Investigating Transformations in the Cartesian Plane using Spreadsheets

D. Allison, A. Didenko, and G. Miller

Abstract—The link between coordinate transformations in the plane and their effects on the graph of a function can be difficult for students studying college level mathematics to comprehend.   To solidify this conceptual link in the mind of a student Microsoft Excel can serve as a convenient graphing tool and pedagogical aid. The authors of this paper describe how various transformations and their related functional symmetry properties can be graphically displayed with an Excel spreadsheet.

Keywords—Mathematics education, Microsoft Excel spreadsheet, technology.

I. Introduction

THE myriad uses of mathematics in scientific and technological fields lay varying emphases upon a great variety of different mathematical techniques and topics. However, there is a core of essential mathematical topics that are common to all these fields of activity. One such topic, or skill, is to be able to make connections between graphs of functions and their associated equations. Students studying towards a college level qualification in a technological/scientific discipline are usually initiated into the subject using simple (for example linear) functions as a basis to investigate:

- The nature of a graph of a function by means of hand-drawn and calculator graphs.
- Solving equations both analytically and using graphing calculators to find solutions via the x-intercept method and/or the intersection of graphs technique.
- Analytical and graphical methods of modeling in applied settings.

It is not a trivial matter for students to appreciate the connection between a certain mathematical coordinate transformation and the symmetry properties possessed by the graph of a function.

Prior to the information technology era a student's introduction to the topic of coordinate transformations and functional symmetry properties was necessarily restricted to a pencil-and-paper approach. This made the exploration of the subject matter error prone and tedious. The onset of computer revolution in the shape of the Internet, multimedia resources and educational software supported and enhanced the delivery of course materials at all educational levels, from junior school through to university [1-5]. However, it is fair to state that the impact of this technology has been greater within the higher education sector [6-8]. In support of this view the National Council of Teachers of Mathematics (NCTM) has issued what it refers to as its "Technology Principle" [9] which states that:

"Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning."

The response to this call for technology to be used in mathematics teaching has been strong and widespread [10].

In a recent paper [11] Baker & Sugden describe the variety of ways that spreadsheets have been useful in the teaching of technological subjects. Today there exist a number of specialized software packages that can aid technologists implement mathematics in their research; for instance *Maple, Matlab* and kindred packages. These packages are also very helpful for the teaching of mathematics, especially at college level. However, the humble spreadsheet has tended to be overlooked as a medium for the transmission and exploration of mathematical concepts. Spreadsheet use is one of the most popular personal computer activities, second only to word processing and Internet browsing, in frequency of use and variety of applications. But it seems that relatively few computer users are aware of the mathematical problem-solving powers inherent in spreadsheets. Microsoft's *Excel* spreadsheet is one such program that can be an effective tool for a variety of purposes:

- Sorting and manipulating data.
- Data presentation.
- Mathematical and statistical calculation.

When it comes to mathematics the Excel spreadsheet has certain features that make it useful for math-based tasks such as the numerical evaluation of integrals and the handling of interpolation problems – this is despite the fact that the software lacks built-in special purpose subroutines to automate such computations. However, the software is especially suited for the presentation of data sets by means of graphs, charts and tables. This quality is important because these days students are more visually orientated than previous generations were. Responding to this state of affairs the authors of this paper have developed Excel-based instructional tools for the communication of mathematical concepts in a graphical environment.

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In previous papers [12-23], the authors described how Excel can be effectively deployed in the classroom as a teaching tool for pre-calculus and calculus courses. In the current paper we continue the theme, focusing on the ability of Excel to efficiently generate graphs of functions useful for the exploration of their symmetry properties.

This paper investigates how spreadsheet software can help the student visualize what happens to the graph of a function when it undergoes certain elementary transformations [24].

