Investigation of SSR Characteristics of SSSC With GA Based Voltage Controller

R. Thirumalaivasan, M.Janaki, and Nagesh Prabhu

Abstract—In this paper, investigation of subsynchronous resonance (SSR) characteristics of a hybrid series compensated system and the design of voltage controller for three level 24-pulse Voltage Source Converter based Static Synchronous Series Compensator (SSSC) is presented. Hybrid compensation consists of series fixed capacitor and SSSC which is a active series FACTS controller. The design of voltage controller for SSSC is based on damping torque analysis, and Genetic Algorithm (GA) is adopted for tuning the controller parameters. The SSR Characteristics of SSSC with constant reactive voltage control modes has been investigated. The results show that the constant reactive voltage control of SSSC has the effect of reducing the electrical resonance frequency, which detunes the SSR.The analysis of SSR with SSSC is carried out based on frequency domain method, eigenvalue analysis and transient simulation. While the eigenvalue and damping torque analysis are based on D-Q model of SSSC, the transient simulation considers both D-Q and detailed three phase nonlinear system model using switching functions.

Keywords— FACTS, SSR, SSSC, damping torque, GA.

I. INTRODUCTION

SERIES compensation of long transmission lines is an economic solution to the problem of enhancing power transfer and improving system stability. However series compensated transmission lines connected to turbo generators can result in Subsynchronous Resonance (SSR) leading to adverse torsional interactions [2]. The hybrid series compensation consisting of suitable combination of passive elements and active FACTS controller such as TCSC or SSSC can be used to mitigate SSR. SSSC is a new generation series FACTS controller based on VSC and has several advantages over TCSC based on thyristor controllers. The voltage injected by SSSC is predominantly reactive (voltage in quadrature with the line current). The SSSC has only one degree of freedom (i.e, reactive voltage control) (unless an energy source connected on the DC side of VSC which allow for real power exchange) which is used to control active power flow in the line [3]. The VSC based on three level converter topology greatly reduces the harmonic distortion on the ac side [1], [4], and [5]. In this paper, the analysis and simulation of a hybrid series compensated system with SSSC based on three level twenty four pulse VSC is presented. The

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major objectives are to investigate SSR Characteristics of the system using both linear analysis and nonlinear transient simulation, and to design voltage controller for SSSC based on damping torque with Genetic Algorithm. The IEEE FBM is adopted for the analysis of SSR. The study is carried out based on frequency domain method, eigenvalue analysis and transient simulation. The modeling of the system neglecting VSC is detailed (including network transients) and can be expressed in DQ variables or (three) phase variables. The modeling of VSC is based on 1) DQ variables (neglecting harmonics in the output voltages of the converters) and 2) phase variables and the use of switching functions. The damping torque analysis, eigenvalue analysis, and the controller design is based on the DQ model while the transient simulation considers both models of VSC. The results based on linear analysis are validated using transient simulation based on nonlinear system model. The paper is organized as follows. Section-II describes the modeling of SSSC whereas the different methods of analysis of SSR are discussed in section-III. Section-IV describes a case study and investigates the SSR characteristics of the system without SSSC. Section-V describes investigation of SSR characteristics of the system with SSSC, and design of voltage controller for SSSC based on Genetic algorithm. The major conclusions of the paper are given in Section-VI.

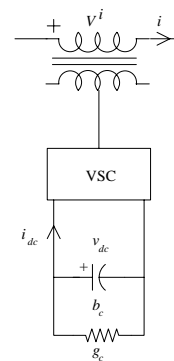## II. MODELING OF SSSC WITH THREE LEVEL VSC

