MAP-Based Image Super-resolution Reconstruction

Xueting Liu, Daojin Song, Chuandai Dong and Hongkui Li

Abstract—From a set of shifted, blurred, and decimated image , super-resolution image reconstruction can get a high-resolution image. So it has become an active research branch in the field of image restoration. . In general, super-resolution image restoration is an ill-posed problem. Prior knowledge about the image can be combined to make the problem well-posed, which contributes to some regularization methods. In the regularization methods at present, however, regularization parameter was selected by experience in some cases and other techniques have too heavy computation cost for computing the parameter. In this paper, we construct a new super-resolution algorithm by transforming the solving of the System $\xi = A\eta$ into the solving of the equations $X + A^* X^{-1} A = I$, and propose an inverse iterative method.

Keywords—High-resolution MAP image Reconstruction Image interpolation Motion Estimation Hermitian positive definite solutions

I. Introduction

DETERIORATIVE images are confined by the available physical conditions, and caused by motion blurring, point spread blurring, under-sampling and sensor noise in imaging process. When imaging systems producing images , it is always with a limited resolution. For application, however, images with higher resolution are more desired.

To produce an image with a higher resolution from a series of images with lower resolution, super-resolution techniques offer a possibility. In the fact, the underlying techniques are implied that different sub-pixel displacement of each low resolution images contains different information of the high resolution image.

Such as CCD and CMOS image sensors, Some image sensors have been widely used. So people already have got high quality images. However, due to hardware cost and fabrication complexity limitations, the current resolution level of the image sensors can not make a captured image have no visible artifacts when it is magnified. Since to increase the current resolution level by improving hardware performance, is expensive, hence it has become a topic of

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very great interest for how to increase the current resolution level by applying low cost software tools. The research is focused to high precision sub-resolution registration algorithm; blind super-resolution methods; robust and efficient reconstruction; real-time processing techniques and so on currently.

The goal of Super-resolution restoration is to reconstruct the original scene from a degraded observation. "super-resolution" refers to removal of blur caused by the image system as well as recovery of spatial frequency information beyond the diffraction limit of the optical system, which is out of focus blur, motion blur, non-ideal sampling, etc.. To many image processing applications, this recovery process is critical.

Such as nonlinear expansion, the classical linear image restoration has been thoroughly studied and also smooth the image data in discontinuous regions, producing a larger image which appears rather blurry. So the super-resolution restoration algorithm has been studied from 60's.

In [7], Sean Borman, Robert StevensonSpatial identified three critical factors affecting super-resolution restoration. At first, reliable subpixel motion information is essential. Secondly, the imaging system and its degradations must be accurately described by observation models. The restoration methods must provide the maximum potential for inclusion of a-priori information and lastly.

So study in guarantee image Super- resolution ratio restore result and is it test prerequisite that the speed does not reduce to charge, how establish image model, reduce computer operation amount is it have important meaning and value to recover as to image.

In tradition single image restoration problem only a single input image is available for processing. Super-resolution image restoration addresses the problem of producing super-resolution still image from several images, which contains additional similar, but not identical information. The additional information makes it possible that construct a higher resolution image form original data.

Super-resolution techniques can be divided into two main divisions: frequency domains and spatial domain. Frequency domain methods are earlier super-resolution methods, they can only deal with image sequences with global translational. Spatial domain methods are very flexible. At present, they are main research direction of super-resolution. Spatial methods include Iterated Back projection (IBP), Projection onto Convex Sets (POCS), Maximum a Posteriori (MAP) estimation and Maximum Likelihood (ML) estimation. Two

powerful classes of spatial domain methods are POCS and MAP.

The pioneer work of super-resolution reconstruction may go back to 1984 by Tsai and Huang. Since then, many researchers have devoted themselves to the work in this area. In [3], the authors discuss SR reconstruction for both motion models from a frequency-domain point of view and presented a noniterative algorithm for SR reconstruction is using spatio-temporal filtering by assuming a priori knowledge of the motion(optical flow). In [4], N.Goldberg, A.Feuer, and G.C.Goodwin proposed a super-resolution technique specifically aimed at enhancing low-resolution text images from handheld devices by the using the Teager filter to highlight high frequencies which are then combined with the warped and interpolated image sequence following motion estimation using Taylor series decomposition.

II. MAP ALGORITHM

Huang and Tsai introduced the image super-resolution technique by fusing multiple images into a higher resolution frame with improved visual quality firstly. Their work was motivated by the need to enhance the image quality of the observed frames captured by the US satellite, Landsat. With the emergence of mobile devices, new text interpretation challenges have arisen particularly in natural scene images. Text in such scenes suffers from different degradations, including uneven lighting, optical and motion blur, low resolution, geometric distortion, sensor noise and complex back-grounds. Using multiple frames of a video sequence and static SR techniques, most of these degradations can be minimized or even suppressed, e.g. one can enhance the resolution of the image by recovering the high frequencies corrupted by the optical system.

