Delay-Dependent Stability Analysis for Neutral Type Neural Networks with Uncertain Parameters and Time-Varying Delay

Qingqing Wang, Shouming Zhong

Abstract—In this paper, delay-dependent stability analysis for neutral type neural networks with uncertain paramters and time-varying delay is studied. By constructing new Lyapunov-Krasovskii functional and dividing the delay interval into multiple segments, a novel sufficient condition is established to guarantee the globally asymptotically stability of the considered system. Finally, a numerical example is provided to illustrate the usefulness of the proposed main results.

Keywords—Neutral type neural networks, Time-varying delay, Stability, Linear matrix inequality(LMI).

I. INTRODUCTION

NEUTRAL type neural networks have been extensively studied in recent years due to its wide application in contemporary society of science and technology such as image processing, automatic control, pattern recognition,and so on[1,2]. Among various behaviors, the stability has proven to be the most important one that has received considerable research attention, see,for example,[3-20] and references cited therein. On the other hand, systems with uncertain paramters have been attracting increasing research attention [20-22]. In [21], the problem of robust stability criteria for recurrent neural networks with time-varying delays are investigated based on linear matrix inequality (LMI) approach. In addition, the authors in [22] discuss the problem of robust stability for Hopfield neural networks of neutral-type via constructing a new Lyapunov-Krasovskii functional. At recent times, the authors in [23-26] have provided a less conservative stability condition for delayed systems by using delay partitioning approach. The advantage of this approach is to get more tighter upper bound of the terms calculated by time-derivative of Lyapunov functional.

Motivated by the above discussion, in this paper, the stability analysis for neutral-type neural networks with uncertain parameters and time-varying delay is considered. Some novel delay-dependent stability criteria based on LMI for neutral-type neural networks will be proposed by partitioning the delay interval into multiple segments,and constructing new Lyapunov-Krasovskii functional. The obtained criteria are less conservatism which can be easily

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checked by using the MATLAB LMI Toolbox. Finally, in order to show the feasibility of the proposed criteria in this paper,a numerical example is considered.

II. PROBLEM STATEMENT

Consider the following neutral-type neural networks with time varying delays described by

$$\dot{y}(t) = -Ay(t) + Bg(y(t)) + Cg(y(t-\tau(t))) + E\dot{y}(t-d) + I_0 \tag{1}$$

where $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^T \in R^n$ is the neuron state vector,$g(y(t)) = [g_1(y_1(t)), g_2(y_2(t)), \ldots, g_n(y_n(t))]^T$ denotes the neuron activation function ,and $I_0 = [I_1, I_2, \ldots, I_n]^T \in R^n$ is a constant input vector,$A = diag\{a_i\} \in R^n$ is a positive diagonal matrix, $B = (b_{ij})_{n \times n} \in R^n$ is the connection weight matrix, $C = (c_{ij})_{n \times n} \in R^n$, and $E = (e_{ij})_{n \times n} \in R^n$ are the delayed connection weight matrices, d is the constant neutral time delay.

The following assumptions are adopted throughout the paper.

Assumption 1: The delay $\tau(t)$ is time-varying continuous function and satisfies:

$$0 \le \tau(t) \le \tau, \dot{\tau}(t) \le \mu \le 1 \tag{2}$$

Assumption 2: Each neuron activation function $g_i(\cdot), i = 1, 2, \ldots, n$,in (1) satisfies the following condition:

$$l_i^- \le \frac{g_i(\alpha) - g_i(\beta)}{\alpha - \beta} \le l_i^+, \forall \alpha, \beta \in R, \alpha \ne \beta \tag{3}$$

where $l_i^-, l_i^+, i = 1, 2, \ldots, n$ are constants.

