International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:1, No:9, 2007

Modeling and Simulating of Gas Turbine Cooled Blades

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***Abstract*—**In contrast to existing methods which do not take into account multiconnectivity in a broad sense of this term, we develop mathematical models and highly effective combination (BIEM and FDM) numerical methods of calculation of stationary and quasi-stationary temperature field of a profile part of a blade with convective cooling (from the point of view of realization on PC). The theoretical substantiation of these methods is proved by appropriate theorems. For it, converging quadrature processes have been developed and the estimations of errors in the terms of A.Ziqmound continuity modules have been received.

For visualization of profiles are used: the method of the least squares with automatic conjecture, device spline, smooth replenishment and neural nets. Boundary conditions of heat exchange are determined from the solution of the corresponding integral equations and empirical relationships. The reliability of designed methods is proved by calculation and experimental investigations heat and hydraulic characteristics of the gas turbine first stage nozzle blade.

***Keywords*—**Modeling, Simulating, Gas Turbine, Cooled Blades.

I. INTRODUCTION

THE development of aviation gas turbine engines (AGTE) at the present stage is mainly reached by assimilation of high values of gas temperature in front of the turbine ($T_Г$). The activities on gas temperature increase are conducted in several directions. Assimilation of high ($T_Г$) in AGTE is however reached by refinement of cooling systems of turbine blades. It is especially necessary to note, that with $T_Г$ increase the requirement to accuracy of results will increase. In other words, at allowed values of AGTE metal temperature $T_{lim} = (1100...1300K)$, the absolute error of temperature calculation should be in limits ($20-30K$), that is no more than 2-3%.

This is difficult to achieve (multiconnected fields with various cooling channels, variables in time and coordinates boundary conditions). Such problem solving requires application of modern and perfect mathematical device.

II. PROBLEM FORMULATION

In classical statement a heat conduction differential equation in common case for non-stationary process with distribution of heat in multi–dimensional area (Fourier-Kirchhoff equation) has a kind [1]:

$$\frac{\partial(\rho C_v T)}{\partial t} = div(\lambda\, grad T) + q_v, \tag{1}$$

where ρ, c_v and λ - accordingly material density, thermal capacity, and heat conduction; q_v - internal source or drain of heat, and T - is required temperature.

Research has established that the temperature condition of the blade profile part with radial cooling channels can be determined as two-dimensional [2]. Besides, if to suppose constancy of physical properties and absence of internal sources (drains) of heat, then the temperature field under fixed conditions will depend only on the skew shape and on the temperature distribution on the skew boundaries. In this case, equation (1) will look like:

$$\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{2}$$

When determining particular temperature fields in gas turbine elements are used boundary conditions of the third kind, describing heat exchange between the skew field and the environment (on the basis of a hypothesis of a Newton-Riemann). In that case, these boundary conditions will be recorded as follows:

$$\alpha_0 (T_0 - T_{\gamma_0}) = \lambda \frac{\partial T_{\gamma_0}}{\partial n} \tag{3}$$

This following equation characterizes the quantity of heat transmitted by convection from gas to unit of a surface of a blade and assigned by heat conduction in a skew field of a blade.

$$-\lambda \frac{\partial T_{\gamma_i}}{\partial n} = \alpha_i (T_{\gamma_i} - T_i) \tag{4}$$

Equation (4) characterizes the heat quantity assigned by convection of the cooler, which is transmitted by heat conduction of the blade material to the surface of cooling channels: where T_0 - temperature of environment at $i = 0$; T_i - temperature of the environment at $i = \overline{1,M}$ (temperature of

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the cooler), where M - quantity of outlines; T_{γ_0} - temperature on an outline γ_i at $i = 0$ (outside outline of blade); T_{γ_i} - temperature on an γ_i at $i = \overline{1, M}$ (outline of cooling channels); α_0 - heat transfer factor from gas to a surface of a blade (at $i = 0$); α_i - heat transfer factor from a blade to the cooling air at $i = \overline{1, M}$; λ - thermal conductivity of the material of a blade; n - external normal on an outline of researched area.