Fig. 1 Schematic representation of SSSC

Fig. 1 shows the schematic representation of SSSC. In the power circuit of a SSSC, the converter is usually either a multi-pulse or a multilevel configuration. Here the SSSC is realized by a combination of 24-pulse with three level configuration. When the DC voltage is constant, the

magnitude of ac output voltage of the converter can be changed by Pulse Width Modulation (PWM) with two level topology which demands higher switching frequency and leads to increased losses. The three level converter topology can achieve the goal by varying dead angle β with fundamental switching frequency [6], [7].
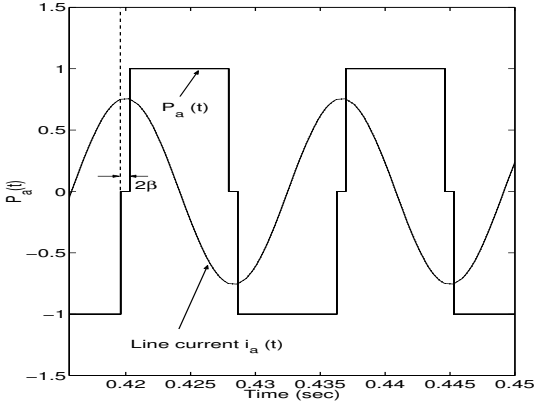

Fig. 2 Switching function for a three level converter

The converters that allow the variation of both magnitude and the phase angle of converter output voltage are classified as TYPE-1 converters [8]. The harmonics are dependent on the capacitance and the operating point of the SSSC. The detailed three phase model of SSSC is developed by modeling the converter operation by switching functions. The switching function for phase 'a' is shown in Fig. 2.

The switching function $P_a(t)$ for phase 'a' is shown in Fig.2. The switching functions of phase b and c are similar but phase shifted successively by 120^0 in terms of the fundamental frequency. Assuming that the DC capacitor voltages

$$Vdc1 = Vdc2 = \frac{Vdc}{2}$$

The converter terminal voltages with respect to the mid point of dc side 'N' can be obtained

$$\begin{bmatrix} V_{aN}^i \\ V_{bN}^i \\ V_{cN}^i \end{bmatrix} = \begin{bmatrix} P_a(t) \\ P_b(t) \\ P_c(t) \end{bmatrix} \frac{V_{dc}}{2} \qquad (1)$$

The converter terminal voltages with respect to the neutral of transformer can be expressed as,

$$\begin{bmatrix} V_{an}^i \\ V_{bn}^i \\ V_{cn}^i \end{bmatrix} = \begin{bmatrix} S_a(t) \\ S_b(t) \\ S_c(t) \end{bmatrix} V_{dc} \qquad (2)$$

where, $$S_a(t) = \frac{P_a(t)}{2} - \left[\frac{P_a(t) + P_b(t) + P_c(t)}{6} \right]$$

$S_a(t)$ is the switching function for phase 'a' of a 6-pulse 3-level VSC and V_{dc} is the dc side capacitor voltage. Similarly for phase 'b', $S_b(t)$ and for phase 'c', $S_c(t)$ can

be derived. The peak value of the fundamental and harmonics in the phase voltage V_{an}^i are found by applying Fourier analysis on the phase voltage and can be expressed as,

$$V_{an}^i = \frac{2}{h\pi} V_{dc} \cos(h\beta) \qquad (3)$$

Where, $h = 1, 5, 7, 11, 13$ and β is the dead angle (period) during which the converter pole output voltage is zero. We can eliminate the 5th and 7th harmonics by using a twelve-pulse VSC, which combines the output of two six-pulse converters using transformers.

The switching functions for first twelve-pulse converter are given by,

$$S_{1a}^{12}(t) = S_{1a}(t) + \frac{1}{\sqrt{3}} (S_{1a}^1(t) - S_{1c}^1(t)),$$

$$S_{1b}^{12}(t) = S_{1b}(t) + \frac{1}{\sqrt{3}} (S_{1b}^1(t) - S_{1a}^1(t)),$$

$$S_{1c}^{12}(t) = S_{1c}(t) + \frac{1}{\sqrt{3}} (S_{1c}^1(t) - S_{1b}^1(t)),$$

Where $$S_{1x}^1(t) = S_{1x}\left[t + \frac{2\pi}{\omega_0} \frac{1}{12} \right]$$