The transformations are of the following types:

i) the x-coordinate in the domain of the function is replaced with $-x$, and a new function $f(-x)$ is generated. The graph of the transformed function corresponds to reflecting the graph of $f(x)$ about the y-axis.

ii) A new function $g(x)$ is generated from $f(x)$ as follows:

$$g(x) = -f(-x)$$

The graph of $g(x)$ can be obtained by rotating the graph of $f(x)$ about the origin half a turn, or 180°.

iii)

A new function $h(x)$ is generated from $f(x)$ as follows:

$$h(x) = -f(x)$$

The graph of $h(x)$ can be obtained by reflecting the graph of $f(x)$ about the x-axis.

II. THE SPREADSHEET

In this section of the paper we shall describe how Excel can be used to illustrate what happens to the graph of a function when it undergoes transformations of the types described in section I.

The spreadsheet can be used as a supplementary aid for use by instructors and students in college level mathematics courses.

What follows are some typical screen shots showing the effects of the transformations on the graphs of various functions.
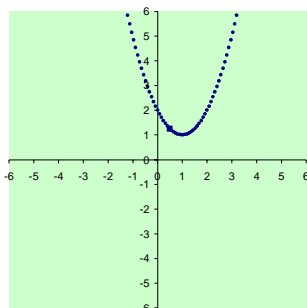

Fig. 1 illustrates the graph of the function $f(x) = (x-1)^2 + 1$

A typical sample point (a,b) lying on the graph is illustrated, which will serve as a reference point when demonstrating the transformation $f(x) \rightarrow -f(x)$.
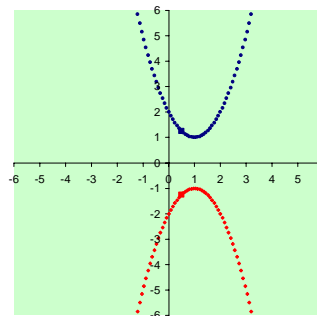

Fig. 2 shows how the reference point undergoes the transformation $(a,b) \rightarrow (a,-b)$, and both the graphs of $y = f(x)$ and $y = -f(x)$ are displayed together by the software

The effect of replacing the independent variable x by $-x$ in the function $f(x)$ is given in Fig. 3, where the graph of $y = f(-x)$ is shown in conjunction with the graph of $y = f(x)$. It should be noted that the spreadsheet renders each separate graph with a different color in order to help the user distinguish between them.
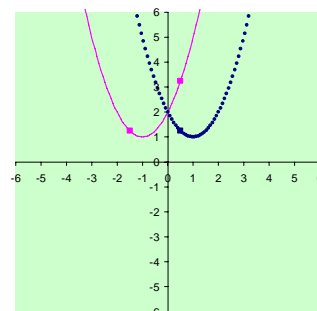

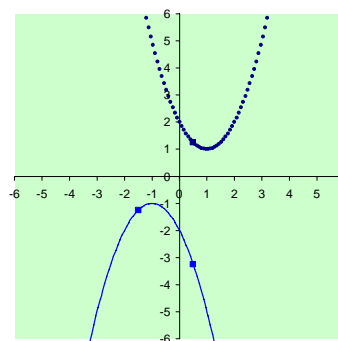Fig. 3 where the graph of $y = f(-x)$ is shown in conjunction with the graph of $y = f(x)$

Fig. 4 shows the graphs of two functions, one being the original function $y = f(x)$ and the other corresponding to the transformed function $y = -f(-x)$
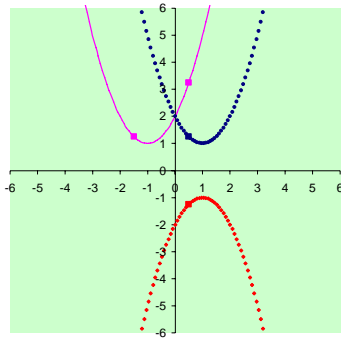
2

Fig. 5 The relationship between the three functions
$f(x) = (x-1)^2 + 1$, $f(-x)$ and $-f(x)$ is on display in the
screenshot corresponding

Again it should be noted by the reader that three different colors are utilized for the curves to distinguish between the different functions.