Firstly we consider the desired HR image η of size

$$\eta = [\eta_1, \eta_2, \cdots, \eta_N]^T$$
$$N = M_1 N_1 \times M_2 N_2,$$

where M_1 and M_2 are the horizontal and vertical down-sampling factors, respectively. And

$$\xi^{(k)} = [\xi_1^{(k)}, \xi_2^{(k)}, \cdots \xi_s^{(k)}]^T$$
$$s = N_1 \times N_2$$

denote a set of K LR images. The imaging process, the observed LR image results, which from warping, blurring, and subsampling operators, performed on η. The imaging process is also corrupted by additive noise. Let

$$\xi_k = DB_k M_k \eta + n_k, \quad for 1 \le k \le p \tag{1}$$

denote the observation model , where M_k is an S×N warp matrix, B_k is an S×N blur matrix, D is the decimation matrix of size $(S \times N)^2 / L \times P$

In general, we rewriting the model in a simpler form:

$$\psi = A\eta + n \tag{2}$$

Let

$$\xi = \psi - \eta$$

Then the equation (2) can be written as

$$\xi = A\eta \tag{3}$$

By [1], let

$$A = \begin{pmatrix} I & M \\ M^* & I \end{pmatrix}$$

in which $A = \widetilde{A} + diag[I - \sqrt[n]{X}, 0]$, where

$$\widetilde{A} = \begin{pmatrix} \sqrt[n]{X} & M \\ M^* & I \end{pmatrix}$$

Furtherly, we decompose $\begin{pmatrix} \sqrt[n]{X} & M \\ M^* & I \end{pmatrix}$ to the LU decomposition

$$\begin{pmatrix} \sqrt[n]{X} & M \\ M^* & I \end{pmatrix} = \begin{pmatrix} I & 0 \\ M^* \sqrt[n]{X^{-1}} & I \end{pmatrix} \begin{pmatrix} \sqrt[n]{X} & M \\ 0 & X \end{pmatrix}$$

Then the solving of the equation (3) is transformed to the solving of the equations

$$X + M^* X^{-1} M = I \tag{4}$$

which matrix X must be a positive definite solution to the matrix equations (4).

III. REGULARIZATION TECHNIQUES

The MAP approach provides a flexible and convenient way to model a priori knowledge to constrain the solution. Usually, Bayesian methods are used when the probability density function of the original image can be established. Given the K LR frames ξ_k, and using the Bayes theorem, the MAP estimator of the SR image η maximizes the a posteriori $P(\eta|\xi_k)$,i.e.:

$$\hat{\eta}_{MAP} = \arg \max_{\eta} [\Pr\{\eta|\xi\}]$$
$$= \arg \max_{\eta} [\Pr\{\xi|\eta\} + \Pr\{\eta\}]$$

The maximum is independent of ξ_k and only the numerator need be considered.

Some scientists, such as Myers. Capel , Zisserman , Donald-son has seen MAP reconstruction in SR text in-depth investigation. Capel and Zisserman used an image gradient penalty defined by the Huber function as a prior model. This encourages local smoothness while preserving any step edge sharpness. Donaldson and Myers used the same Huber gradient penalty function with an additional prior probability distribution based on the bimodal characteristic of text.

In this paper, we assume the motion estimation error between images is independent and noise is an independent identically distributed zero mean Gaussian distribution, hence

the conditional density can be expressed in the compact form

$$\Pr(\xi|\eta) = \prod \frac{1}{\sigma_k \sqrt{2\pi}} \exp(-\frac{(\hat{\xi}_k - \xi_k)^2}{2\sigma_k^2})$$

where σ^2 is the error variance.

$$\Pr(\xi) = \exp\left[-(\frac{\lambda}{2}\|Q\xi\|^2)\right]$$

Regularization techniques can either be used during the reconstruction process or the deblurring and denoizing step as shown.

Pepole have proposed a variety of direct and iterative numerical regularization methods since the discrete ill-posedness, SRR needs well designed regularization algorithms. Such as

$$\hat{\eta}_{MAP} = \arg \max_{\eta}[\log \frac{1}{\sigma_k \sqrt{2\pi}} - \sum \frac{(\hat{\xi}_k - \xi_k)^2}{2\sigma_k^2} - \frac{\lambda}{2}\|Q\eta\|^2]$$

Let

$$f(\eta, \xi) = \min\left[\|\xi - H\eta\|^2 + \kappa \|Q\eta\|^2\right] \qquad (5)$$

$$\therefore \qquad \hat{\eta}_{MAP} = \arg f(\eta, \xi)$$

where κ is the regularization parameter for balancing the first term against the regularization term., The choice of η is then obtained by minimizing (5.

