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Creep Transition in a Thin Rotating Disc Having Variable Density with Inclusion

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Abstract**—**Creep stresses and strain rates have been obtained for a thin rotating disc having variable density with inclusion by using Seth's transition theory. The density of the disc is assumed to vary radially, i.e. $\rho = \rho_0 \left(r/b\right)^{-m}$; ρ_0 and m being real positive constants. It has been observed that a disc, whose density increases radially, rotates at higher angular speed, thus decreasing the possibility of a fracture at the bore, whereas for a disc whose density decreases radially, the possibility of a fracture at the bore increases.

Keyword**—**Elastic-Plastic, Inclusion, Rotating disc, Stress, Strain rates, Transition, variable density.

I. Introduction

ROTATING discs have a wide range of applications in engineering, such as high speed gears, turbine rotors, compressors, flywheel and computer's disc drive. The analytical procedures presently available are restricted to problems with simplest configurations. The use of rotating disc in machinery and structural applications has generated considerable interest in many problems in domain of solid mechanics. Solutions for thin isotropic discs can be found in most of the standard creep text books [1-5]. Reddy and Srinath [6] investigate the influence of material density on the stresses and displacement of a rotating disc. It has been shown that the existence of density gradient in a rotating disc influences the stresses and displacements significantly. Change [7] has developed a closed-form elastic solution for an anisotropic rotating disc with variable density. Wahl [8] has obtained creep stresses in a rotating disc by assuming small deformation, incompressibility condition, Tresca'a yield condition, a power strain law and it associated flow rule. Seth's transition theory [9] does not require these assumptions and thus solves a more general problem, from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at the critical points of the differential equations defining the deformed field. It has been successfully applied to several problems [14-19, 21].

Seth [10] has defined the generalized principal strain measure as,

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1-2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1-\left(1-2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right], \ (i = 1,2,3) \qquad (1)$$

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where n is the measure and $\overset{A}{e}_{ii}$ is the Almansi finite strain components. In this paper, we investigate elastic-plastic transition in a thin rotating disc of variable density with rigid inclusion by using Seth's transition theory. The density of the disc vary along the radius in the form

$$\rho = \rho_0 \left(r/b\right)^{-m} \qquad (2)$$

where ρ_0 is the constant density at $r = b$ and m is the density parameter. Results have been discussed numerically and depicted graphically.

II. Governing Equations

We consider a thin disc of variable density with central bore of radius a and external radius b. The annular disc is mounted on a rigid shaft. The disc is rotating with angular speed ω of gradually increasing magnitude about an axis perpendicular to its plane and passed through the center as shown in figure 1. The thickness of disc is assumed to be constant and is taken to be sufficiently small so that it is effectively in a state of plane stress, that is, the axial stress T_{zz} is zero. The displacement components in cylindrical polar co- ordinate are given by [10]

$$u = r\left(1-\beta\right), \ v = 0, \ w = dz, \qquad (3)$$

where β is function of $r = \left(x^2+y^2\right)^{1/2}$ only and d is a constant. The finite strain components are given by Seth [9] as,

$$\overset{A}{e}_{rr} = \frac{1}{2}\left[1-\left(r\beta'+\beta\right)^2\right],$$
$$\overset{A}{e}_{\theta\theta} = \frac{1}{2}\left[1-\beta^2\right],$$
$$\overset{A}{e}_{zz} = \frac{1}{2}\left[1-(1-d)^2\right],$$
$$\overset{A}{e}_{r\theta} = 0, \ \overset{A}{e}_{\theta z} = 0, \ \overset{A}{e}_{zr} = 0, \qquad (4)$$

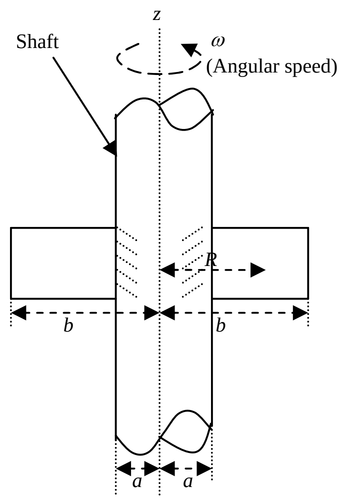

Fig. 1 Geometry of Rotating Disc

where $\beta' = d\beta/dr$.
Substituting equation (4) in equation (1), the generalized components of strain are

$$e_{rr} = \frac{1}{n}\left[1-\left(r\beta'+\beta\right)^n\right],$$
$$e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right],$$
$$e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right],$$
$$e_{r\theta} = 0, \ e_{\theta z} = 0, \ e_{zr} = 0, \qquad (5)$$

where $\beta' = d\beta/dr$.

The stress–strain relations for isotropic material are given by [20]

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} , \quad (i , j = 1, 2, 3), \tag{6}$$

where T_{ij} and e_{ij} are stress and strain tensor respectively, λ , μ are LAME'S constants and $I_1 = e_{kk}$ is the first strain invariant. δ_{ij} is the KRONECKER'S delta.

Equation (6) for this problem becomes

$$T_{rr} = \frac{2\lambda\mu}{\lambda + 2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2\mu e_{rr} , T_{\theta\theta} = \frac{2\lambda\mu}{\lambda + 2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2e_{\theta\theta} ,$$
$$T_{r\theta} = 0,\ T_{\theta z} = 0,\ T_{zr} = 0,\ T_{zz} = 0 , \tag{7}$$

where $\beta' = d\beta/dr$.

Substituting equation (5) in equation (7) we get the stresses as

$$T_{rr} = \frac{2\mu}{n}\left[3 - 2C - \beta^n\left\{1 - C + (2 - C)(P + 1)^n\right\}\right],$$
$$T_{\theta\theta} = \frac{2\mu}{n}\left[3 - 2C - \beta^n\left\{2 - C + (1 - C)(P + 1)^n\right\}\right],$$
$$T_{r\theta} = 0, T_{\theta z} = 0, T_{zr} = 0, T_{zz} = 0 , \tag{8}$$

where $r\beta' = \beta P$ and $C = 2\mu / \lambda + 2\mu$.

Equations of equilibrium are all satisfied except

$$\frac{d}{dr}(rT_{rr}) - T_{\theta\theta} + \rho\omega^2 r^2 = 0 , \tag{9}$$

where ρ is the density of the material of the disc.