III. Problem Solution

At present for the solution of this boundary problem (2)-(4) four numerical methods are used: Methods of Finite Differences (MFD), Finite Element Method (FEM), probabilistic method (Monte-Carlo method), and Boundary Integral Equations Method (BIEM) (or its discrete analog — Boundary Element Method (BEM)).

Let us consider BIEM application for the solution of problem (2)-(4).

3.1. In contrast to [4], we offer to decide the given boundary value problem (2)-(4) as follows. We locate the distribution of temperature $T = T(x, y)$ as follows:

$$T(x, y) = \int_{\Gamma} \rho \ell n R^{-1} ds, \tag{5}$$

where $\Gamma = \bigcup_{i=0}^{M} \gamma_i$ -smooth closed Jordan curve; M -quantity of cooled channels; $\rho = \bigcup_{i=0}^{M} \rho_i$ - density of a logarithmic potential uniformly distributed on γ_i $S = \bigcup_{i=0}^{M} s_i$.

Thus curve $\Gamma = \bigcup_{i=0}^{M} \gamma_i$ are positively oriented and are given in a parametric kind: $x = x(s)$; $y = y(s)$; $s \in [0, L]$; $L = \int_{\Gamma} ds$.

Using BIEM and expression (5) we shall put problem (2)-(4) to the following system of boundary integral equations:

$$\rho(s) - \frac{1}{2\pi} \int_{\Gamma} (\rho(s) - \rho(\xi)) \frac{\partial}{\partial n} \ell n R(s, \xi) d\xi = \frac{\alpha_i}{2\pi\lambda} (T - \int_{\Gamma} \rho(s) \ell n R^{-1} ds), \tag{6}$$

where

$$R(s, \xi) = ((x(s) - x(\xi))^2 + (y(s) - y(\xi))^2)^{1/2}.$$

For the singular integral operator's evaluation, which are included in (6) the discrete operators of the logarithmic potential with simple and double layer are investigated. Their connection and the evaluations in modules term of the continuity (evaluation such as assessments by A. Zigmound are obtained) is shown [1,6]. Thus are developed effective from the point of view of realization on computers the numerical methods basing on constructed two-parametric quadratute processes for the discrete operators logarithmic potential of the double and simple layer. Their systematic errors are estimated, the methods quadratures mathematically are proved for the approximate solution Fredholm I and II boundary integral equations using Tikhonov regularization and are proved appropriate theorems [1].

3.2. The given calculating technique of the blade temperature field can be applied also to blades with the plug–in deflector. On consideration blades with deflectors in addition to boundary condition of the III kind adjoin also interfaces conditions between segments of the outline partition as equalities of temperatures and heat flows

$$T_v(x, y) = T_{v+1}(x, y),$$

$$\frac{\partial T_v(x, y)}{\partial n} = \frac{\partial T_{v+1}(x, y)}{\partial n},$$

where v - number of segments of the outline partition of the blade cross-section; *x, y* - coordinates of segments. At finding of cooler T best values, is necessary to solve the inverse problem of heat conduction. For it is necessary at first to find solution of the heat conduction direct problem with boundary condition of the III kind from a gas leg and boundary conditions I kinds from a cooling air leg

$$T_v(x, y)\big|_{\gamma_0} = T_{i_0},$$

where T_{i_0} -the unknown optimum temperature of a wall of a blade from a leg of a cooling air.

3.3. The developed technique for the numerical solution of stationary task of the heat conduction in cooled blades can be distributed also to quasistationary case.