Based on Assumption 1-2, it can be easily proven that there exists one equilibrium point for (1) by Brouwer's fixed-point theorem. Assuming that $y^* = [y_1^*, y_2^*, \ldots, y_n^*]^T$is the equilibrium point of (1) and using the transformation $x(\cdot) = y(\cdot) - y^*$,system (1) can be converted to the following system :

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Cf(x(t-\tau(t))) + E\dot{x}(t-d) \tag{4}$$

where
$x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T, f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T$,
$f_i(x_i(\cdot)) = g_i(y_i(\cdot) + y_i^*) - g_i(y_i^*), i = 1, 2, \ldots, n.$
From $Eq.(3), f_i(\cdot)$ satisfies the following condition:

$$l_i^- \le \frac{f_i(\alpha)}{\alpha} \le l_i^+, \forall \alpha \ne 0, i = 1, 2, \ldots, n. \tag{5}$$

Due to the disturbance frequent occurs in many applications,so by translating A, B, C and E to function $A(t), B(t), C(t)$ and $E(t)$ respectively,we have

$$\dot{x}(t) = -A(t)x(t) + B(t)f(x(t)) + C(t)f(x(t - \tau(t)))$$
$$+E(t)\dot{x}(t - d) \quad (6)$$

Assumption 3: Assumption that $A(t) = A + \Delta A(t), B(t) = B + \Delta B(t), C(t) = C + \Delta C(t)$,and $E(t) = E + \Delta E(t)$ $\Delta A(t), \Delta B(t), \Delta C(t), \Delta E(t)$ are unknown constant matrices representing time-varying parametric uncertainties, and are of linear fractional forms:

$$[\Delta A(t), \Delta B(t), \Delta C(t), \Delta E(t)] = DF(t)[\tilde{E}_a, \tilde{E}_b, \tilde{E}_c, \tilde{E}_e] \quad (7)$$

with

$$F^T(t)F(t) \le I \quad (8)$$

where $D, \tilde{E}_a, \tilde{E}_b, \tilde{E}_c, \tilde{E}_e$ are known constant matrices of appropriate dimensions.

Lemma 1 [27]. For any constant positive-definite matrix $M \in R^{n \times n}$ and $h_1 \le h_2$, the following inequalities hold:

$$(h_2 - h_1)\int_{h_1}^{h_2} \dot{x}^T(s)M\dot{x}(s)ds \ge (\int_{h_1}^{h_2}\dot{x}(s)ds)^T M(\int_{h_1}^{h_2}\dot{x}(s)ds) \quad (9)$$

Lemma 2 (Schur complement [28]).For any constant matrix H_1, H_2, H_3,where $H_1 = H_1^T$ and $H_2 = H_2^T > 0$.Then $H_1 + H_3^T H_2^{-1} H_3 < 0$ if and only if $\begin{bmatrix} H_1 & H_3^T \\ H_3 & -H_2 \end{bmatrix} < 0$ or $\begin{bmatrix} -H_2 & H_3 \\ H_3^T & H_1 \end{bmatrix} < 0.$

Lemma 3 [29]. Given symmetric matrices Ω and D_1, D_2, of appropriate dimensions, $\Omega + D_1 F(t) D_2 + D_2^T F^T(t) D_1^T < 0$ for all $F(t)$ satisfying $F^T(t)F(t) \le I$, if and only if there exists some $\varepsilon > 0$ such that $\Omega + \varepsilon^{-1} D_1 D_1^T + \varepsilon D_2^T D_2 < 0.$

III. MAIN RESULTS

In this section, a less conservative delay-dependent stability criterion is obtained on the condition of $\Delta A(t) = \Delta B(t) = \Delta C(t) = \Delta E(t) = 0$ in system (6). For representation convenience, the following notations are introduced:

$$L_1 = diag\{l_1^-, l_2^-, \ldots, l_n^-\}$$
$$L_2 = diag\{l_1^+, l_2^+, \ldots, l_n^+\}$$
$$L_3 = diag\{l_1^- l_1^+, l_2^- l_2^+, \ldots, l_n^- l_n^+\}$$
$$L_4 = diag\{\frac{l_1^- + l_1^+}{2}, \frac{l_2^- + l_2^+}{2}, \ldots, \frac{l_n^- + l_n^+}{2}\}$$

Theorem 1 Given that the Assumption 1-2 hold,the system (6) globally asymptotically stable if there exist symmetric positive definite matrices $P, Q_1, Q_2, Q_3, Q_4, R_1, R_2, R_3, \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}$ positive diagonal matrices $W = diag\{w_i\}, \Lambda = diag\{\delta_i\},$