$$S_{1x}(t) = S_x\left[t + \frac{\pi}{\omega_0} \frac{1}{24} \right],$$

$$x = a, b \text{ and } c \qquad (4)$$

The switching functions for second twelve-pulse converter are given by,

$$S_{2a}^{12}(t) = S_{2a}(t) + \frac{1}{\sqrt{3}} (S_{2a}^1(t) - S_{2c}^1(t)),$$

$$S_{2b}^{12}(t) = S_{2b}(t) + \frac{1}{\sqrt{3}} (S_{2b}^1(t) - S_{2a}^1(t)),$$

$$S_{2c}^{12}(t) = S_{2c}(t) + \frac{1}{\sqrt{3}} (S_{2c}^1(t) - S_{2b}^1(t)),$$

Where $$S_{2x}^1(t) = S_{2x}\left[t + \frac{2\pi}{\omega_0} \frac{1}{12} \right]$$

$$S_{2x}(t) = S_x\left[t - \frac{\pi}{\omega_0} \frac{1}{24} \right]$$

$$x = a, b \text{ and } c \qquad (5)$$

The switching functions for a twenty-pulse converter are given by,

$$S_a^{24}(t) = S_{1x}^{12}(t) + S_{2x}^{12}(t),$$

$$x = a, b \text{ and } c \qquad (6)$$

If the switching functions are approximated by their fundamental components (neglecting harmonics) for a 24-pulse three-level converter, we get,

$$V_{an}^i = \frac{8}{\pi} V_{dc} \cos(\beta) \sin(\omega_0 t + \phi + \gamma) \qquad (7)$$

and V_{bn}^i, V_{cn}^i are phase shifted successively by 120°.

The line current is given by,

$$i_a = \sqrt{\frac{2}{3}} I_a \sin(\omega_0 t + \phi) \text{ and } i_b, i_c \text{ are phase shifted}$$

successively by 120°. Note that γ is the angle by which the

fundamental component of converter output voltage leads the line current. It should be noted that is nearly equal to $\pm\frac{\pi}{2}$ depending upon whether SSSC injects inductive or capacitive voltage.

Neglecting converter losses the DC capacitor current is given by,

$$\left[i_{dc}\right] = -\left[S_a^{24}(t) \quad S_b^{24}(t) \quad S_c^{24}(t)\right]\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (8)$$

A particular harmonics reaches zero, when $2\beta = \dfrac{180^o}{h}$. At $\beta_{optimum} = 3.75^o$ a three level 24-pulse converter behaves nearly like a two level 48-pulse converter as 23rd and 25th harmonics are negligibly small.

A. Mathematical Model of SSSC in D-Q frame [9] and [10]

When switching functions are approximated by their fundamental frequency components neglecting harmonics, SSSC can be modeled by transforming the three-phase voltages and currents to D-Q variables using Kron's transformation [9]. The SSSC can be represented functionally, as shown in Fig. 3.

In Fig. 3, Rs and Xs are the resistance and reactance of the interfacing transformer of VSC. The magnitude control of converter output voltage V^i is achieved by modulating the conduction period affected by dead angle β of a converter while the dc voltage is maintained constant.
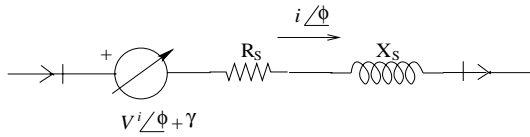

Fig. 3. equivalent circuit of SSSC as viewed from AC side

The converter output voltage can be represented in the D-Q frame of reference as

$$V^i = \sqrt{V_D^{i\,2} + V_Q^{i\,2}} \quad (9)$$

$$V_D^i = k_m V_{dc} \sin(\phi + \gamma) \quad (10)$$

$$V_Q^i = k_m V_{dc} \cos(\phi + \gamma) \quad (11)$$

Where k_m is modulation index and for a three-level converter it is a function of dead angle β and given by

$$k_m = k\rho\cos\beta_{se}; \; k = \frac{4\sqrt{6}}{\pi} \text{ for a 24-pulse converter.}$$

ρ is the transformation ratio of SSSC interfacing transformer. From control point of view it is convenient to

define the active voltage $(V_{P(se)})$ and $(V_{R(se)})$ reactive voltage injected by SSSC in terms of variables in D-Q frame $(V_D^i \text{ and } V_Q^i)$ as follows.

$$V_{R(se)} = V_D^i\cos\phi - V_Q^i\sin\phi \quad (12)$$

$$V_{P(se)} = V_D^i\sin\phi + V_Q^i\cos\phi \quad (13)$$

The dc side capacitor is described by the dynamical equation as,

$$\frac{dV_{dc}}{dt} = -\frac{\omega_b}{b_c}i_{dc} - \frac{g_c\omega_b}{b_c}V_{dc} \quad (14)$$

Where $i_{dc} = -[k_m\sin(\phi+\gamma)i_D + k_m\cos(\phi+\gamma)i_Q]$,

i_D and i_Q are the D-Q components of the line current. ϕ is the phase angle of line current and γ phase angle of converter output voltage.