Using equation (8) in equation (9), we get a non- linear differential equation in β as

$$(2 - C)n\beta^{n+1}P(P + 1)^{n-1}\frac{dP}{d\beta} = \frac{n\rho\omega^2 r^2}{2\mu} + \beta^n\left[1 - (P + 1)^n - nP\left\{1 - C + (2 - C)(P + 1)^n\right\}\right], \tag{10}$$

where $r\beta' = \beta P$ (P is function of β and β is function of r).

Transition points of β in equation (10) are $P \to -1$ and $P \to \pm\infty$ .The boundary conditions are

$$u = 0 \text{ at } r = a \text{ and } T_{rr} = 0 \text{ at } r = b. \tag{11}$$

III. Solution through the Principal Stresses Difference

For finding the creep stresses, the transition function through principal stress difference [11-19, 21] at the transition point $P \to$ -1 leads to the creep state .The transition function R is defined as

$$R = T_{rr} - T_{\theta\theta} = \frac{2\mu\beta^n}{n}\left[1 - (P + 1)^n\right] . \tag{12}$$

Taking the logarithmic differentiating of equation (12) with respect to r, we get

$$\frac{d}{dr}(\log R) = \frac{nP}{r[1 - (P + 1)^n]}\left\{1 - (P + 1)^n - \beta(P + 1)^{n-1}\frac{dP}{d\beta}\right\} . \tag{13}$$

Substituting the value of $dP/d\beta$ from equation (10) in equation (13) and taking asymptotic value $P \to$ -1, we get

$$\frac{d}{dr}(\log R) = -\frac{1}{r(2 - C)}\left[n(3 - 2C) + 1 + \frac{n\rho\omega^2 r^{2+n}}{2\mu D^n}\right] . \tag{14}$$

Asymptotic value of β as $P \to$ -1 is D/r; D being a constant.

Substituting equation (2) in equation (14) after integrating with respect to r, we get

$$R = T_{rr} - T_{\theta\theta} = Ar^k \exp\left(Fr^{n-m+2}\right) . \tag{15}$$

where $k = -\left[\frac{n(3 - 2C) + 1}{(2 - C)}\right]$, $F = -\frac{n\omega^2\rho_0}{2\mu D^n b^{-m}(2 - C)(n + 2)}$ = $-\left[\frac{n\omega^2\rho_0(3 - 2C)}{ED^n b^{-m}(2 - C)^2(n + 2)}\right]$ and A is a constant of integration.

From equations (12) and (15), we get

$$T_{rr} - T_{\theta\theta} = Ar^k \exp\left(Fr^{n-m+2}\right) . \tag{16}$$

Substituting equation (16) in equation (9), we get

$$T_{rr} = -A\int r^{k-1}\exp\left(Fr^{n-m+2}\right)dr - \frac{\rho_0\omega^2 r^{2-m}}{(2 - m)b^{-m}} + B . \tag{17}$$

where B is a constant of integration.

Using boundary condition (11) in equation (17), we get

$$B = A\int_{r=b} r^{k-1}\exp\left(Fr^{n-m+2}\right)dr + \frac{\rho_0\omega^2 b^2}{(2 - m)} .$$

Substituting the value of B in equation (17), we get

$$T_{rr} = A\int_r^b r^{k-1}\exp\left(Fr^{n-m+2}\right)dr + \frac{\rho_0\omega^2\left(b^{2-m} - r^{2-m}\right)}{(2 - m)b^{-m}} . \tag{18}$$

From equation (16) and (18), we get

$$T_{\theta\theta} = A\left(\int_r^b r^{k-1}\exp\left(Fr^{n-m+2}\right)dr - r^k\exp\left(Fr^{n-m+2}\right)\right) + \frac{\rho_0\omega^2\left(b^{2-m} - r^{2-m}\right)}{2} . \tag{19}$$

From equation (12) and (16), taking asymptotic value $P \to -1$, we get

$$\beta = \left[\left(\frac{n}{2\mu}\right)\left[T_{rr} - T_{\theta\theta}\right]\right]^{\frac{1}{n}} = \left[\frac{n(3 - 2C)}{E(2 - C)}Ar^k\exp\left(Fr^{n-m+2}\right)\right]^{\frac{1}{n}} . \tag{20}$$

Substituting equation (20) in equation (3), we get

$$u = r - r\left[\frac{n(3 - 2C)}{E(2 - C)}Ar^k\exp\left(Fr^{n-m+2}\right)\right]^{\frac{1}{n}} . \tag{21}$$

Using boundary condition (11) in equation (21), we get

$$A = \frac{E(2-C)}{n(3-2C)a^k \exp\left(Fa^{n-m+2}\right)}.$$

Substituting the value of A in equation (18), (19) and (21), we get

$$T_{rr} = \left[\frac{E(2-C)}{n(3-2C)a^k \exp\left(Fa^{n-m+2}\right)}\left(\int_r^b r^{k-1}\exp\left(Fr^{n-m+2}\right)dr\right) + \frac{\rho_0\omega^2\left(b^{2-m}-r^{2-m}\right)}{(2-m)b^{-m}}\right]. \quad (22)$$

$$T_{\theta\theta} = \left[\frac{E(2-C)}{n(3-2C)a^k \exp\left(Fa^{n-m+2}\right)}\left(\begin{array}{l}\int_r^b r^{k-1}\exp\left(Fr^{n-m+2}\right)dr \\ -r^k \exp\left(Fr^{n-m+2}\right)\end{array}\right) + \frac{\rho_0\omega^2\left(b^{2-m}-r^{2-m}\right)}{(2-m)b^{-m}}\right]. \quad (23)$$

$$u = r - r\left[\frac{r^k \exp\left(Fr^{n-m+2}\right)}{a^k \exp\left(Fa^{n-m+2}\right)}\right]^{\frac{1}{n}}. \quad (24)$$

We introduce the following non-dimensional components as

$R = r/b$, $R_0 = a/b$, $\sigma_r = T_{rr}/E$, $\sigma_\theta = T_{\theta\theta}/E$, $\bar{u} = u/b$ and $\Omega^2 = \rho_0\omega^2 b^2/E$.