K_1, K_2 with appropriate dimensions, such that the following LMIs holds:

$$\Omega = \begin{bmatrix} \Omega_{11} & 0 & \Omega_{13} & R_2 & R_3 & \Omega_{16} & \Omega_{17} & \Omega_{18} & -A^T R \\ * & \Omega_{22} & 0 & 0 & 0 & 0 & K_2 L_4 & 0 & 0 \\ * & * & \Omega_{33} & -G_{12} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & B^T R \\ * & * & * & * & * & * & \Omega_{77} & 0 & C^T R \\ * & * & * & * & * & * & * & -Q_4 & E^T R \\ * & * & * & * & * & * & * & * & -R \end{bmatrix} < 0 \quad (10)$$

where

$$\Omega_{11} = -PA - AP - 2(L_2\Lambda - L_1 W)A + Q_1 + Q_3 + G_{11} - R_1 - R_2 - R_3 - K_1 L_3$$

$$\Omega_{13} = G_{12} + R_1$$

$$\Omega_{16} = PB - A(W - \Lambda) + (L_2\Lambda - L_1 W)B + K_1 L_4$$

$$\Omega_{17} = PC + (L_2\Lambda - L_1 W)C$$

$$\Omega_{18} = PE + (L_2\Lambda - L_1 W)E$$

$$\Omega_{22} = -(1 - \mu)Q_1 - K_2 L_3$$

$$\Omega_{33} = G_{22} - G_{11} - R_1$$

$$\Omega_{44} = -G_{22} - R_2, \quad \Omega_{55} = -Q_3 - R_3$$

$$\Omega_{66} = 2(W - \Lambda)B + Q_2 - K_1, \quad \Omega_{67} = (W - \Lambda)C$$

$$\Omega_{68} = (W - \Lambda)E, \quad \Omega_{77} = -(1 - \mu)Q_2 - K_2,$$

$$R = \frac{\tau^2}{9}R_1 + \frac{4\tau^2}{9}R_2 + dR_3 + Q_4$$

Proof: Construct a new class of Lyapunov functional candidate as follow:

$$V(x_t) = \sum_{i=1}^{4} V_i(x_t)$$

with

$$V_1(x_t) = x^T(t)Px(t) + 2\sum_{i=1}^{n} w_i \int_0^{x_i(t)} (f_i(s) - l_i^- s)ds$$

$$+ 2\sum_{i=1}^{n} \delta_i \int_0^{x_i(t)} (l_i^+ s - f_i(s))ds$$

$$V_2(x_t) = \int_{t-\tau(t)}^t x^T(s)Q_1 x(s)ds + \int_{t-\tau(t)}^t f^T(x(s))Q_2 f(x(s))ds$$

$$\int_{t-d}^t x^T(s)Q_3 x(s)ds + \int_{t-d}^t \dot{x}^T(s)Q_4 \dot{x}(s)ds$$

$$V_3(x_t) = \int_{t-\frac{\tau}{3}}^{t} \begin{bmatrix} x(s) \\ x(s-\frac{\tau}{3}) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-\frac{\tau}{3}) \end{bmatrix} ds$$

$$V_4(x_t) = \frac{\tau}{3} \int_{-\frac{\tau}{3}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta$$
$$+ d \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds d\theta$$
$$+ \frac{2\tau}{3} \int_{-\frac{2\tau}{3}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta$$

Then, taking the time derivative of $V(x_t)$ with respect to t along the system (6) yield

$$\dot{V}(x_t) = \sum_{i=1}^{4} \dot{V}_i(x_t)$$

where

$$\dot{V}_1(x_t) = 2x^T(t) P \dot{x}(t) + 2(f(x(t)) - L_1 x(t))^T W \dot{x}(t)$$
$$+ (L_2 x(t) - f(x(t)))^T \Lambda \dot{x}(t) \tag{11}$$

$$\dot{V}_2(x_t) = x^T(t)(Q_1 + Q_3)x(t) - x^T(t-d)Q_3 x(t-d)$$
$$- (1-\mu)x^T(t-\tau(t))Q_1 x(t-\tau(t)) + f^T(x(t))Q_2 f(x(t))$$
$$- (1-\mu)f^T(x(t-\tau(t)))Q_2 f(x(t-\tau(t)))$$
$$+ \dot{x}^T(t)Q_4 \dot{x}(t) - \dot{x}^T(t-d)Q_4 \dot{x}(t-d) \tag{12}$$