B. SSSC Voltage control (Three-Level VSC)

In Type-1 controller both magnitude (modulation index k_m) and phase angle of converter output voltage (γ) are controlled. The dc side capacitor voltage is maintained at a constant voltage by controlling the active component of the injected voltage $V_{P(se)}$. The real voltage reference $V_{P(se)(ord)}$ is obtained as the output of DC voltage controller.

The reactive voltage reference $V_{R(se)(ord)}$ may be kept constant or obtained from a power scheduling controller. However, for the SSR analysis constant reactive voltage control is considered.

It should be noted that harmonic content of the SSSC injected voltage would vary depending upon the operating point since magnitude control will also govern the switching.

The capacitor voltage reference can be varied (depending on reactive voltage reference) so as to give optimum harmonic performance. In three level 24-pulse converter, dc voltage reference may be adjusted by a slow controller to get optimum harmonic performance at $\beta_{optimum} = 3.75^o$ in steady state.
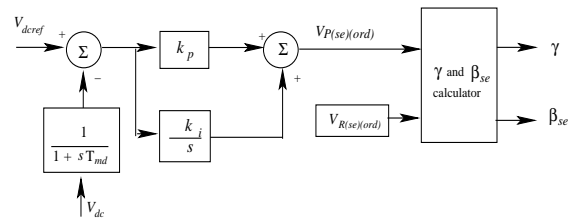

Fig. 4 Type-1 controller for SSSC

The structure of type-1 controller for SSSC is given in Fig. 4. In this figure, γ and β_{se} are calculated as,

$$\gamma = \tan^{-1}\left[\frac{V_{R(se)(ord)}}{V_{P(se)(ord)}}\right] \tag{15}$$

$$\beta_{se} = \cos^{-1}\left[\frac{\sqrt{V_{P(se)(ord)}^2 + V_{R(se)(ord)}^2}}{k_m V_{dc}}\right] \tag{16}$$

III. ANALYSIS OF SSR

The two aspects of SSR are [4]: 1) steady state SSR [(induction generator effect (IGE) and torsional interaction (TI)] and 2) transient torques. The analysis of steady-state SSR can be done by linearized models at the operating point and include damping torque analysis and eigenvalue analysis.

The analysis of transient SSR requires transient simulation of the nonlinear model of the system. For the analysis of SSR, it is adequate to model the transmission line by lumped resistance and inductance where the line transients are also considered. The generator stator transients are also considered by using detailed (2.2) model of the generator [11]. The analysis of SSR with SSSC is carried out based on damping torque analysis, eigenvalue analysis, and transient simulation.

A. Damping Torque Analysis

Damping torque analysis is a frequency-domain method which can be used to screen the system conditions that give rise to potential SSR problems involving torsional interactions. It also enables the planners to decide upon a suitable countermeasure for the mitigation of the detrimental effects of SSR. The damping torque method gives a quick check to determine the torsional-mode stability. The system is assumed to be stable if the net damping (including electrical and mechanical) at any torsional-mode frequency is positive [12].
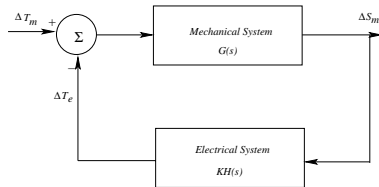

Fig. 5 Interaction between mechanical and electrical system

The interaction between the electrical and mechanical system can be represented by the block diagram shown in Fig. 5. (ΔS_m) is the per unit deviation in generator rotor speed and (ΔT_e) is the per unit change in electric torque termed as damping torque.

