

The Effects of Misspecification of Stochastic Processes on Investment Appraisal

George Yungchih Wang¹

National Kaohsiung University of Applied Sciences, Taiwan

Abstract—For decades financial economists have been attempted to determine the optimal investment policy by recognizing the option value embedded in irreversible investment whose project value evolves as a geometric Brownian motion (GBM). This paper aims to examine the effects of the optimal investment trigger and of the misspecification of stochastic processes on investment in real options applications. Specifically, the former explores the consequence of adopting optimal investment rules on the distributions of corporate value under the correct assumption of stochastic process while the latter analyzes the influence on the distributions of corporate value as a result of the misspecification of stochastic processes, i.e., mistaking an alternative process as a GBM. It is found that adopting the correct optimal investment policy may increase corporate value by shifting the value distribution rightward, and the misspecification effect may decrease corporate value by shifting the value distribution leftward. The adoption of the optimal investment trigger has a major impact on investment to such an extent that the downside risk of investment is truncated at the project value of zero, thereby moving the value distributions rightward. The analytical framework is also extended to situations where collection lags are in place, and the result indicates that collection lags reduce the effects of investment trigger and misspecification on investment in an opposite way.

Keywords—GBM, real options, investment trigger, misspecification, collection lags

I. INTRODUCTION

FOR decades financial economists have been attempted to determine the optimal investment policy by recognizing the option value embedded in irreversible investment. Along the line of real options theory, numerous studies explore the optimal investment timing to pay an investment cost in return for an irreversible project whose value is a major source of uncertainty, evolving as a geometric Brownian motion (GBM).² Research is then extended to determine the optimal investment policy under a variety of stochastic processes, which has been shown to have a major impact on irreversible investment decisions in literature. Yet, among abundant literature few studies are directed at exploring the actual effect of adopting the optimal investment policy on investment, particularly on corporate value. Furthermore, the optimal investment policy in the context of real options is built on the foundation of maximizing the value of managerial flexibility rather than of maximizing the expected project payoffs. It is thus interesting to investigate how the optimal investment policy influences the distribution of corporate value under a

specific stochastic process.

On the other hand, since most real options models make the common assumption that the underlying variable follows a GBM for tractable solutions, it is possible that management may take the GBM assumption for granted without further examining the appropriateness of the GBM assumption. Another reason of management accepting the GBM assumption is possibly the difficulty of distinguishing a GBM from an alternative process because of the short-sampled data. In any case, the literature has documented that there are plenty of practical examples that practitioners explicitly apply the GBM-based models in the practical investment decisions. These examples include Merck [7, 14], British Telecommunications [8]: the former applies the Black-Scholes model to evaluate pharmaceutical development projects and the biotech stock index to estimate project volatility; the latter uses Geske's [5, 6] compound options model to managing R&D investment in the telecommunication service industry. It is important to point out that both the Black-Scholes and Geske models rest on the assumption that the underlying process follows a GBM. More applications building on the GBM assumption can be found in Eastman Kodak [4], New England Electric, Enron [2], Mycogen [19], and Phillips Electronics [9].

This paper aims to examine the effects of optimal investment triggers and of the misspecification of stochastic process on investment in real options applications. Specifically, the former is to explore the consequence of adopting optimal investment rules on the distributions of corporate value under the correct assumptions of stochastic process; the latter is to analyze the influence on the distributions of corporate value as a result of the misspecification of stochastic processes, particularly, mistaking an alternative process as a GBM. Research methodology is based on Monte Carlo simulation, from which several performance measures are also constructed to gauge the misspecification effect. In addition, a numerical procedure is developed to simulate capital investment decisions and realized project payoffs under a GBM and an alternative process. These alternative processes of interest are mixed diffusion-jump (MX), mean reversion (MR), and jump amplitude (JA).

As most real options models, with few exceptions, assume that a project is brought on line immediately without collection delays after the investment decision is made, the study of the trigger price effect and the misspecification effect will start by making the same assumption. The analytical framework is then extended in the presence of delays in collecting project payoffs. The rest of the paper is organized into four sections. Section 2 presents the base case for analyzing these two effects on investment. Section 3 describes research methodology involving a simulation procedure to approach the problem.

