> harmonic	0,11	3,02
Descending, 4 <sup>th</sup> harmonic	0,01	-90,3
Descending, 5 <sup>th</sup> harmonic	0,07	34,7
Descending, 6 <sup>th</sup> harmonic	0,01	149

The character of the obtained mean walking time histories of different individuals are very close although there were noticeable differences in individual DLF's especially the phase values. The large scatter of DLF values can also be seen in S.C. Kerr's and N.W.M. Bishop's [5] work. The reason of it is dissimilarities in harmonic ratios (for example, DLF value of the 1th harmonic to DLF value of the 2th harmonic) among individuals. But it does not change the character of walking time history itself. Therefore the mean value of the whole

history seems to be reasonable way to obtain approximated function but not averaging individual DLF's of the harmonics.

Fig. 12 - 15 is a plot of DLF values from S.C. Kerr's and N.W.M. Bishop's [5] work for the vertical direction. Their

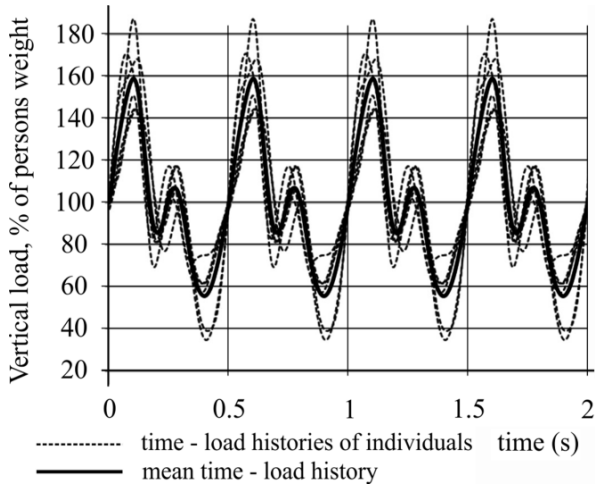


Fig. 6 Analytical function of continuous human walking force time history in vertical direction during stair ascend

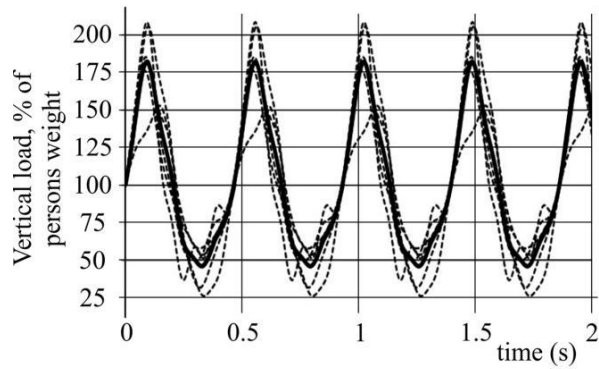


Fig. 7 Analytical function of continuous human walking force time history in vertical direction during stair descend

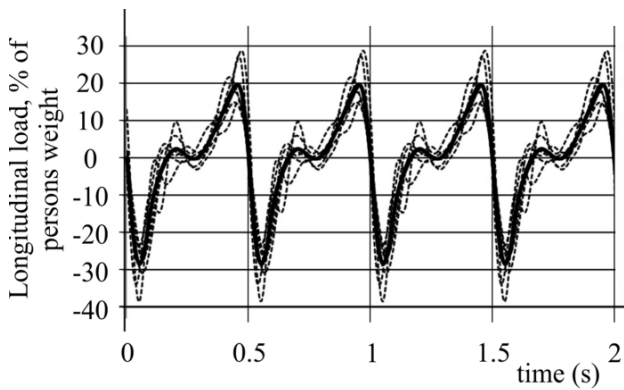


Fig. 8 Analytical function of continuous human walking force time history in longitudinal direction during stair ascend

