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An Eulerian Numerical Method and its Application to Explosion Problems

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Abstract**—**The Eulerian numerical method is proposed to analyze the explosion in tunnel. Based on this method, an original software M-MMIC2D is developed by C++ program language. With this software, the explosion problem in the tunnel with three expansion-chambers is numerically simulated, and the results are found to be in full agreement with the observed experimental data.

Keywords**—**Eulerian method, numerical simulation, shock wave, tunnel

I. Introduction

THE process of explosion and impact under extreme conditions of high speed, high temperature and high pressure often involves material large deformation, multi-material coupling, interface treatment between multi-material, strong discontinuity and nonlinearities. It is a complicated physical and chemical process because of its instantaneous characters, and it is very difficult to get accurate results. Besides theoretical and experimental approaches, numerical simulations begin to play an important role in studying these phenomena with the rapid progress in computer software. The commonly used methods for these large distortion problems are Eulerian method. It uses a fixed mesh and can eliminate distorted mesh which is caused by interface large deformation. Therefore, Eulerian numerical method has obvious advantages for explosion and impact problems involving large deformation.

II. Governing Equations

A. Conservation Equations

The explosion problems can be described by 2-D unsteady elastic-plastic hydrodynamics equations as follows: .
The equation of conservation of mass:

$$\frac{\partial \rho}{\partial t}+u_z\frac{\partial \rho}{\partial t}+u_r\frac{\partial \rho}{\partial t}+\rho\left(\frac{\partial u_z}{\partial z}+\frac{\partial r u_r}{r\partial r}\right)=0 \quad (1)$$

The equation of conservation of momentum:

$$\rho\left(\frac{\partial u_z}{\partial t}+u_z\frac{\partial u_z}{\partial z}+u_r\frac{\partial u_z}{\partial r}\right)=-\frac{\partial p}{\partial z}+\frac{\partial S_{zz}}{\partial z}+\frac{\partial (r S_{rz})}{r\partial r} \quad (2)$$

$$\rho\left(\frac{\partial u_r}{\partial t}+u\frac{\partial u_r}{\partial z}+v\frac{\partial u_r}{\partial r}\right)=-\frac{\partial p}{\partial r}+\frac{\partial S_{rz}}{\partial z}+\frac{\partial (r S_{rr})}{r\partial r}-\frac{S_{\theta\theta}}{r} \quad (3)$$

The equation of conservation of energy:

$$\rho\left(\frac{\partial e}{\partial t}+u_z\frac{\partial e}{\partial z}+u_r\frac{\partial e}{\partial r}\right)=-\rho\left(\frac{\partial (r u_r)}{r\partial r}+\frac{\partial u_z}{\partial z}\right)+S_{zz}\frac{\partial u_z}{\partial z}+S_{rr}\frac{\partial u_r}{\partial r}+S_{rz}\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)+\frac{u_r S_{\theta\theta}}{r} \quad (4)$$

Where t is time; r and z are radial and axial coordinate of the cylindrical coordinate system respectively; u_r , u_z are the radial and axial component of velocity; ρ, p ,e are density, pressure and specific internal energy respectively; S_{rr}, S_{zz}, S_{rz} are components of stress deviator tensor.

B. Constitutive relation

Within the range of elasticity, partial stresses are defined by the general Hooke principle.

$$\dot{S}_{rr}=2G\left(\dot{\varepsilon}_{rr}-\frac{1}{3}D\right)+2R'S_{rz} \quad (5)$$

$$\dot{S}_{zz}=2G\left(\dot{\varepsilon}_{zz}-\frac{1}{3}D\right)+2R'S_{rz} \quad (6)$$

$$\dot{S}_{rz}=2G\dot{\varepsilon}_{rz}+R'(S_{zz}-S_{rr}) \quad (7)$$

Here G is shear modulus; $\dot{\varepsilon}_{rr},\dot{\varepsilon}_{zz},\dot{\varepsilon}_{rz}$ the components of strain rates. In the cylindrical coordinate, $\dot{\varepsilon}_{rr},\dot{\varepsilon}_{zz},\dot{\varepsilon}_{rz}$, D, and R can be expressed as follows:

$$\dot{\varepsilon}_{rr}=\frac{\partial u_r}{\partial r} \quad (8)$$

$$\dot{\varepsilon}_{zz}=\frac{\partial u_z}{\partial z} \quad (9)$$

$$\dot{\varepsilon}_{rz}=\frac{1}{2}\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right) \quad (10)$$

$$D=\frac{\partial (r u_r)}{r\partial r}+\frac{\partial (u_z)}{\partial z} \quad (11)$$

$$R'=\frac{1}{2}\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right) \quad (12)$$

C. State Equations

For air, the state equation of perfect gas is adopted:

$$P=(k_a-1)\rho\cdot e \quad (13)$$

Where k_a is the isentropic index of the air, $k_a=1.4$

For the detonation products, the JWL state equation is adopted:

$$P=A\left(1-\frac{\omega}{R_1 V}\right)e^{-R_1 V}+B\left(1-\frac{\omega}{R_2 V}\right)e^{-R_2 V}+\frac{\omega E}{V} \quad (14)$$

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Where P is the pressure; E is the internal energy; V is the relative volume. A, B, R_1, R_2, ω are known constants. The parameters of JWL equation of state are listed in Table I.