¹Email: gwang@cc.kuas.edu.tw

² See [3] and [20] for a complete literature review.

Three performance measures are also constructed to examine the misspecification effect of stochastic processes. Section 4 explains simulation results with/out collection lags. Finally, Section 5 gives concluding remarks.

II. BASIC INVESTMENT PROBLEM

Consider a firm that is currently holding a license, expiring in τ years, to manufacture a widget and trying to decide whether to invest in the widget factory. Once the project is launched, the firm needs to invest a direct cost, I , which is assumed to be irreversible, in return for a value, V , with a growth rate of α and a volatility of σ . The project life is assumed to be T , the risk-free rate is currently r , and the opportunity cost of holding a project is δ .

To examine the effects of optimal investment triggers and misspecification of stochastic processes, it is assumed that there are three financial managers, each of which represents a particular value distribution. For the reason of comparison, Manager A assumes that the investment opportunity should be taken after the licensing period. Consequently, Manager A essentially reflects the value distribution based on a given stochastic process. Manager B and C both are assumed to be rational managers who make investment decisions by following the optimal decision rules. The difference between Manager B and C is that the former can correctly identify the actual stochastic process and make optimal investment decisions accordingly while the latter makes investment decisions based on the mistaken assumption that V follows a GBM. Let V_B^* and V_C^* denote the optimal investment rules adopted by Manager B and C, respectively. Since Manager B adopts the optimal investment rule based on the actual process, V_B^* can be V_{MX}^* , V_{MR}^* , or V_{JA}^* , depending on the actual process assumed. In addition, Manager C's optimal investment policy, V_C^* , is equal to V_{GBM}^* as Manager C assumes that the underlying stochastic process evolves as a GBM.

It is important to point out that these three types of different investment behavior are designed to conveniently investigate the trigger price effect and the misspecification effect. As mentioned earlier, the trigger price effect describes the consequence of the actions of adopting the optimal investment triggers on the distribution of corporate value. It is therefore obvious that the effect of adopting an investment trigger can be easily observed by comparing the distributions of realized project payoffs caused by Manager A and B, given a specific stochastic process. In addition, the misspecification effect of stochastic process depicts the impact on the corporate value due to the misspecification of stochastic process, i.e., mistaking an alternative process for a GBM. In our setting, the misspecification effect can be examined by comparing the distributions of realized project payoffs of Manager B and C. To further explore the misspecification effect on investment, three performance measures are constructed to measure the likelihood of making mistaken decisions and the loss in corporate values. These performance measures are the probability of making mistaken decisions, the unconditional

loss ratio, and the conditional loss ratio, which will be illustrated in the next section.

For the purpose of simplifying the problem, when facing an investment opportunity, management is given two alternatives only: to invest or to defer the project. Let Ψ and Ω denote binary variables which represent investment decisions made by Manager B and C, respectively. Therefore, Ψ represents the optimal decision, given the correct specification of stochastic process, and Ω is the mistaken decision, given the misspecification of stochastic process. Suppose both Ω and Ψ can either take 1 (invest) or 0 (defer) in the binary setting. Since Ω and Ψ are independent decisions for the same investment opportunity under consideration, there are four possible outcomes, two of which describe the situations in which have consistent investment decisions and the other two state the situations where mistaken decisions are in place. Table 1 outlines these four possible outcomes due to the misspecification of stochastic process.