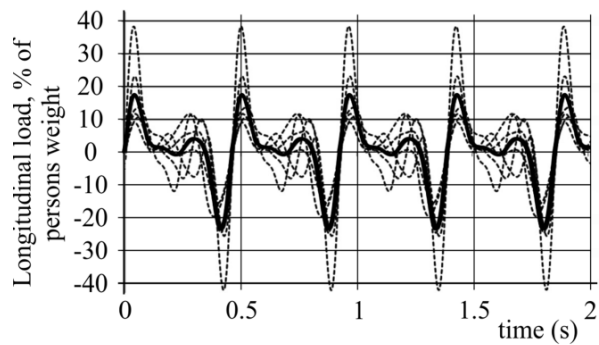


Fig. 9 Analytical function of continuous human walking force time history in longitudinal direction during stair descend

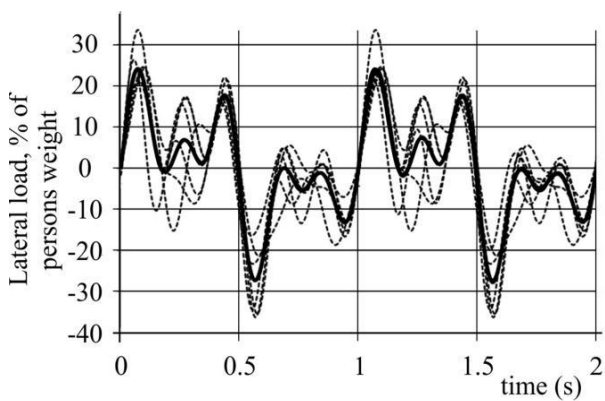


Fig. 10 Analytical function of continuous human walking force time history in lateral direction during stair ascend

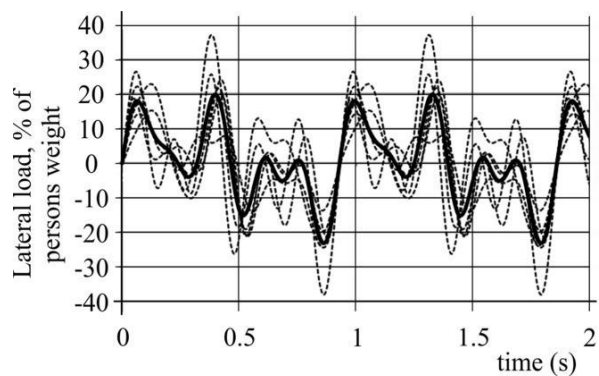


Fig. 11 Analytical function of continuous human walking force time history in lateral direction during stair descend

plot is marked with DLF values obtained from the mean walking time histories that were found during this research. Authors are not aware of other researches where the DLF and phase shift values for lateral and longitudinal directions of stair ascent and descent case can be found.

There is a good agreement with S.C. Kerr's obtained results of the vertical force component for the first harmonic. The obtained second harmonic's DLF value is slightly higher than S.C. Kerr's obtained average value. It is proposed that the second harmonic is more sensible to structures slenderness changes and should be investigated more closely.