TABLE I
PARAMETERS OF STATE EQUATION FOR THE DETONATION PRODUCTS

ρ /(g.cm^{-3})	1.60
P_{CJ}/GPa	18.5
D_{CJ}/(m.s^{-1})	6900
A/GPa	371.2
B/ GPa	3.231
R_1	4.15
R_2	0.95
ω	0.30

D. The Calculation of Multi-Material Interface

There are explosive, concrete and air in the computational field. Two or three kinds of mediums may exist in a same grid. For the mixed grid with two materials, Young's interface technology is adopted [1]: using a line to stand for the interface, the slope of the line is determined by the material distribution in the surrounding eight cells, and the position of the line is determined by the portion of volume in a cell. For the mixed grid with three materials, SLIC (simple line interface calculation) is adopted [2].

III. NUMERICAL METHOD

In computation, the "sum" splitting scheme is used, and the forward difference scheme and center difference scheme are adopted for time and space respectively. For each direction, the computation of equations can be divided into two steps: Lagrange step and Transport step (Euler step).

In Lagrange step, considering the pressure gradient's and partial stress's effect, the Lagrange velocity and internal energy can be obtained. We take the r direction for example. We have the following difference equation:

$$\tilde{u}_{ri,j}^{n+1} = u_{ri,j}^n + \frac{\Delta t^n}{\rho_{i,j}^n}[-\frac{1}{\Delta r_{i,j}}\left(p_{i,j+\frac{1}{2}}^n + q_{ri,j}^n - p_{i,j-\frac{1}{2}}^n - q_{ri,j-1}^n\right) + \frac{1}{r_{i,j}\Delta r_{i,j}}\left(r_{i,j+\frac{1}{2}} S^n_{rri,j+\frac{1}{2}} - r_{i,j-\frac{1}{2}} S^n_{rri,j-\frac{1}{2}}\right) + \frac{1}{\Delta z_{i,j}}\left(S^n_{rzi+\frac{1}{2},j} - S^n_{rzi-\frac{1}{2},j}\right) + \frac{S^n_{zzi,j} + S^n_{rri,j}}{r_{i,j}}] \quad (15)$$

We can get $\tilde{u}_{zi,j}^{n+1}$, $\tilde{e}_{i,j}^{n+1}$ by the same method.

The increase of specific internal energy $\Delta e_{i,j}$ is:

$$\Delta e_{i,j} = \tilde{e}_{i,j}^{n+1} - e_{i,j}^n \quad (16)$$

Where t is time; r ,z are Euler coordinate system, r is radical direction, z is axial direction; u_r , u_z are the velocity components in r direction and z direction; ρ, p ,e are density, pressure and specific internal energy respectively; S_{rr}, S_{zz}, S_{rz}, are deviatoric stresses.

In Transport step, calculating the transportation of mass, momentum, energy carried in or out through the grid boundaries, and the final values of these physical parameters of the mediums in grid can be gained.

First, we need to calculate the transportation velocity $\hat{u}_{ri,j}$, that is:

$$\hat{u}_{ri,j} = \frac{\tilde{u}_{ri,j+1}\Delta r_{i,j} + \tilde{u}_{ri,j}\Delta r_{i,j+1}}{\Delta r_{i,j} + \Delta r_{i,j+1} + 2\delta\Delta t\left(\tilde{u}_{ri,j+1} - \tilde{u}_{ri,j}\right)} \quad (16)$$

Then, the transportation volume is gained:

$$\Delta V_r = \hat{u}_{ri,j} S_{i,j}^r \Delta t \quad (18)$$

Through the interface of mediums in the grid, we can get the transportation volume of every medium. After calculating the transportation volume, the values of transportation mass, momentum and energy can be finally obtained.

IV. NUMERICAL RESULTS

In engineering practice, it is often required to evaluate the risk of explosion accidents and take the corresponding protection measures to alleviate the damage of buildings, engineering facilities and human beings from explosions. The shock wave interactions with barriers are crucial in the protection design of mines, tunnels, factories and so on.