$$\dot{V}_3(x_t) = \begin{bmatrix} x(t) \\ x(t-\frac{\tau}{3}) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\frac{\tau}{3}) \end{bmatrix}$$
$$- \begin{bmatrix} x(t-\frac{\tau}{3}) \\ x(t-\frac{2\tau}{3}) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} x(t-\frac{\tau}{3}) \\ x(t-\frac{2\tau}{3}) \end{bmatrix} \tag{13}$$

$$\dot{V}_4(x_t) = \dot{x}^T(t) R \dot{x}(t) - \frac{\tau}{3} \int_{t-\frac{\tau}{3}}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds$$
$$- \frac{2\tau}{3} \int_{t-\frac{2\tau}{3}}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds - d \int_{t-d}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds \tag{14}$$

Using Lemma 1, we can obtain that

$$- \frac{\tau}{3} \int_{t-\frac{\tau}{3}}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds \tag{15}$$
$$\leq -[x(t) - x(t-\frac{\tau}{3})]^T R_1 [x(t) - x(t-\frac{\tau}{3})]$$

$$- \frac{2\tau}{3} \int_{t-\frac{2\tau}{3}}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds \tag{16}$$
$$\leq -[x(t) - x(t-\frac{2\tau}{3})]^T R_2 [x(t) - x(t-\frac{2\tau}{3})]$$

$$- d \int_{t-d}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds \tag{17}$$
$$\leq -[x(t) - x(t-d)]^T R_3 [x(t) - x(t-d)]$$

For positive diagonal matrices $K_i, i = 1, 2$, we can get from (5) that

$$\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} -K_1 L_3 & K_1 L_4 \\ * & -K_1 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \geq 0 \tag{18}$$

$$\begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} -K_2 L_3 & K_2 L_4 \\ * & -K_2 \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix} \geq 0 \tag{19}$$

From (11)-(19), we can obtain that:

$$\dot{V}(x_t) \leq \xi^T(t) \Pi \xi(t) \tag{20}$$

where

$$\xi^T(t) = [x^T(t), x^T(t-\tau(t)), x^T(t-\frac{\tau}{3}), x^T(t-\frac{2\tau}{3}),$$
$$x^T(t-d), f^T(x(t)), f^T(x(t-\tau(t))), \dot{x}^T(t-d)]$$

$$\Pi = \begin{bmatrix} \pi_{11} & 0 & \pi_{13} & R_2 & R_3 & \pi_{16} & \pi_{17} & \pi_{18} \\ * & \pi_{22} & 0 & 0 & 0 & 0 & K_2 L_4 & 0 \\ * & * & \pi_{33} & -G_{12} & 0 & 0 & 0 & 0 \\ * & * & * & \pi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \pi_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \pi_{66} & \pi_{67} & \pi_{68} \\ * & * & * & * & * & * & \pi_{77} & 0 \\ * & * & * & * & * & * & * & \pi_{88} \end{bmatrix}$$

$$\pi_{11} = -PA - AP - 2(L_2 \Lambda - L_1 W)A + Q_1 + Q_3 + G_{11}$$
$$- R_1 - R_2 - R_3 - K_1 L_3 + A^T RA$$

$$\pi_{13} = G_{12} + R_1$$

$$\pi_{16} = PB - A(W - \Lambda) + (L_2 \Lambda - L_1 W)B + K_1 L_4 - A^T RB$$

$$\pi_{17} = PC + (L_2 \Lambda - L_1 W)C - A^T RC$$

$$\pi_{18} = PE + (L_2 \Lambda - L_1 W)E - A^T RE$$

$$\pi_{22} = -(1-\mu)Q_1 - K_2 L_3$$

$$\pi_{33} = G_{22} - G_{11} - R_1$$

$$\pi_{44} = -G_{22} - R_2, \; \pi_{55} = -Q_3 - R_3$$

$$\pi_{66} = 2(W - \Lambda)B + Q_2 - K_1 + B^T RB$$

$$\pi_{67} = (W - \Lambda)C + B^T RC$$

$$\pi_{68} = (W - \Lambda)E + B^T RE$$

$$\pi_{77} = -(1-\mu)Q_2 - K_2 + C^T RE,$$

$$\pi_{88} = -Q_4 + E^T RE$$

Using Lemma 2, we can obtain that $\dot{V}(x_t) < 0$ on the condition of (10), therefore, the system (6) is asymptotically stable. This completes the proof. ∎

Remark 1 Firstly,in this paper,dividing the delay interval $[0, \tau]$ into three different ones $[0, \frac{\tau}{3}], [\frac{\tau}{3}, \frac{2\tau}{3}], [\frac{2\tau}{3}, \tau]$; Secondly, constructing new Lyapunov functional which contains some new integral terms.It have potential to yield less conservative results.