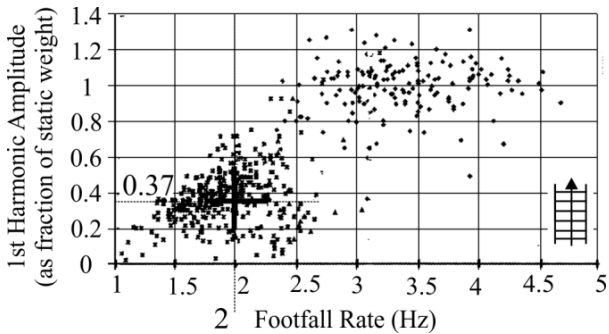


Fig. 12 First harmonic values for ascending stairs

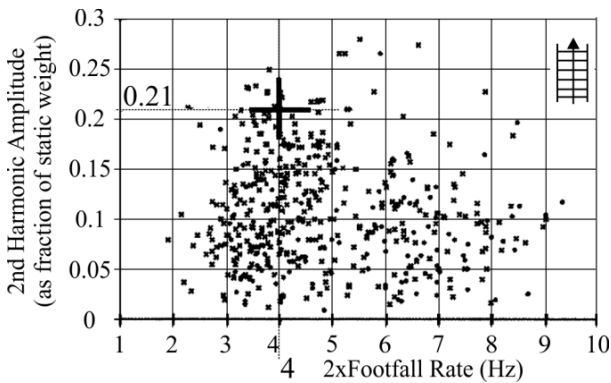


Fig. 13 Second harmonic values for ascending stairs

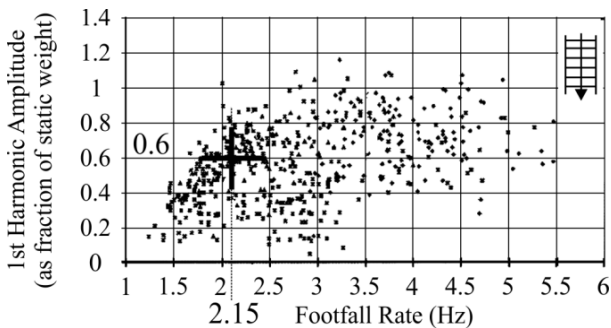


Fig. 14 First harmonic values for descending stairs

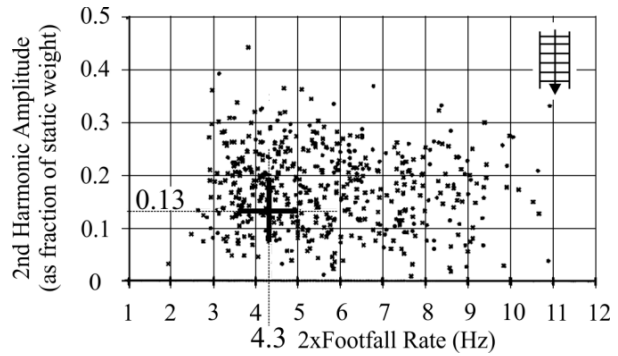


Fig. 15 Second harmonic values for descending stairs

Authors suggest that the obtained analytical mean functions of walking force histories could be more successfully used in numerical assessment of the structure's dynamic response.

It is found that the minimum number of harmonics to obtain approximate replication of the real walking force time history for vertical, longitudinal and lateral directions is three. Those are harmonics with DLF's above 0.05. In case of vertical and longitudinal directions it should be taken first three harmonics (2Hz, 4Hz and 6Hz), but for lateral direction it should be first, third and fifth harmonic (1Hz, 3Hz, 5Hz).

To have better results while performing the numerical analysis of the structure, in Table II – IV the given DLF values should be used together with the appropriate phase values. Although by using modal decomposition, each vibration mode can be analyzed separately using harmonic loads modulated by the appropriate mode shape to account for the moving load. This is true when structure is linearly elastic [12]. When calculating the structure's response to human walking, the use of DLF and phase shift values together, ensures model behavior closer to the real structure behavior since it's response time is simulated more accurately.

During the stair ascent at 2Hz the averaged vertical reaction's force peak amplitude is 1.6 times body weight, during descent - 1.8 times body weight.

Longitudinal reaction force peak amplitude is 0.28 times body weight during the ascent and 0.23 times body weight during the descent.

Lateral reaction force peak amplitude is 0.28 times body weight during the ascent and 0.24 times body weight during the descent.

V.CONCLUSION

There are structures such as very slender sightseeing towers that should be checked if the dynamic response from human induced dynamic loads is satisfactory to meet the serviceability criteria.

This paper gives the necessary parameters for obtaining continues walking force function in vertical, lateral and longitudinal directions for human movement on stairs. The obtained parameters of the vertical force are within agreement

of other researchers' work. Authors are not aware of any information which could be compared with the results of the obtained ground reaction forces in lateral and longitudinal directions.

The obtained analytical mean functions of human walking force histories during the stair ascent and descend may be used in numerical assessments of structure's dynamic response.

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