Let us consider a third boundary-value problem for the heat conduction quasilines equation:

$$\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda(T)\frac{\partial T}{\partial y}\right) = 0 \tag{7}$$

$$\alpha_i(T_{ci} - T_{\gamma i}) - \lambda(T)\frac{\partial T_{\gamma i}}{\partial n} = 0 \tag{8}$$

For linearization of tasks (7) - (8) we shall use the Kirchhoff permutation:

$$A = \int_0^T \lambda(\xi) d\xi \tag{9}$$

Then equation (7) is transformed into the following Laplace equation:

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = 0 \tag{10}$$

For preserving convection additives in boundary-value condition (8) we shall accept in initial approximation $\lambda(T) = \lambda_c$. Then from (9) we have

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:1, No:9, 2007

$$T = A / \lambda c \tag{11}$$

And the regional condition (8) will be transformed as follows:

$$\alpha_i (T_{ci} - A_{\gamma i} / \lambda_c) - \frac{\partial A_{\gamma i}}{\partial n} = 0 \tag{12}$$

So, the stationary problem (10) with (12) is solved by boundary integrated equations method. If the solution $L(x, y)$ in the (x, y) point of the linear third boundary-value problem (10), (12) for the Laplace equation substitute in (9) and after integration to solve the appropriate algebraic equation, which degree is higher than the degree of function $\lambda(T)$ with digit, we shall receive meaning of temperature $T(x, y)$ in the same point. Thus in radicals is solved the algebraic equation with non-above fourth degree

$$a^0 T^4 + a^1 T^3 + a^2 T^2 + a^3 T + a^4 = A. \tag{13}$$

This corresponds to the $\lambda(T)$ which is the multinomial with degree non-above third. In the result, the temperature field will be determined on the first approximation, as the boundary condition (8) took into account constant meaning heat conduction λ_c in convective thermal flows. According to it we shall designate this solution $T^{(I)}$ (accordingly $A^{(I)}$). For determining consequents approximations $A^{(2)}$ (accordingly $T^{(2)}$), the function $A(T)$ is decomposing in Taylor series in the neighborhood of $T^{(I)}$ and the linear members are left in it only. In result is received a third boundary-value problem for the Laplace equation relatively function $A^{(2)}$. The temperature $T^{(2)}$ is determined by the solution of the equation (11).

The multiples computing experiments with the using BIEM for calculation the temperature fields of nozzle and working blades with various amount and disposition of cooling channels, having a complex configuration, is showed, that for practical calculations in this approach, offered by us, the discretization of the integrations areas can be conducted with smaller quantity of discrete points. Thus the reactivity of the algorithms developed and accuracy of evaluations is increased. The accuracy of temperatures calculation, required consumption of the cooling air, heat flows, and losses from cooling margins essentially depends on reliability of boundary conditions, included in calculation of heat exchange.

3.4 Piece-polynomial smoothing of cooled gas-turbine blade structures with automatic conjecture is considered: the method of the least squares, device spline, smooth replenishment, and neural nets are used.

A new approach of mathematical models' parameters identification is considered. This approach is based on Neural Networks (Soft Computing) [7-9].

Let us consider the regression equations:

$$Y_i = \sum_{j=1}^{n} a_{ij} \otimes x_j; \ i = \overline{1, m} \tag{14}$$

$$Y_i = \sum_{r,s} a_{rs} \otimes x_1^r \otimes x_2^s; \ r = \overline{0, l}; \ s = \overline{0, l}; \ r + s \le l \tag{15}$$

where a_{rs} are the required parameters (regression coefficients).

The problem is put definition of values a_{ij} and a_{rs} parameters of equations (14) and (15) based on the statistical experimental data, i.e. input x_j and x_1, x_2, output coordinates Y of the model.

Neural Network (NN) consists from connected between their neurons sets. At using NN for the solving (14) and (15) input signals of the network are accordingly values of variables $X = (x_1, x_2, ..., x_n)$, $X = (x_1, x_2)$ and output Y. As parameters of the network are a_{ij} and a_{rs} parameters' values.

At the solving of the identification problem of parameters a_{ij} and a_{rs} for the equations (14) and (15) with using NN, the basic problem is training the last.