Equations (22) to (24) in non-dimensional form become

$$\sigma_r = \frac{(2-C)}{n(3-2C)R_0^k \exp\left(F_1 R_0^{n-m+2}\right)}\int_R^1 R^{k-1}\exp\left(F_1 R^{n-m+2}\right)dR + \frac{\Omega^2}{(2-m)}\left(1-R^{2-m}\right), \quad (25)$$

$$\sigma_\theta = \left[\frac{(2-C)}{n(3-2C)R_0^k \exp\left(F_1 R_0^{n-m+2}\right)}\left(\begin{array}{l}\int_R^1 R^{k-1}\exp\left(F_1 R^{n-m+2}\right)dR \\ -R^k \exp\left(F_1 R^{n-m+2}\right)\end{array}\right)\right] + \frac{\Omega^2}{(2-m)}\left(1-R^{2-m}\right) \quad (26)$$

$$\bar{u} = R - R\left[\frac{R^k \exp\left(F_1 R^{n-m+2}\right)}{R_0^k \exp\left(F_1 R_0^{n-m+2}\right)}\right]^{\frac{1}{n}}, \quad (27)$$

where $F_1 = -\frac{n\Omega^2(3-2C)b^n}{(2-C)^2 D^n (n-m+2)}$; $k = -\left[\frac{n(3-2C)+1}{(2-C)}\right]$; $2-m \neq 0$ and $n-m+2 \neq 0$.

For $m=2$ and $m=n+2$ equations (25) to (27) become,

$$\sigma_r = \left[\frac{(2-C)}{n(3-2C)R_0^k \exp(F_2)}\left(\int_R^1 R^{k-1}\exp(F_2)dR\right) - \Omega^2 \log R\right] \quad (28)$$

$$\sigma_\theta = \left\{\begin{array}{l}\left[\frac{(2-C)}{n(3-2C)R_0^k \exp(F_2)}\left(\int_R^1 R^{k-1}\exp(F_2)dR - R^k \exp(F_2)\right)\right] \\ -\Omega^2 \log R\end{array}\right\}, \quad (29)$$

$$\bar{u} = R - R\left[\frac{R^k \exp(F_2)}{R_0^k \exp(F_2)}\right]^{\frac{1}{n}} \quad (30)$$

where $F_1 R^{n-m+2} = -\frac{n\Omega^2(3-2C)b^n l \log R}{(2-C)^2 D^n} = F_2$, $k = -\left[\frac{n(3-2C)+1}{(2-C)}\right]$.

For a disc made of incompressible material *i.e* ($C \to 0$) the stresses given by equations (25) to (27) become

$$\sigma_r = \left\{\begin{array}{l}\left[\frac{2}{3nR_0^{k_1}\exp\left(F_2 R_0^{n-m+2}\right)}\left(\int_R^1 R^{k_1-1}\exp\left(F_3 R^{n-m+2}\right)dR\right)\right] \\ +\frac{\Omega^2}{(2-m)}\left(1-R^{2-m}\right)\end{array}\right\} \quad (31)$$

$$\sigma_\theta = \left[\frac{2}{3nR_0^{k_1}\exp\left(F_2 R_0^{n+2}\right)}\left(\begin{array}{l}\int_R^1 R^{k_1-1}\exp\left(F_3 R^{n-m+2}\right)dR \\ -R^{k_1}\exp\left(F_3 R^{n-m+2}\right)\end{array}\right)\right] + \frac{\Omega^2}{(2-m)}\left(1-R^{2-m}\right) \quad (32)$$

$$\bar{u} = R - R\left[\frac{R^{k_1}\exp\left(F_3 R^{n-m+2}\right)}{R_0^{k_1}\exp\left(F_3 R_0^{n-m+2}\right)}\right]^{\frac{1}{n}}, \quad (33)$$

where $F_3 = -\frac{3n\Omega^2 b^n}{4D^n(n-m+2)}$; $k_1 = -\left[\frac{3n+1}{2}\right]$; $2-m \neq 0$ and $n-m+2 \neq 0$. For $m = 2$ and $m = n+2$ equations (32) to (34) become

$$\sigma_r = \left\{\begin{array}{l}\left[\frac{2}{3nR_0^{k_1}\exp\left(F_2 R_0^{n-m+2}\right)}\left(\int_R^1 R^{k_1-1}\exp\left(F_4 R^{n-m+2}\right)dR\right)\right] \\ +\frac{\Omega^2}{(2-m)}\left(1-R^{2-m}\right)\end{array}\right\} \quad (34)$$

$$\sigma_\theta = \left\{\begin{array}{l}\left[\frac{2}{3nR_0^{k_1}\exp\left(F_2 R_0^{n+2}\right)}\left(\begin{array}{l}\int_R^1 R^{k_1-1}\exp\left(F_3 R^{n-m+2}\right)dR \\ -R^{k_1}\exp\left(F_3 R^{n-m+2}\right)\end{array}\right)\right] \\ +\frac{\Omega^2}{(2-m)}\left(1-R^{2-m}\right)\end{array}\right\} \quad (35)$$

$$\bar{u} = R - R\left[\frac{R^{k_1}\exp\left(F_3 R^{n-m+2}\right)}{R_0^{k_1}\exp\left(F_3 R_0^{n-m+2}\right)}\right]^{\frac{1}{n}}, \quad (36)$$

where $F_3 R^{n-m+2} = -\frac{3n\Omega^2 b^n \log R}{4D^n} = F_4$ and $k_1 = -\left[\frac{3n+1}{2}\right]$.

For the having constant density (m = 0)

The stresses given by equation (25)-(27) for a disc having constant density become

$$\sigma_r = \left[\begin{array}{l}\frac{(2-C)}{n(3-2C)R_0^k \exp\left(F_1 R_0^{n+2}\right)}\left(\int_R^1 R^{k-1}\exp\left(F_1 R^{n+2}\right)dR\right) \\ +\frac{\Omega^2}{2}\left(1-R^2\right)\end{array}\right]. \quad (37)$$

$$\sigma_\theta = \left\{ \begin{array}{l} \left[\dfrac{(2-C)}{n(3-2C)R_0^k \exp\left(F_1 R_0^{n+2}\right)} \left(\begin{array}{l} \int_R^1 R^{k-1} \exp\left(F_1 R^{n+2}\right) dR \\ -R^k \exp\left(F_1 R^{n+2}\right) \end{array} \right) \right] \\ + \dfrac{\Omega^2}{2}\left(1-R^2\right) \end{array} \right\}. \quad (38)$$

$$\bar{u} = R - R\left[\frac{R^k \exp\left(F_1 R^{n+2}\right)}{R_0^k \exp\left(F_1 R_0^{n+2}\right)} \right]^{\frac{1}{n}}. \quad (39)$$

where $F_1 = -\dfrac{n\Omega^2 (3-2C) b^n}{(2-C)^2 D^n (n+2)}$ and $k = -\left[\dfrac{n(3-2C)+1}{(2-C)} \right]$.