Based on Theorem 1, we have the following result for uncertain neutral-type neural networks with time-varying delay.

Theorem 2 Given that the Assumption 1-3 hold,the system (6) globally asymptotically stable if there exist symmetric positive definite matrices $P, Q_1, Q_2, Q_3, Q_4, \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}$, R_1, R_2, R_3,positive diagonal matrices $W = diag\{w_i\}, K_1,$ $K_2, \Lambda = diag\{\delta_i\}$ with appropriate dimensions, and a scalar $\varepsilon > 0$ such that the following LMIs holds:

$$\begin{bmatrix} \Omega & \Psi & \varepsilon \Phi^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \tag{21}$$

where

$$\Psi = [PD + (L_2\Lambda - L_1 W)D, 0_{n \times 4n}, (W - \Lambda)D, 0_{n \times 2n}, D^T R]^T$$

$$\Phi = [-\tilde{E}_a, 0_{n \times 4n}, \tilde{E}_b, \tilde{E}_c, \tilde{E}_e, 0_n]$$

Proof: Replacing A, B, C, E in (10) with $A + DF(t)\tilde{E}_a, B + DF(t)\tilde{E}_b, C + DF(t)\tilde{E}_c, E + DF(t)\tilde{E}_e$,respectively,(10) is equivalent to the following condition:

$$\Omega + \Psi F(t)\Phi + \Phi^T F^T(t)\Psi^T < 0 \tag{22}$$

According to Lemma 3, one can obtain (22) is equivalent to the following inequalities on the condition of $F^T(t)F(t) \leq I$:

$$\Omega + \varepsilon^{-1}\Psi\Psi^T + \varepsilon\Phi^T\Phi < 0 \tag{23}$$

Using Lemma 2,we know that (23) is equivalent to (21). This completes the proof. ∎

Remark 2 In this paper, Theorem 1 and Theorem 2 require the upper bound μ of the time-varying delay $\tau(t)$ to be known. However, in many cases μ is unknown, considering this situation, we can set $Q_1 = Q_2 = 0$ in $V(x_t)$, and employ the similar methods in Theorem 1 and Theorem 2, we can derive the delay-derivative-independent and delay-dependent stability criteria.

IV. Example

Example 1 Consider delayed neutral-type neural networks (6) with the following parameters:

$$A = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.7 \end{bmatrix}, B = \begin{bmatrix} 0.0503 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix}, C = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix}$$

$$E = \begin{bmatrix} 1.2135 & 0 \\ -0.3412 & 0.2257 \end{bmatrix}, \tilde{E}_a = \tilde{E}_b = \tilde{E}_c = \tilde{E}_e = I$$

The neuron activation functions are assumed to satisfy Assumption 2 with $L_1 = diag\{0, 0\}, L_2 = diag\{0.3, 0.8\}$.

TABLE I
ADMISSIBLE UPPER BOUND τ FOR DIFFERENT d AND μ.

Method	Theorem 1
$d = 0.1, \mu = 0.2$	3.4055
$d = 0.3, \mu = 0.4$	4.0657
$d = 0.5, \mu = 0.6$	4.3121
$d = 0.7, \mu = 0.8$	4.5602

In this example, by applying Theorem 1 and solving the LMI (10) using MATLAB LMI Control ToolBox, we can obtain the maximum allowable upper bounds of delay for various of d and μ. From Table I, it can be seen that our results show significant improvements and less conservative.

V. Conclusion

In this paper, the problem of stability analysis for delayed neutral-type neural networks with uncertain paramters has been investigated. By choosing new Lyapunov-Krasovskii functional, dividing the delay interval into multiple segments , and combining linear matrix inequalities (LMI) techniques, two new sufficient criteria ensuring the global stability asymptotic stability of delayed neutral-type neural networks is obtained. Finally, one example is given to show the effectiveness of our obtained criteria.

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