The necessary condition of (5) meeting the minimum is

$$\kappa = -\frac{(Q^T Q\eta)^T H^T H(H\eta - \xi)}{(Q^T Q\eta)^T (Q^T Q\eta)}$$

We select the conjugate gradient optimization technique for it avoids the complex computation on the Hessian matrix in the Newton gradient approach and converges faster than the steepest descent gradient approach.

Let $X = Y^{-1}$, then equation(4) is equivalent to

$$Y = \|M\|(I + Y^2) \qquad (6)$$

$\|M\| = \sqrt{\max_i \lambda_i}$, where the λ_i are the eigenvalues of MM^*. It is easy to the solving of the equation(2) is transformed to the solving of the equation(6). Now, we construce an inverse iterative method.

We consider the inverse iteration

$$Y_0 = I$$

$$Y_k = I + \|M\|(I + Y_{k-1}^2) \qquad (7)$$

The following we prove that if $\|M\| < \frac{2}{7}$ and equation(4) has a Hermitian positive definite solution, then it has maximal one X_L. Moreover, the sequence (Y_k) in (7) is monotonically increasing and converges to

$$Y_L = X_L^{-1} \le (\frac{2+\sqrt{2}}{2})^2 I .$$

Firstly , let

$$f(x) = \|M\|(1 + x^2)$$

where $x \in [0, \frac{\sqrt{2}}{2}]$, and sequence (x_k) inductively defined by

$$x_0 = 0$$

$$x_{k+1} = T(x_k), n \ge 0, \qquad (8)$$

We can assume the matrix sequence (x_k) in (8) is monotonically increasing and bounded above by $\frac{\sqrt{2}}{2}$, Since the following reason. We prove that the matrix sequence (x_k) in (8) is monotonically increasing. We compute

$$x_1 = (1 + x_0^2) > 0 = x_0$$

Assuming that $x_k \ge x_{k-1}$, for x_{k+1} we have

$$x_{k+1} = \|M\|(1 + x_k^2)$$

$$\ge \|M\|(1 + x_{k-1}^2)$$

$$= x_k$$

So

$$x_{k+1} \ge x_k$$

for $k = 0, 1, 2, \cdots,$ hence the sequence (x_k) is monotonically increasing.

At last, we prove (x_k) in (8) is bounded above by $\frac{\sqrt{2}}{2}$.

It is easy to $x_0 = 0 < \frac{\sqrt{2}}{2}$, and

$$x_1 = \|M\|(1 + x_0^2) = \|M\| < \frac{2}{7} < \frac{\sqrt{2}}{2}$$

Now we suppose, that $x_k < \frac{\sqrt{2}}{2}$. We can obtain easily

$$x_{k+1} = \|M\|(1 + x_k^2) < \frac{\sqrt{2}}{2}$$

Therefore (x_k) converges to $x \in [0, \frac{\sqrt{2}}{2}]$.

$$\| Y_k \| \le (1 + x_k^2) \le \sqrt{\frac{2+\sqrt{2}}{2}}$$

$$\| Y_0 \| = \| I \| = 1 \le \sqrt{\frac{2+\sqrt{2}}{2}}$$

If

$$\| Y_{k-1} \| \le (1 + x_{k-1}{}^2) \le \sqrt{\frac{2+\sqrt{2}}{2}}$$

then

$$\| Y_k \| \quad = \| M(1+Y_{k-1}^2) \| \le (1 + x_k{}^2)$$

Due to $Y_k => I$.

By [14] we have

$$\| Y_{k+1} - Y_k \|$$

$$\le \| \|M\|(1+Y_k^2) - \|M\|(1+Y_{k-1}^2) \|$$

$$\le \|M\| \| (1+Y_k^2) - (1+Y_{k-1}^2) \|$$

$$= \|M\| \| Y_k^2 - Y_{k-1}^2 \|$$

$$= \|M\| \| (Y_k + Y_{k-1})(Y_k - Y_{k-1}) \|$$

$$\le \frac{2}{7}(\frac{2+\sqrt{2}}{2} + \frac{2+\sqrt{2}}{2}) \| Y_k - Y_{k-1} \|$$

$$\le 1$$

So Y_k is a cauchy sequence in $(I, \sqrt{\frac{2+\sqrt{2}}{2}}I)$, so there

exists Y_L in $(I, \sqrt{\frac{2+\sqrt{2}}{2}}I)$, such that

$$Y_L = \lim_{n \to \infty} Y_k$$

IV. CONCLUSION

Experimental results demonstrate this new technique can give excellent reconstruction result in Image Super-resolution Reconstruction.

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