The transfer function relating (ΔT_e) to (ΔS_m) is $KH(s)$. The damping torque coefficient ($T_{de}(\omega)$) is defined as follows:

$$T_{de}(\omega) = \Re\left[\frac{\Delta T_e(j\omega)}{\Delta S_m(j\omega)}\right] = \Re\left[H(j\omega)\right]_{K=1} \tag{17}$$

It can be shown that the change in the decrement factor due to the electrical system ($\Delta\sigma = \sigma_e$) is approximately given by

$$\sigma_{ei} = \frac{T_{de}(\omega_i)}{4H_{mi}} \tag{18}$$

Where σ_{ei} corresponds to the torsional oscillation frequency (ω_i) and (H_{mi}) is the modal inertia for the i th mode. For the computation of the damping torque with VSC-based SSSC, the equations representing the system are linearized about an operating point and the equivalent admittance seen at the generator internal bus in the D-Q reference frame is calculated.

B. Eigenvalue Analysis

In this analysis, the detailed generator model (2.2) [11] is considered. The electromechanical system consists the multi-mass mechanical system, the generator, the excitation system, power system stabilizer (PSS) and the transmission line with SSSC. The SSSC equations (9)–(16) along with the equations representing electromechanical system [2], [11] (in D-Q variables), are linearized at the operating point and eigenvalues of system matrix are computed. The stability of the system is determined by the location of the eigenvalues of system matrix. The system is stable if the eigenvalues have negative real parts.

C. Transient Simulation

The eigenvalue analysis uses equations in D-Q variables where the switching functions are approximated by their fundamental frequency components (converter switchings are neglected). To validate the results obtained from damping torque and eigenvalue analysis, the transient simulation should be carried out using detailed nonlinear three-phase model of SSSC which considers the switching in the three-phase converters.

IV. A CASE STUDY

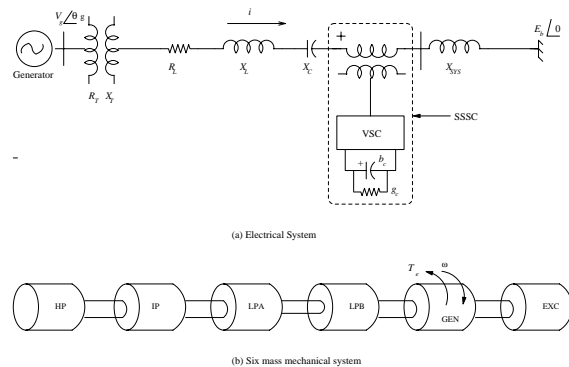(a) Electrical System

(b) Six mass mechanical system

Fig. 6 Modified IEEE first benchmark model with SSSC

The system considered is a modified IEEE FBM [13]. The system is represented schematically in Fig. 6, which consists of a generator, turbine, and series compensated long

transmission line with SSSC injecting a reactive voltage in series with the line. The electrical system data is taken from [11]. The modeling aspects of the electromechanical system comprising the generator, the mass-spring mechanical system, the excitation system, power system stabilizer (PSS), the transmission line containing the conventional series capacitor are discussed in [2]. The Analysis is carried out based on the following initial operating condition and assumptions.

1) The generator delivers 0.9 p.u. power to the transmission system.

2) The input mechanical power to the turbine is assumed constant.

3) The total series compensation level is set at 0.6 p.u. The study is carried out for the following cases

Case-1: Without SSSC

Case-2: With SSSC

In Case-1, the series compensation of 0.6 p.u is completely met by fixed capacitor and in Case-2, hybrid compensation is used wherein 0.45 p.u of compensation is met by fixed capacitor and the remaining 0.15 p.u by SSSC.

For transient simulation, a step decrease of 10% mechanical input torque applied at 0.5 sec and removed at 1 sec is considered in all case studies.