For a disc made of incompressible material $(C \to 0)$ equations (31)-(33) become

$$\sigma_r = \left\{ \begin{array}{l} \left[\dfrac{2}{3nR_0^{k_1} \exp\left(F_2 R_0^{n+2}\right)} \left(\int_R^1 R^{k_1 - 1} \exp\left(F_3 R^{n+2}\right) dR \right) \right] \\ + \dfrac{\Omega^2}{2}\left(1-R^2\right) \end{array} \right\}. \quad (40)$$

$$\sigma_\theta = \left\{ \begin{array}{l} \left[\dfrac{2}{3nR_0^{k_1} \exp\left(F_2 R_0^{n+2}\right)} \left(\begin{array}{l} \int_R^1 R^{k_1 - 1} \exp\left(F_3 R^{n+2}\right) dR \\ -R^{k_1} \exp\left(F_3 R^{n+2}\right) \end{array} \right) \right] \\ + \dfrac{\Omega^2}{2}\left(1-R^2\right) \end{array} \right\}. \quad (41)$$

$$\bar{u} = R - R\left[\frac{R^{k_1} \exp\left(F_3 R^{n+2}\right)}{R_0^{k_1} \exp\left(F_3 R_0^{n+2}\right)} \right]^{\frac{1}{n}}. \quad (42)$$

where $F_3 = -\dfrac{3n\Omega^2 b^n}{4D^n (n+2)}$ and $k_1 = -\left[\dfrac{3n+1}{2} \right]$.

These equations are the same as obtained by Gupta and Pankaj [15].

IV. Strain Rates

When creep sets in, the strains should be replaced by strain rate. The stress-strain relations (6) become

$$\dot{e}_{ij} = \frac{1+\nu}{E} T_{ij} - \frac{\nu}{E} \delta_{ij} \Theta. \quad (43)$$

where $\dot{e}_{ij}$ is the strain rate tensor with respect to flow parameter t and $\Theta = T_{11} + T_{22} + T_{33}$.

Differentiating equation (5) with respect to time t, we get

$$\dot{e}_{\theta\theta} = -\beta^{n-1}\dot{\beta}. \quad (44)$$

For SWAINGER measure (n = 1), we have from equation (44)

$$\dot{\varepsilon}_{\theta\theta} = \dot{\beta}. \quad (45)$$

The transition value of equation (13) at $P \to -1$, gives

$$\beta = \left[\frac{n(3-2C)}{(2-C)} \right]^{\frac{1}{n}} (\sigma_r - \sigma_\theta)^{\frac{1}{n}}. \quad (46)$$

Using equation (44), (45) and (46) in equation (43), we get

$$\dot{\varepsilon}_{rr} = \left[\frac{n(\sigma_r - \sigma_\theta)(3-2C)}{(2-C)} \right]^{\frac{1}{n}-1} (\sigma_r - \nu\sigma_\theta),$$

$$\dot{\varepsilon}_{\theta\theta} = \left[\frac{n(\sigma_r - \sigma_\theta)(3-2C)}{(2-C)} \right]^{\frac{1}{n}-1} (\sigma_\theta - \nu\sigma_r),$$

$$\dot{\varepsilon}_{zz} = -\left[\frac{n(\sigma_r - \sigma_\theta)(3-2C)}{(2-C)} \right]^{\frac{1}{n}-1} \left[\nu(\sigma_r + \sigma_\theta) \right]. \quad (47)$$

For incompressible material ($C \to 0$) equations (47) become

$$\dot{\varepsilon}_{rr} = \left[\frac{3n(\sigma_r - \sigma_\theta)}{2} \right]^{\frac{1}{n}-1} \left(\frac{2\sigma_r - \sigma_\theta}{2} \right),$$

$$\dot{\varepsilon}_{\theta\theta} = \left[\frac{3n(\sigma_r - \sigma_\theta)}{2} \right]^{\frac{1}{n}-1} \left(\frac{2\sigma_\theta - \sigma_r}{2} \right),$$

$$\dot{\varepsilon}_{zz} = -\left[\frac{3n(\sigma_r - \sigma_\theta)}{2} \right]^{\frac{1}{n}-1} \left[\frac{1}{2}(\sigma_r + \sigma_\theta) \right]. \quad (48)$$

These constitutive equations are same as obtained by Odquist [22] provided we put $n = 1/N$.

V. Numerical Illustration and Discussion

For calculating stresses and strain-rates distribution based on the above analysis, the following values have been taken: $\Omega^2 = \rho_0 \omega^2 b^2 / E = 50$, 75; m = -1, 0, 1; C = 0.00, 0.25, 0.5; n = 1/3, 1/5 (*i.e* N = 3, 7) and D = 1.In classical theory measure N is equal to $1/n$. Definite integrals in the equations (25) - (26) have been solved by using Simpson's rule .

Curves have been drawn in Figs. 2, 3 and 4 between stresses σ_r , σ_θ and radii ratio R = r/b for a rotating disc made of incompressible/ compressible material having variable density. It is seen from Figs. 2 to 4 that the radial stress has maximum value at the internal surface of disc as compare to circumferential stress. It is also observed that the radial stress has maximum value at the internal surface of the rotating disc with inclusion made of incompressible material as compare to compressible material for measure n = 1/7 or (N = 7) at angular speed Ω^2 = 50, whereas circumferential stress is maximum at the internal surface for measure n = 1/3 or (N = 3) at this angular speed. The values of radial/ circumferential stress further increases at the internal surface with the increase in angular speed (Ω^2 = 75) for measure n = 1/7 or (N = 7) and n = 1/3 or (N = 3) respectively .With the effect of density variation , it is seen from Figs. 2, 3 and 4 that the values of radial/ circumferential stress must be decrease at the internal surface of a disc. As reported by Rimrott [23], a material tends to fracture by cleavage. It is likely to begin as a sub-surface fracture close to the bore, because the largest tensile stress occurs, at this location. This means that for a disc rotating with higher angular speed and whose density increases radially, the possibility of a fracture at the bore decreases, whereas for a disc whose density decreases radially, the possibility of a fracture at the bore increases.