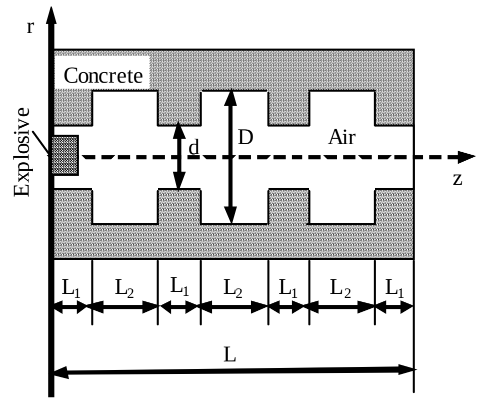


Fig. 1 The physical model

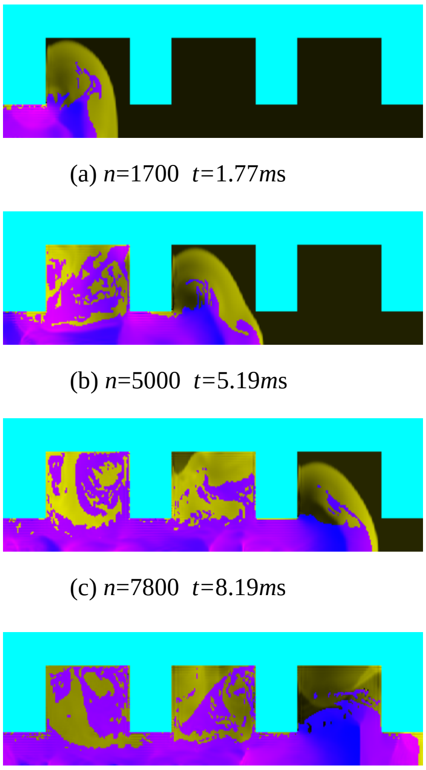
To evaluate the efficiency of the proposed Eulerian method and hydrocode in the simulation of these problems, the simulation of explosions in a tunnel is discussed.

The simplified model is shown in Fig.1. The size of computational region is 25m×5.2m. The diameter of the tunnel is 2.5m. L_1 =2.5m, L_2 =5m. The mass of the explosive that placed on the entrance of the tunnel is 450kg. The computational region is divided into 500×104 rectangular Eulerian mesh. The length of the mesh is 0.05m. The simulation results of shock waves propagation in tunnel are visualized[3] in Fig.2. The pressure-time curves are shown in Fig.3 and Fig.4.

The reflection and disturbance of the shock wave in the expansion-chamber can be seen clearly from the Fig.2. When shock wave enters the expansion-chamber, air streams move around the corners and form the swirl.

As time passes, the expansion disturbance of the shock wave in expansion-chambers and the several reflections of this disturbance in tunnel induce the complicated wave systems and present many wave tops.

It can be seen from the Fig.3 and Fig.4 that the peak of the pressure decreases gradually in the propagation process of the shock wave along tunnel. Obviously, expansion-chamber makes the energy of shock waves dissipate quickly.



(a) n=1700 t=1.77*ms*

(b) n=5000 t=5.19*ms*

(c) n=7800 t=8.19*ms*

(d) n=10000 t=12.09*ms*

Fig. 2 Propagation of shock waves in tunnel

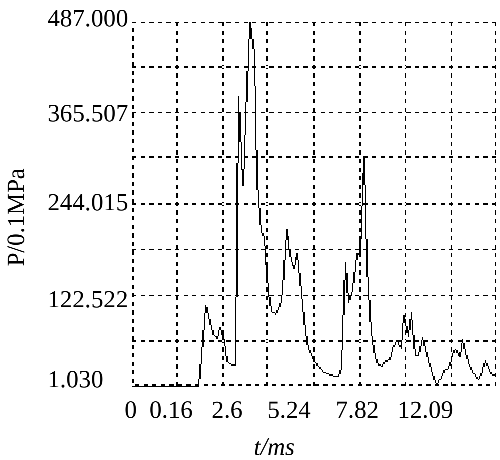


Fig. 3 The presser-time curve at Exit of the first expansion-chamber

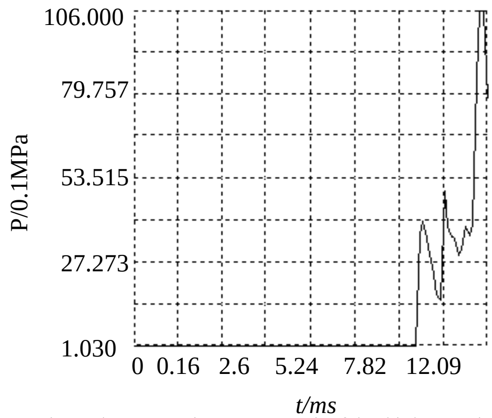


Fig. 4 The presser-time curve at exit of the third expansion-chamber

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