A. Analysis of SSR without SSSC

i) Damping torque analysis

The damping torque due to electrical network is evaluated in the range of frequency of 10-360 rad/sec. It is to be noted that for case-1, the damping torque is maximum negative at a frequency of around 127 rad/sec which matches with the natural frequency of torsional mode-2 and adverse torsional interactions are expected.
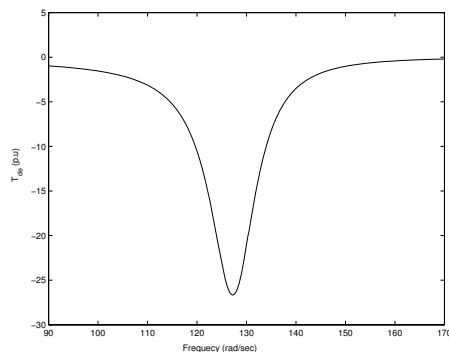

Fig. 7 Damping torque with admittance function in D-Q axes for case-1

ii) Eigenvalue analysis

In this analysis generator model (2.2) is considered. Considering mechanical damping, the system equations are linearized at the operating point. The eigenvalues of system matrix for case-1 are computed and are given in Table I. It is to be noted that mode-2 is unstable.

TABLE I
EIGEN VALUES OF THE SYSTEM

Torsional Mode	Case-1: Without SSSC $Xc=0.6p.u$
0	$-1.85980 \pm j\ 8.959i$
1	$-0.15050 \pm j\ 99.45i$
2	$0.2435 \pm j\ 127i$
3	$-0.638570 \pm j\ 160.43i$
4	$-0.363900 \pm j\ 202.82i$
5	$-1.850400 \pm j\ 298.17i$
Network mode (sub)	$-4.295100 \pm j\ 126.92i$
Network mode (sup)	$-5.740000 \pm j\ 626.74i$

iii) Transient Simulation

To validate the results obtained from damping torque and eigenvalue analysis, the transient simulation should be carried out. The transient simulation for case-1 has been carried out with linearized system using MATLAB-SIMULINK [14].
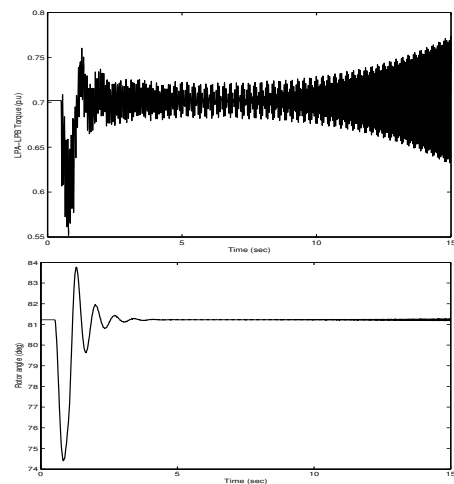

Fig. 8 Variation of LPA-LPB section torque and rotor angle for pulse change in input mechanical torque without SSSC (case-1)

The system response for simulation is shown in Fig. 8. It is clear from Fig.8 that, the system is unstable as the oscillations in rotor angle and LPA-LPB section torque grow with time.

Hence it is obvious that, to mitigate SSR, the damping of torsional mode should be improved by reducing the negative damping.

V. ANALYSIS OF SSR WITH SSSC

A long transmission line needs controllable series as well as shunt compensation for power flow control and voltage regulation. This can be achieved by a suitable combination of passive elements and active FACTS controllers. In the present work, hybrid compensation with series fixed capacitor and active compensation provided by a static synchronous series compensator (SSSC) is considered. SSSC Voltage Controller parameters are tuned based on damping torque to improve the

negative damping.A systematic method for tuning the Voltage Controller parameters for SSSC using GA is presented next.

A. Design of Voltage Controller for SSSC using GA

i) Introduction to GA

GA has been used as optimizing the parameters of control system that are complex and difficult to solve by conventional optimization methods [13]. It maintains a set of candidate solutions called population and repeatedly modifies them. Each member of the population is evaluated using a fitness function. The population undergoes reproduction in a number of iterations. One or more parents are chosen stochastically, but strings with higher fitness values have higher probability of contributing the offspring Genetic operators, such as crossover and mutation are applied to parents to produce offspring. The offspring are inserted into the population and the process is repeated. Given a random initial population, GA operates in cycles called generations, as follows:

The basic steps involved in GA are

Step 1: Begin with a randomly generated population of chromosome- encoded "solutions" to a given problem

Step 2: Calculate the fitness of each chromosome, where fitness is a measure of how well a member of the population performs at solving the problem