Curves have been drawn in Figs. 5 and 6 between strain rates and radius R = r/b at angular speed Ω^2 = 50, 75 and measures n = 1/7, 1/3 or (N = 7, 3). It has been seen from Figs. 5 and 6 that rotating disc made of compressible material the has maximum value at the internal surface as compared to incompressible material for measure $n = 1/7$ or $(N = 7)$ and $n = 1/3$ or $(N = 3)$ at angular speed $\Omega^2 = 50$. The values of strain rates further increases at the internal surface with the increase in angular speed $\Omega^2 = 75$ for measure $n = 1/7$ or $(N = 7)$ and $1/3$ or $(N = 3)$ respectively. With the effect of density variation, strain rates must be decrease.

VI. Conclusion

It has been observed that a disc, whose density increases radially, rotates at higher angular speed, thus decreasing the possibility of a fracture at the bore, whereas for a disc whose density decreases radially, the possibility of a fracture at the bore increases. Radial stress has maximum value at the internal surface of the rotating disc made of incompressible material a compares to circumferential stress and this value of radial stress further increases with increase in angular speed. Strain rates have maximum values at the internal surface for compressible material.

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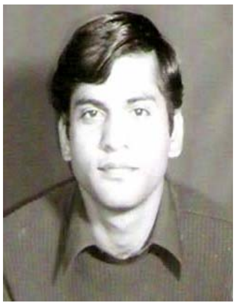

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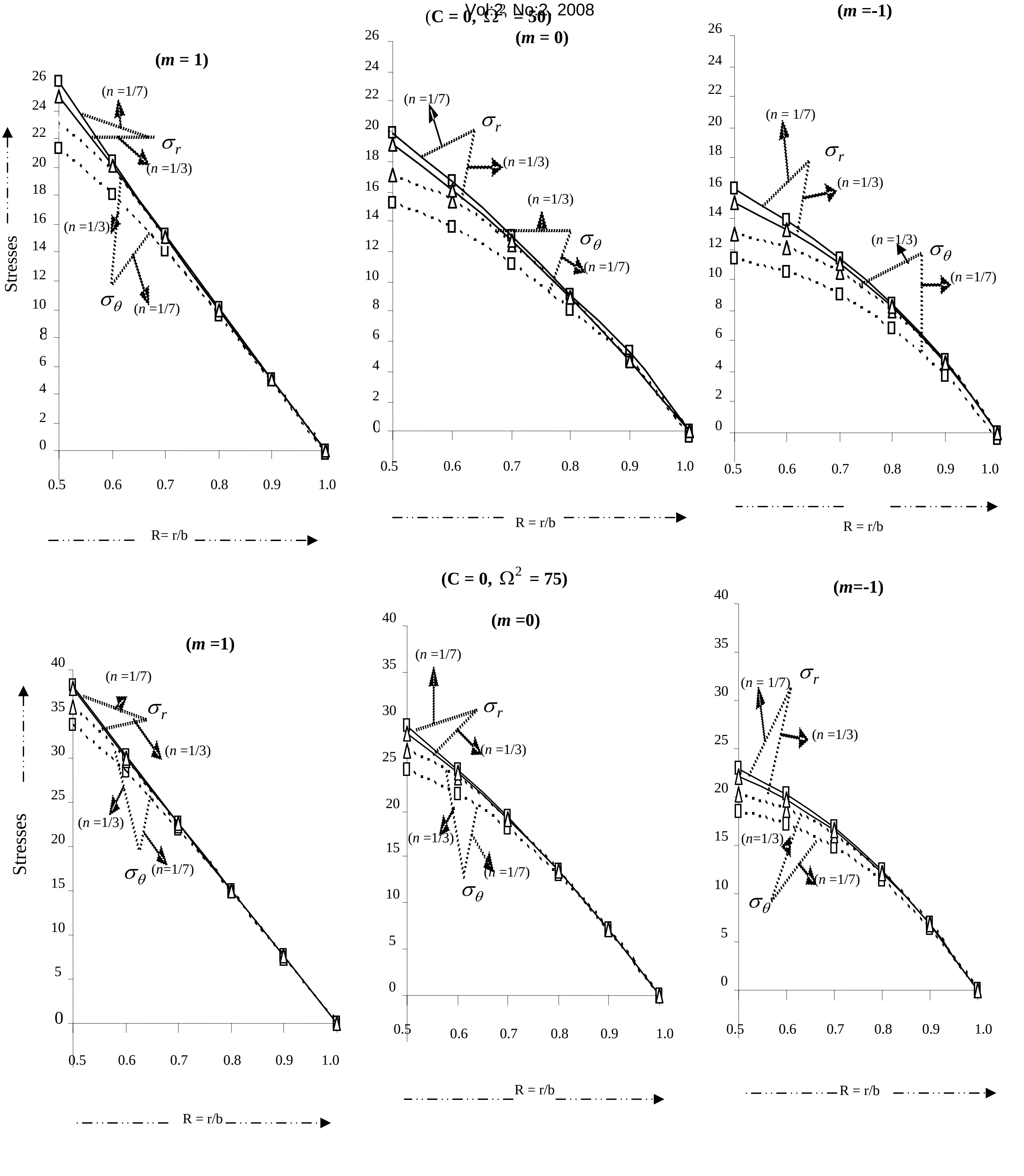


Fig. 2 Creep stresses in a thin rotating disc with rigid inclusion having variable density for incompressible material at different angular speed Ω^2 = 75, 50 along the radius R = r/b