We allow, there are statistical data from experiments. On the basis of these input and output data we making training pairs (X, T) for network training. For construction of the model process on input of NN input signals X move and outputs are compared with reference output signals T.

After comparison, the deviation value is calculating by formula

$$E = \frac{1}{2} \sum_{j=1}^{k} (Y_j - T_j)^2$$

If for all training pairs, deviation value E less given then training (correction) parameters of a network comes to end. In opposite case it continues until value E will not reach minimum.

Correction of network parameters for left and right part is carried out as follows:

$$a_{rs}^{н} = a_{rs}^{c} + \gamma \frac{\partial E}{\partial a_{rs}},$$

where $a_{rs}^{c}, a_{rs}^{н}$ are the old and new values of NN parameters and γ is training speed.

3.5. For determining of the temperature fields of AGTE elements, the problem of gas flow distribution on blades' profile of the turbine cascade is considered. The solution is based on the numerical realization of the Fredholm boundary integrated equation II kind.

On the basis of the theory of the potential flow of cascades, distribution of speed along the profile contour can be found by solving of the following integrated equation [10]:

$$\varphi(x_k, y_k) = V_\infty (x_k \cos\alpha_\infty + y_k \sin\alpha_\infty) \pm \frac{1}{2\pi}\Gamma\theta_B \mp \frac{1}{2\pi}\oint_{S+}\varphi(S)d\theta \quad (16)$$

where $\varphi(x_k, y_k)$ -the value of speeds potential; V_∞ - the gas speed vectors mean on the flowing; α_∞ - the angle between the vector V_∞ and the profile cascade axis; Γ - the circulation of speed; θ_B - the angle that corresponds to the outlet edge of the profile.

The value of the gas flow speed is determined by the derivation of speeds potential along the contour s, i.e. $V(s) = d\varphi/ds$.

Distribution of speed along the profile contour can be determined by solving the integral equation for the current function ψ [10, 11]:

$$\psi = V_\infty (y\cos\alpha_\infty - x\sin\alpha_\infty) \mp \frac{1}{2\pi}\oint_{S+} V \ln\sqrt{sh^2\tfrac{\pi}{t}(x - x_k) + \sin^2\tfrac{\pi}{t}(y - y_k)}ds \quad (17)$$

3.6. The data of speed distribution along the profile contour are incoming for determining outer boundary heat exchange conditions.

The problem of determining inner boundary heat exchange conditions is necessary. To calculate heat transfer in the cooling channel track of the vanes usually is applied criterial relationships [5, 12, 13].

At known geometry of the cooling scheme, for definition of the convective heat exchange local coefficients α_B of the cooler by the standard empirical formulas, is necessary to have income values of air flow distribution in cooling channels.

To determine the distribution of flow in the blade cooling system, an equivalent hydraulic scheme is built.

The construction of the equivalent hydraulic tract circuit of the vane cooling is connected with the description of the cooled vane design. The whole passage of coolant flow is divided in some definite interconnected sections, the so-called typical elements, and every one has the possibility of identical definition of hydraulic resistance. The points of connection of typical elements are changed by node points, in which the streams, mergion or division of cooler flows is taking places proposal without pressure change. All the typical elements and node points are connected in the same sequence and order as the tract sites of the cooled vane.

To describe the coolant flow at every inner node the 1st low by Kirchhoff is used:

$$f_1 = \sum_{j=1}^{m} G_{ij} = \sum_{j=1}^{m} sign(\Delta p_{ij}) k_{ij}\sqrt{\Delta p_{ij}}; \quad i = 1,2,3...n \quad (18)$$

where G_{ij} is the discharge of coolant on the element, $i - j$, m are the e number of typical elements connected to i node of the circuit, n is the number of inner nodes of hydraulic circuit, Δp_{ij} - losses of total pressure of the coolant on element $i - j$. In this formula the coefficient of hydraulic conductivity of the circuit element ($i - j$) is defined as:

$$k_{ij} = \sqrt{2 f_{ij}^2 \cdot p_{ij} / \xi_{ij}}, \quad (19)$$

where f_{ij}, p_{ij}, ξ_{ij} are the mean area of the cross-section passage of elements ($i - j$), density of coolant flow in the element, and coefficient of hydraulic resistance of this element. The system of nonlinear algebraic equations (18) is solved by the Zeidel method with acceleration, taken from:

$$p_i^{k+1} = p_i^k - f_i^k / (\partial f / \partial p)^k,$$

where k is the iteration number, p_i^k is the coolant pressure in i node of the hydraulic circuit. The coefficients of hydraulic resistance ξ_{ij} used in (19) are defined by analytical dependencies, which are in the literature available at present [12].

IV. Results

The developed techniques of profiling, calculation of temperature fields and parameters of the cooler in cooling systems are approved at research of the gas turbine first stage nozzle blades thermal condition. Thus the following geometrical and regime parameters of the stage are used: step of the cascade - $t = 41.5$ мм, inlet gas speed to cascade - $V_1 = 156$ м$/s$, outlet gas speed from cascade - $V_2 = 512$ м$/s$, inlet gas speed vector angle - $\alpha_1 = 0.7^0$, gas flow temperature and pressure: on the entrance to the stage - $T_г^* = 1333\,K$, $p_г^* = 1.2095 \cdot 10^6\,Pa$, on the exit from stage - $T_{г1} = 1005\,K$, $p_{г1} = 0.75 \cdot 10^6\,Pa$; relative gas speed on the exit from the cascade - $\lambda_{1ad} = 0.891$.

The geometrical model of the nozzle blades (Fig. 1), diagrams of speed distributions V and convective heat exchange local coefficients of gas $\alpha_г$ along profile contour (Fig. 2) are received.

The geometrical model (Fig. 3) and the cooling tract equivalent hydraulic scheme (Fig. 4) are developed. Cooler basics parameters in the cooling system and temperature field of blade cross section (Fig. 5) are determined.

The reliability of the methods was proved by experimental investigations heat and hydraulic characteristics of blades in "Turbine Construction" (Laboratory in St. Petersburg, Russia). Geometric model, equivalent hydraulic schemes of cooling tracks have been obtained, cooler parameters and temperature field of "Turbo machinery Plant" enterprise (Yekaterinburg, Russia) gas turbine nozzle blade of the 1st stage have been determined. Methods have demonstrated high efficiency at repeated and polivariant calculations, on the basis of which has been offered the way of blade cooling system modernization.

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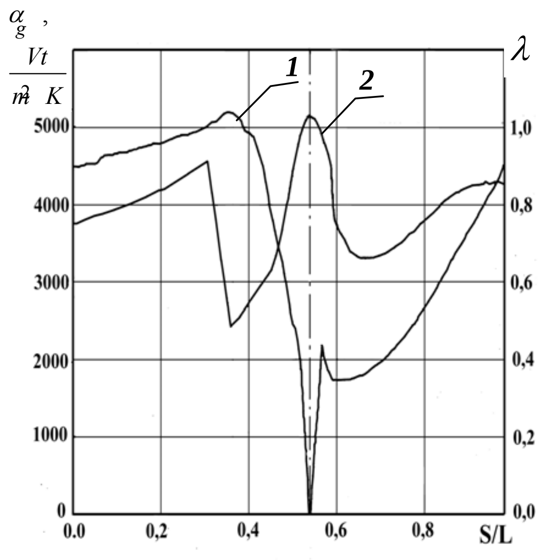

Fig. 2 Distribution of the relative speeds (1) and of gas convective heat exchange coefficients (2) along the periphery of the profile contour