Step 3: Retain only the fittest members and discard the least fit members

Step 4: Generate a new population of chromosomes from the remaining members of the old population by applying the operations reproduction, crossover, and mutation

Step 5: Calculate the fitness of these new members of the population, retain the fittest, discard the least fit, and re-iterate the process.

ii) Objective Function

The GA based optimization guarantees the system stability under varying operating conditions. The design of voltage controller refers to the tuning of controller parameters k_p and

k_i, and it is done based on damping torque analysis [16], [17] using Genetic Algorithm. The damping of torsional mode is improved by reducing the negative damping. This is achieved by minimizing the deviations between the desired damping torque $(T_{de}(des))$ and actual damping torque (T_{de}) to reduce the negative damping at the series compensation considered.

On the basis of these facts, the objective function is defined as

$$Minimize, E = \sum((T_{de}(des)) - (T_{de}(\omega)))^2 \quad (19)$$

Where E is the summation of Squared Error at the series compensation considered.

In order that, SSSC minimizes the negative damping the desired damping torque is taken positive. However, it was observed that, the large positive value of $(T_{de}(des))$ causes network mode unstable.

The parameters used with GA are given in Table II. The SSSC voltage controller parameters obtained with GA are

$k_p = 0.01$ and $k_i = 0.12$.

TABLE II
PARAMETERS USED FOR OPTIMIZATION WITH GENETIC ALGORITHM

Parameter	Value / Type
Maximum Generations	20
Population Size	200
Type of Selection	Normal Geometric [0 0.08]
Type of Crossover	Arithmetic [4]
Type of mutation	Non uniform [4 20 3]
Termination method	Maximum Generation

B. Damping torque analysis

The variation of damping torque for case-2 (with SSSC) is shown in Fig.9. It is observed that maximum undamping occurs at a frequency about 150 rad/sec. Since this network frequency mode is not coinciding with any of the torsional modes, the system is stable.
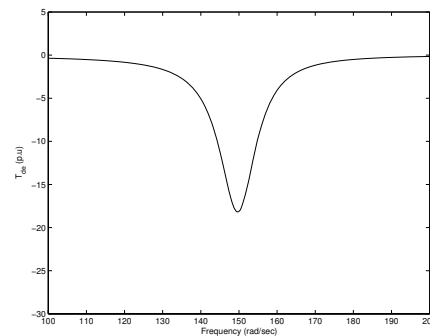

Fig. 9 Damping torque with admittance function in D-Q axes for case-2

TABLE III
EIGEN VALUES OF THE COMBINED SYSTEM

Torsional Mode	Case-2: With SSSC Xc=0.45p.u and Xsssc=0.15p.u
0	-1.38940 ± j 8.1626i
1	-0.19515 ± j 99.135i
2	-0.06269 ± j 127.05i
3	-0.56702 ± j 160.24i
4	-0.36692 ± j 202.81i
5	-1.85040 ± j 298.17i
Network mode (sub)	-4.49730 ± j 149.94i
Network mode (sup)	-5.20050 ± j 583.04i

C. Eigenvalue analysis

The system equations along with SSSC equations representing electromechanical system considering mechanical damping are linearized at the operating point. The eigenvalues of system matrix for case-2 are computed and are given in Table III. It is to be noted that, inclusion of SSSC leads to a stable system and reduces the potential risk of SSR problem.

D. Transient Simulation

The transient simulation of the combined nonlinear system including SSSC is carried out using both D-Q and 3-phase model using MATLAB-SIMULINK [14] and is shown in Fig.10. It is clear that, the system is stable as the oscillations in rotor angle and LPA-LPB section torque reduce with time.