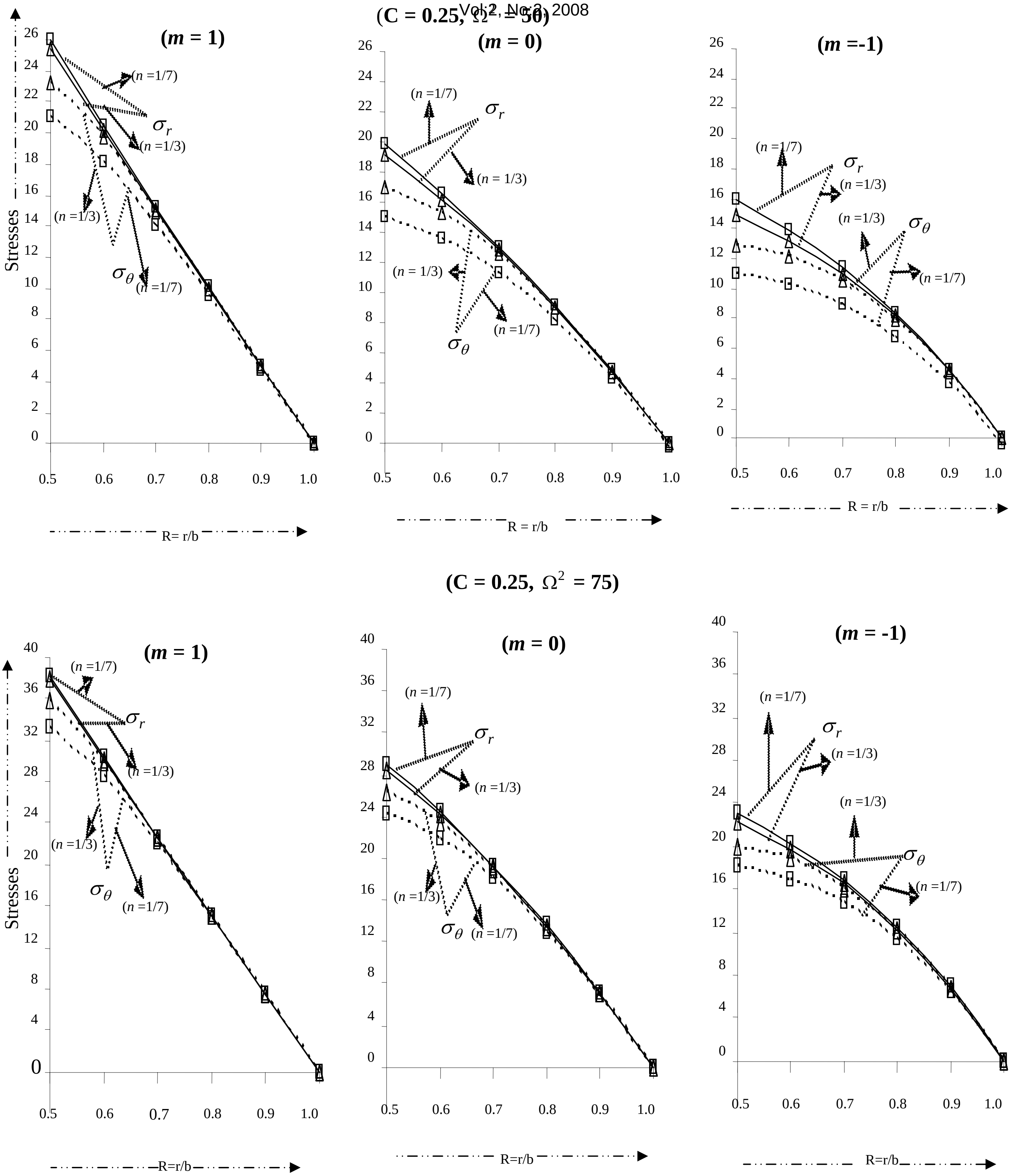
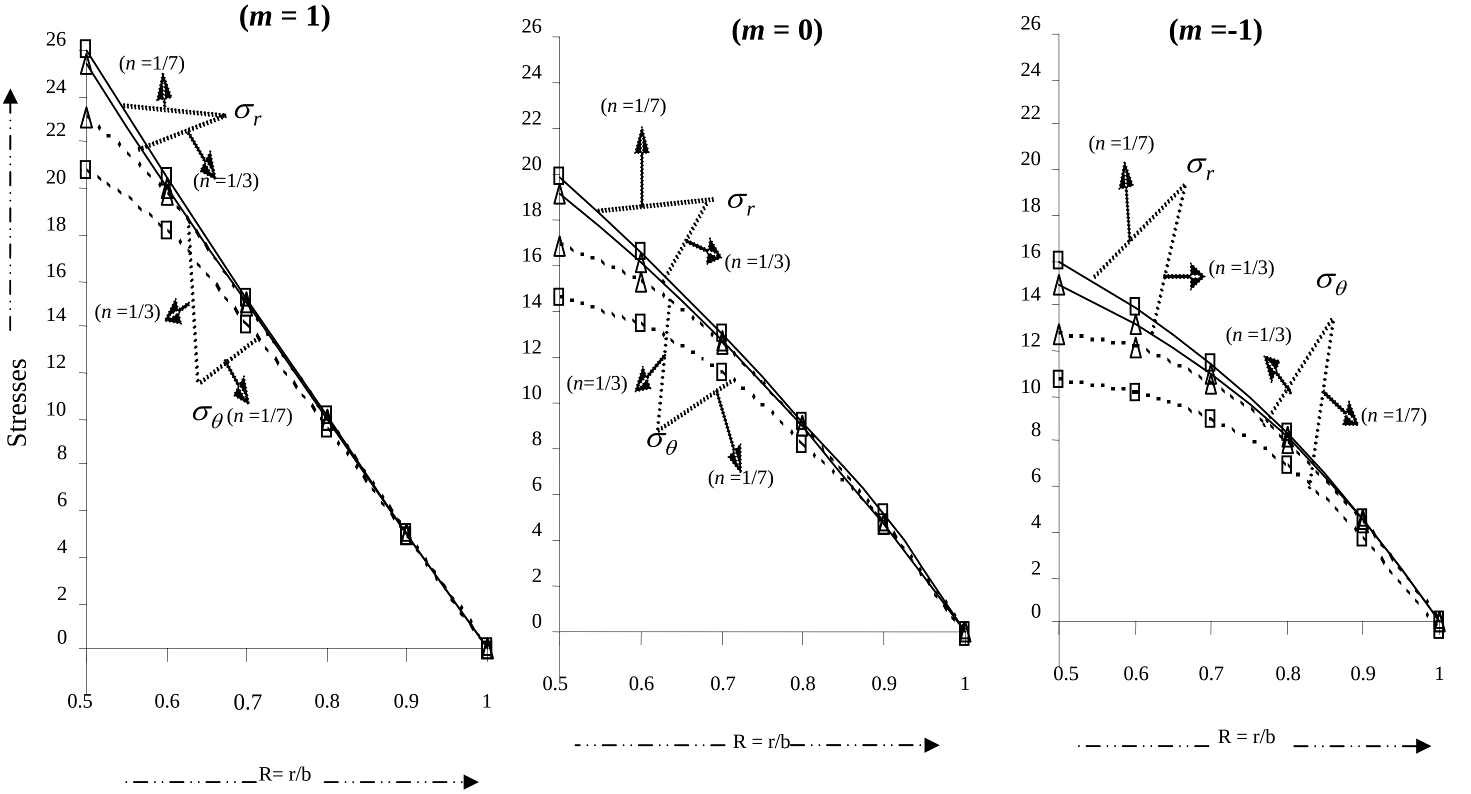