The next function to come under consideration is linear:
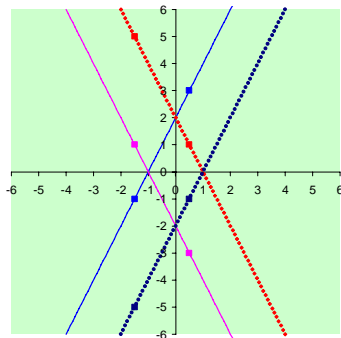$f(x) = 2x - 2$.

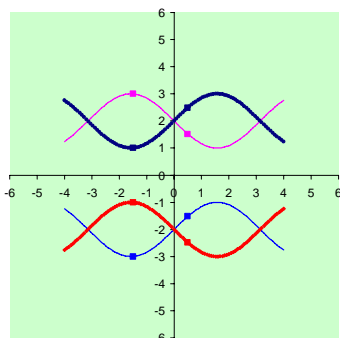Fig. 6 The relationships between the graphs $f(x)$, $f(-x)$,
$-f(-x)$ and $-f(x)$

Fig. 7 shows the graphical relationship between $f(x), f(-x)$
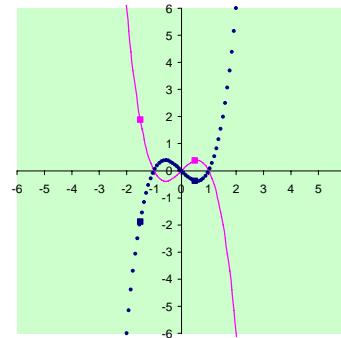, $-f(-x)$ and $-f(x)$ for the case when $f(x) = \sin x + 2$

Fig. 8 has the graph of $f(x) = x^3 - x$ being shown by the dashed
curve, and the graph of its companion function $f(-x)$ shown by the
solid curve

f(x) = x^2
f(x) = |x|
f(x) = 2x-2
f(x) = sin(x)
f(x) = (x-1)^2+1
f(x) = sin(x)+2
f(x) = x^3-x

Fig. 9 The spreadsheet allows the user to select one function $f(x)$ at
a time from a selection table of functions

REFERENCES

[1]   R.M. Reis, Development of Educational Software, *International Journal of Education and Information Technologies*, Vol. 1, Iss. 3 [2007], pp. 167-171.
[2]   R. Fang, H.J. Yang, H.L. Tsai, C.J. Lee and D. Li, The Digital Instrument Support Teacher's Instructional Innovation, *International Journal of Education and Information Technologies*, Vol. 1, Iss. 4 [2007], pp. 215-222.
[3]   R. Fang, H.J. Yang, H.L. Tsai, C.J. Lee, T. Tsai and D. Li, A Type of Technological Course with using Information Technology, *International Journal of Education and Information Technologies*, Vol. 1, Iss. 4 [2007], pp. 181-187.
[4]   I.I. Siller-Alcala, R. Alcantara-Ramirez, J. Jaimes-Ponce and J.P. Vaca-Pliego, How to link theory and practice of control engineering: Practices in real time and simulation, *International Journal of Education and Information Technologies*, Vol. 1, Iss. 2 [2007], pp. 114-120.
[5]   O.V. Vejar, Implementation of Schooling Based on Technologies for Communication and Information in the Subject Physics II, Distance Learning, Multimedia and Video Technologies, *8th WSEAS International Conference on Multimedia, Internet and Video Technologies/8th WSEAS International Conference on Distance Learning and Web Engineering,* University of Cantabria, Santander, Spain, 2008, pp. 61-66.
[6]   H. Z. Poor, Administration of Faculties by Information and Communication Technology and Its Obstacles, *International Journal of Education and Information Technologies*, Vol. 2, Iss. 1 [2008], pp. 24-30.
[7]   R. Matusu, J. Vojtesek and T. Dulik, Technology-Enhanced Learning Tools in European Higher Education, Distance Learning, Multimedia and Video Technologies, *8th WSEAS International Conference on Multimedia, Internet and Video Technologies/8th WSEAS International Conference on Distance Learning and Web Engineering,* University of Cantabria, Santander, Spain, 2008, pp. 51-54.
[8]   H.M. El-Bakry and N. Mastorakis, Realization of E-University for Distance Learning, Distance Learning, Multimedia and Video Technologies, *8th WSEAS International Conference on Multimedia, Internet and Video Technologies/8th WSEAS International Conference*