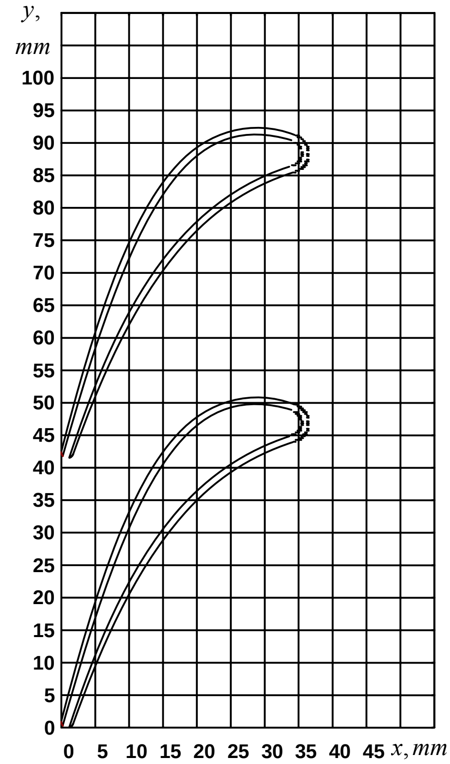

Fig. 1 The cascade of profiles of the nozzle cooled blade

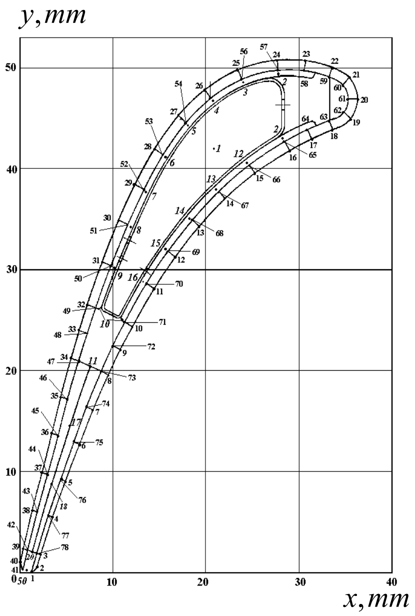

Fig. 3 Geometrical model with foliation of design points of contour (1-78) and equivalent hydraulic schemes reference sections *(1-50)* of the experimental nozzle blade

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:1, No:9, 2007

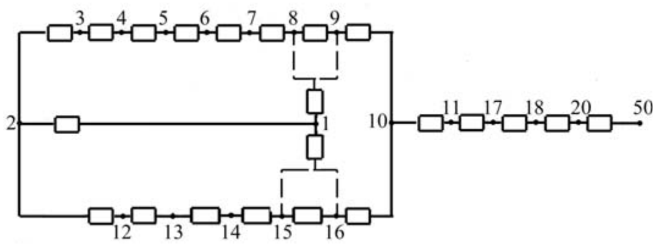

Fig. 4 The equivalent hydraulic scheme of experimental nozzle blade cooling system

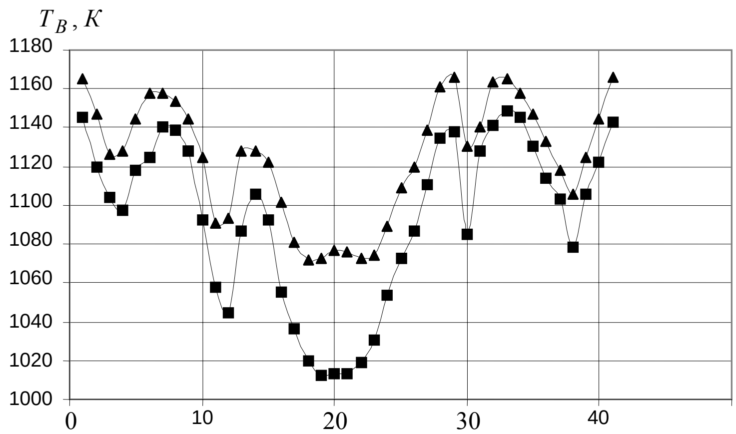

Fig. 5 Distribution of temperature along outside (▲) and internal (■) contours of the cooled nozzle blade