Fig. 3 Creep stresses in a thin rotating disc with rigid inclusion having variable density for compressible material at different angular speed Ω^2 = 75, 50 along the radius R = r/b



(C = 0.5, Ω^2 = 75)

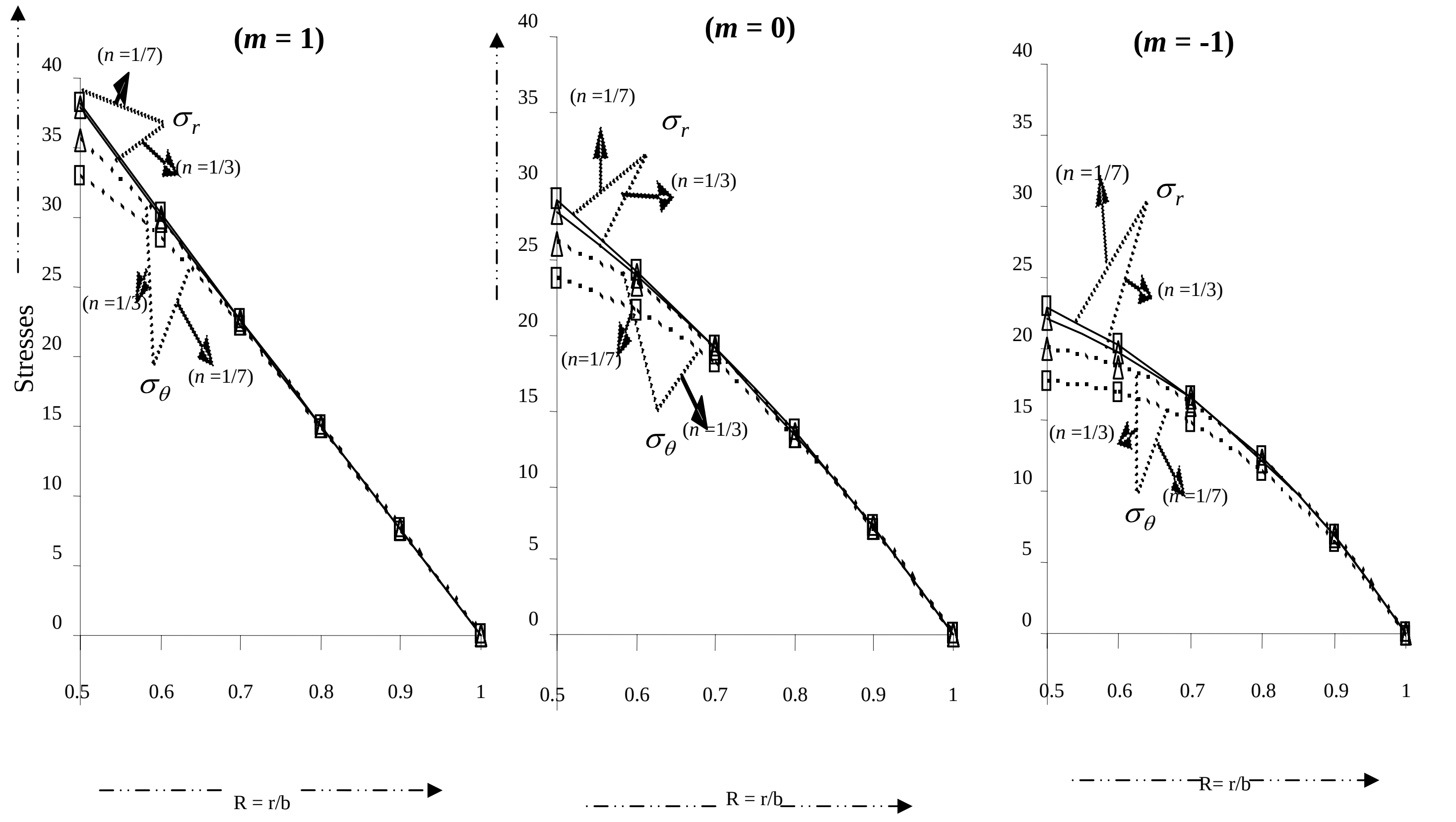


Fig. 4 Creep stresses in a thin rotating disc with rigid inclusion having variable density for compressible material at different angular speed Ω^2 = 75, 50 along the radius R = r/b

(C = 0.5, Ω^2 = 50)