on Distance Learning and Web Engineering, University of Cantabria, Santander, Spain,2008, pp. 17-31.

[9] http://www.nctm.org/standards/content.aspx?id=26809

[10] M. Kathleen Heid and G.W. Blume (Editors) (2008), *Research on Technology and the Teaching and Learning of Mathematics:* Volume 1. Information Age Publishing, Inc., Charlotte, NC, USA, 978-1-931576-19-2.

[11] J. Baker and S. Sugden (2003), Spreadsheets in Education – The First 25 Years, *Spreadsheets in Education (eJSiE):* Vol. 1: Iss. 1, Article 2

[12] A. Didenko, D. Allison, S. Steward, Using technology to forge the link between formal mathematical definitions and related problem-solving techniques, *3rd Annual Conference for Middle East Teachers of Science, Mathematics and Computing,* Abu Dhabi, UAE, pp.119-126, 2006.

[13] A. Didenko, G. Miller, Algebraic and linear algebraic algorithms with EXCEL, *3rd Annual Conference for Middle East Teachers of Science, Mathematics and Computing,* Abu Dhabi, UAE, 2006, p.4.

[14] Gary Miller, Andriy Didenko, David Allison, Orthonormalization by Spreadsheet, *6th UAE MATHDAY Conference,* PI, Abu Dhabi, UAE, 2008.-3pp.

[15] A. Didenko, D. Allison, and G. Miller, Graphing Parametric Curves Using Spreadsheets, *34th Conference on Applications of Mathematics in Engineering and Economics - AMEE 2008,* M. Todorov, Ed. New York: American Institute of Physics, 2008, pp. 187-195.

[16] A. Didenko, D. Allison and G. Miller, Using Spreadsheets as Graphing Tools, *6th UAE Mathday Conference,* The Petroleum Institute, Abu Dhabi, UAE, 2008.

[17] A. Didenko, D. Allison, Using Microsoft Excel as a tool for teaching calculus to engineering students, *3rd International Conference on Mathematical Sciences (ICM-2008),* UAE University, Al Ain, UAE, 2008.

[18] A. Didenko, D. Allison, G. Miller, The Technology of Drawing a Straight Line, *7th UAE Mathday,* Sharjah University, Sharjah, UAE, 2009, p.19

[19] A. Didenko, D. Allison, G. Miller, Graphing Polar Equations Using Spreadsheets, *6th WSEAS International Conference on Engineering Education,* Rodos, Greece, 2009, 108-110.

[20] A. Didenko, D. Allison, G. Miller, Exploring Polar Equations with Interactive Spreadsheets, *International Journal of Education and Information Technologies,* Issue 2, Volume 4, 2010, 57-64.

[21] A. Didenko, D. Allison, G. Miller, Exploring Taylor Polynomials Using Spreadsheets, *International Conference on Interactive Computer Aided Learning,* Villach, Austria, 2009, 99-102.

[22] A. Didenko, D. Allison, G. Miller, Computer Aided Teaching of Engineering Mathematics, *International Journal of Arts and Sciences* 4(2): pp.146-157, 2011.

[23] A. Didenko, D. Allison, G. Miller, A pedagogical software tool to explore the effects of symmetry operations on graphs, *International Journal of Arts and Sciences*, vol. 4, no. 11, pp.35-45, 2011.

[24] J. Hornsby, M.L. Lial and G.K. Rockswold, *A Graphical Approach to Precalculus,* Boston, AddisonWesley, 2007.