E. Discussion

The representation of impedance function of SSSC in single phase basis $(Z_{s(1ph)}(j\omega))$ from that of D-Q axes $[Z_s]$ [11] is approximate and is given below.

$$Z_s(1ph) = \frac{1}{2}[\{Z_{sDD}(j(\omega-\omega_0)+Z_{sQQ}(j(\omega-\omega_0)+ \quad (20)$$
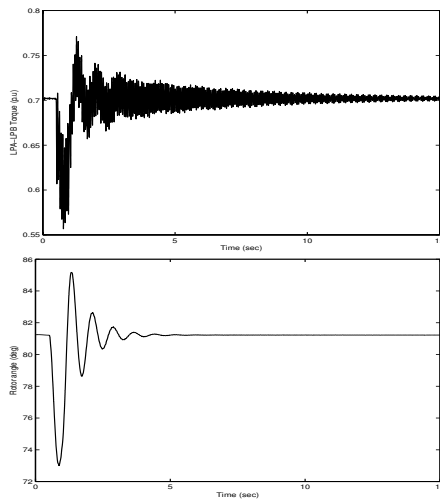$$j\{Z_{sDQ}(j(\omega-\omega_0)-Z_{sQD}(j(\omega-\omega_0))\}]$$

Fig. 10 Variation of LPA-LPB section torque and rotor angle for pulse change in input mechanical torque with SSSC (case-2)

The resistance R_{se} and reactance X_{se} of SSSC on single phase basis as a function of frequency ω_{er} is computed for $X_{sssc} = 0.15$ with constant reactive voltage control. It is found that, the resistance is negligible while the reactance X_{se} is practically constant with frequency. The effect of inclusion of SSSC on the resonance frequency is shown in Fig. 11 for cases-1 and 2. When the fixed capacitor provides 45% compensation, the resonance occurs at $\omega_{er} = 216$ rad/sec where $X_C = X_L$. When the additional compensation of 15% is provided by SSSC, the effective capacitive reactance $(X_C + X_{se})$ is obtained by adding the constant reactance offered by SSSC to that offered by fixed capacitor. The variation of effective capacitive reactance $(X_C + X_{se})$ with frequency is also shown in Fig. 11.Now the resonance occurs at a higher frequency of

$\omega_{er} = 227$ rad/sec where $(X_C + X_{se}) = X_L$ and this is consistent with the subsynchronous network mode frequency $((\omega_0 - \omega_{er}) = 377\text{-}227 = 150$ rad/sec) of about 150 rad/sec as obtained with damping torque analysis with SSSC. The effect of providing additional series compensation by SSSC to supplement the existing fixed capacitor is to increase the electrical resonance frequency of the network. However, this increase in frequency is not significant as compared to that obtained with the equivalent fixed capacitor offering additional compensation (case-1) $\omega_{er} = 250$ rad/sec in this case. This indicates that, the SSSC is not strictly SSR neutral however, it offers a reactance which remain practically constant with frequency.
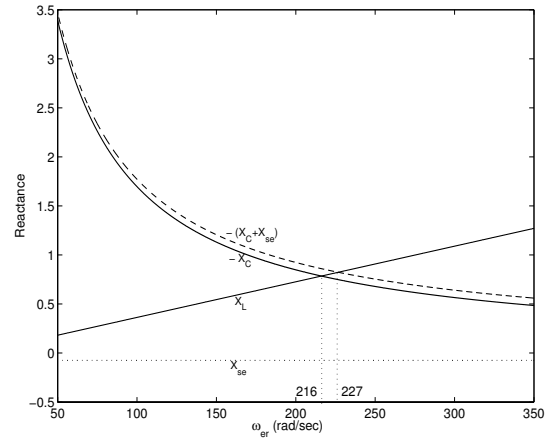

Fig. 11 Graphical representation of resonance conditions with and without SSSC

VI. CONCLUSION

In this paper we have presented the SSR analysis and simulation of a hybrid series compensated system with SSSC. The modeling details of 24-pulse three level VSC based SSSC is presented. The application of D-Q model is validated by the transient simulation of the three phase model of SSSC. There is no appreciable difference in the resonance frequency of the electrical network as the total series compensation (in a hybrid compensation scheme) is increased by increasing the series reactive voltage injected, instead of the series capacitor. This reduces the risk of SSR as the fixed capacitor can be chosen such that the electrical resonance frequency does not coincide with the complement of the torsional modal frequency (which is practically independent of the electrical network). It is observed that the injected reactive voltage can be adjusted to detune the SSR. The case studies indicated that, the SSSC is not strictly SSR neutral however, it offers a reactance which remain practically constant with frequency.

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