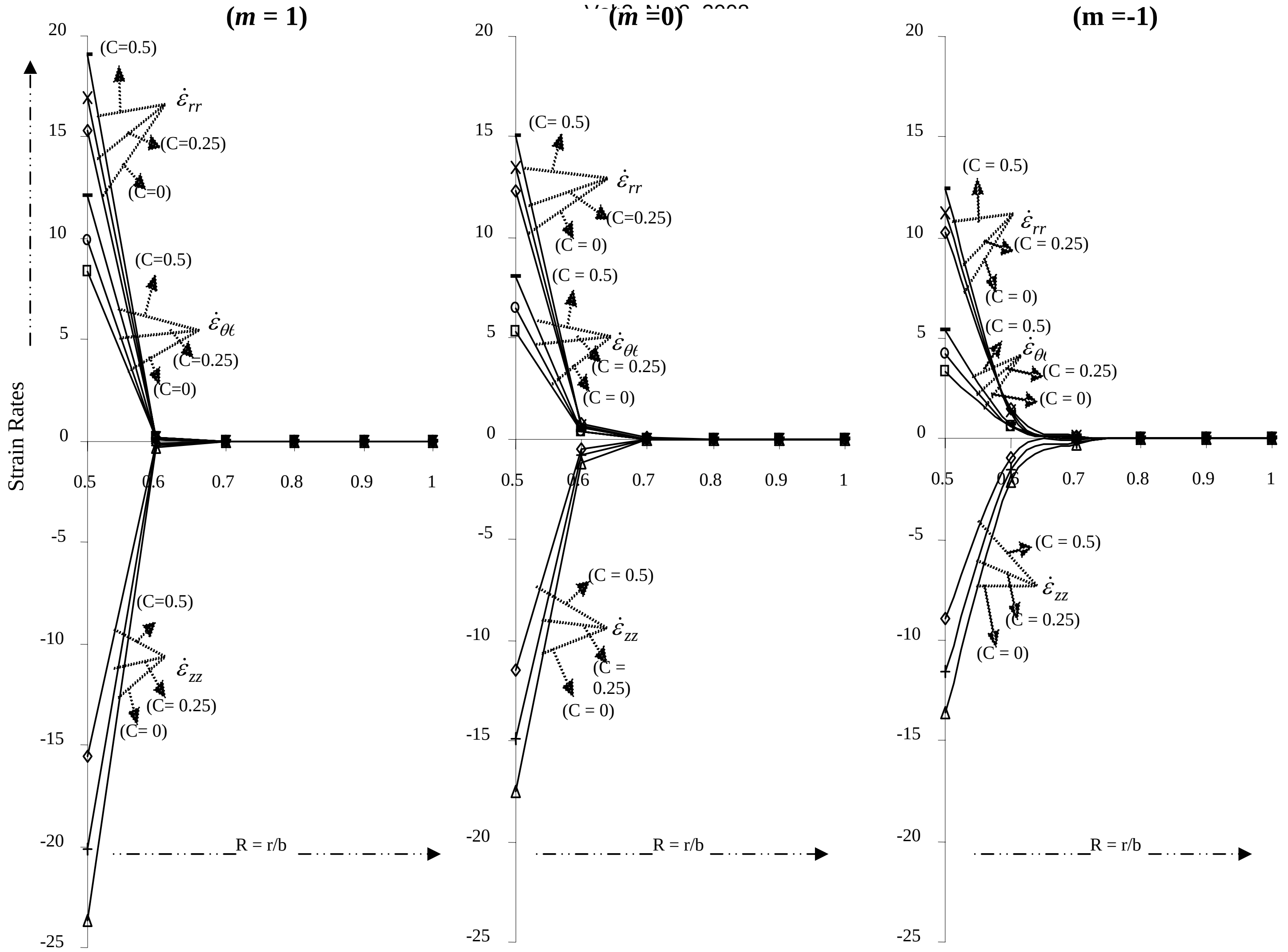


Fig. 5 Strain rate components for a thin rotating disc with inclusion having variable density for measure n = 1/7 at angular speed Ω^2 = 50 along the radius R = r/b

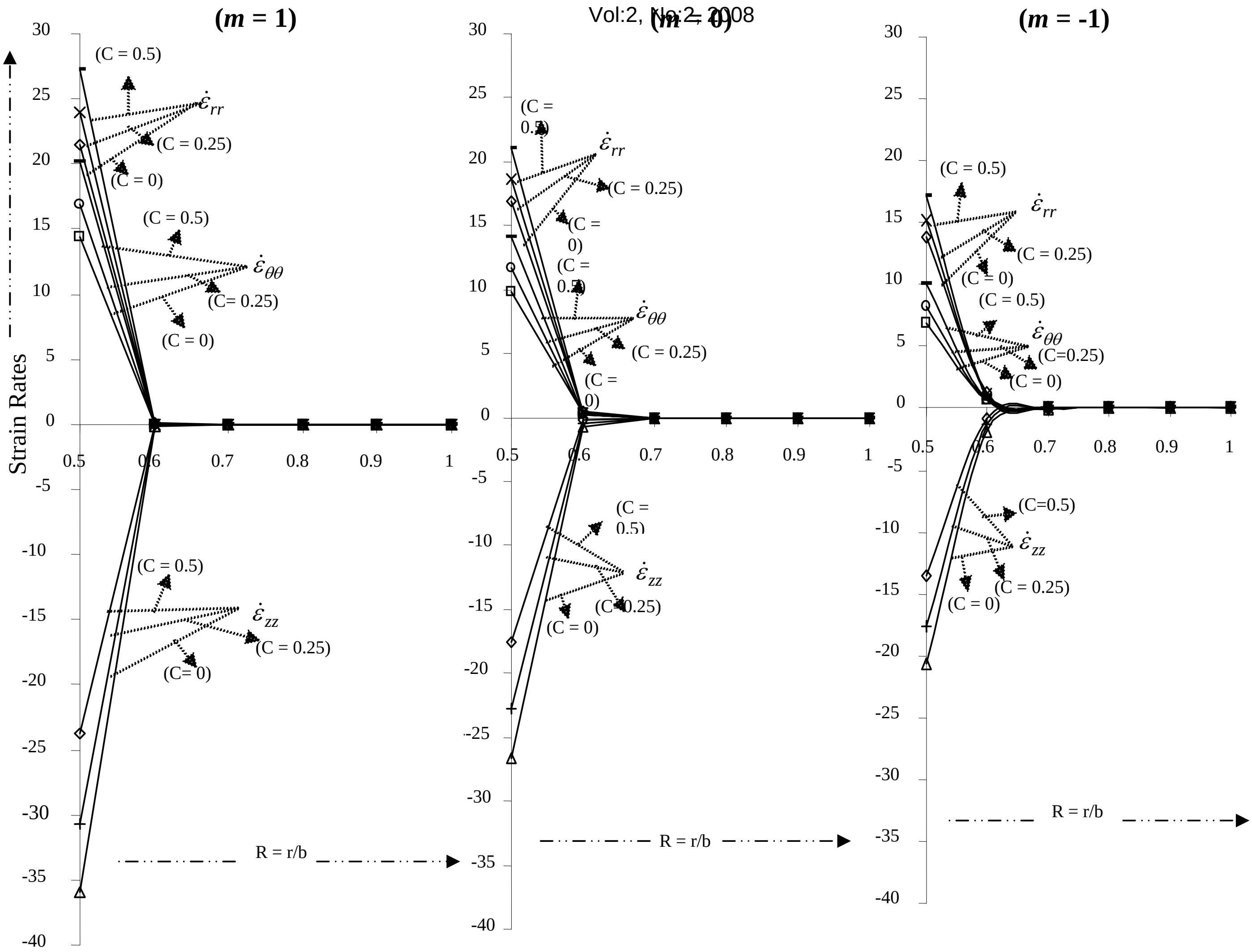


Fig. 6 Strain rate components for a thin rotating disc with inclusion having variable density for measure n = 1/7 at angular speed Ω^2 =75 along the radius R = r/b