TABLE I. FOUR POSSIBLE OUTCOMES DUE TO MISSPECIFICATION OF STOCHASTIC PROCESS

Decisions Based on the Actual Process

		Decisions Based on the Actual Process	
		Invest $\Psi = 1$	Defer $\Psi = 0$
Decisions Based on the GBM	Invest $\Omega = 1$	Consistent Decision	Realized Loss
	Defer $\Omega = 0$	Forgone Profit	Consistent Decision

According to Table 1 it is possible that Manager C may still make investment decisions which are "consistent" with the optimal decisions made by Manager B, even though an underlying process is not correctly specified. On the other hand, it is also possible that Manager C may make mistaken decisions in the situations that he decides to invest as opposed to the optimal decisions to defer the project, or that he decides to defer the project as opposed to the optimal decisions to invest. It is therefore important to distinguish between $P(\Omega = \Psi)$ and $P(\Omega \neq \Psi)$, both of which are defined as the probability of making consistent decisions and the probability of making mistaken decisions, respectively. In the situations where the misspecification of stochastic process occurs, i.e., $\Omega \neq \Psi$, there are two types of investment losses, which are regarded as realized losses and forgone profits. Realized losses are referred to as negative payoffs due to the actions of launching a project as it should be deferred optimally, i.e., $\Omega = 1$ and $\Psi = 0$. On the other hand, forgone profits are defined as relinquished positive profits due to the actions of deferring a project as it should be taken optimally, i.e., $\Omega = 0$ and $\Psi = 1$. Obviously, both realized losses and forgone profits are investment losses which accrue to Manager C due to the misspecification of the stochastic process assumption.

III. RESEARCH METHODOLOGY

A. Simulation Procedure

Given the basic investment problem described in the preceding section, a numerical procedure based on Monte Carlo simulation is developed to investigate the trigger price effect and the misspecification effect of stochastic process on the distributions of corporate values. As the first step of simulation, two different stochastic processes, a GBM and an alternative process, are simulated, given the same parameter values. The former is what Manager C believes the underlying process is, while the latter is the actual process which is correctly identified by Manager B. The specifications of stochastic processes of interest are discussed in the next section. To make both processes look like a non-stationary random walk over the licensing period, the Augmented Dickey-Fuller (ADF) p test is conducted to test the null hypothesis that the value process is characterized by a random walk with a possible drift at the level of significance 5%.³ If any of the value process is rejected in the ADF test, the simulation procedure goes back to the beginning and re-simulates a new process according to the same parameter values until the null hypothesis is accepted. Note that the ADF test is conducted only within the licensing period of τ such that both processes may locally resemble a random walk but globally are two different stochastic processes.

The next step is to compute optimal investment triggers based on an alternative process and a GBM, V_B^* and V_C^* , serving as investment decision-making tools. Real options literature suggests that irreversibility and uncertainty complicate investment in that the closed-form solutions of trigger prices are mostly unavailable except those under the assumption of a GBM and a specific form of a mixed diffusion-jump process. If the solution of investment trigger for an alternative process is not available, the general investment framework based on Monte Carlo simulation in [20] is applied to iteratively derive optimal investment trigger.

Once both V_B^* and V_C^* are derived, the terminal payoffs of both simulated stochastic processes over the waiting period are used to determine the optimal decision and the actual decision. In the meanwhile, the net terminal payoffs under an actual process, denoted by f , are also computed as a percentage of investment cost, given three types of investment behavior. The net terminal payoffs of Manager A, B, and C, are measured by the following equations, respectively:

$$f_A = \frac{V_T - I}{I} \quad (1)$$

³ Generally, there are two most commonly used tests for stationarity: the (Augmented) Dickey-Fuller test and the Phillips-Parron test. The former is essentially a unit root test, where the null hypothesis is a unit root and the alternative is a stationary AR(p) process (the Augmented Dickey-Fuller test is used to test higher order AP process) while the latter is an extension of the Dickey-Fuller test allowing for non-white-noise errors. The DF test is chosen for the reason of ease of computer programming. Refer to [7] for the procedure of hypothesis testing on a random walk.

$$f_B = \frac{\Psi(V_T - I)}{I}, \quad \forall \Psi = 0 \text{ or } 1 \quad (2)$$

$$f_C = \frac{\Omega(V_T - I)}{I}, \quad \forall \Omega = 0 \text{ or } 1 \quad (3)$$

where V_T is the realized project value, according to the actual stochastic process.

It is obvious that when $\Omega = 1$, $\Psi = 0$, and $V_T < I$, Manager C has a realized investment loss. On the other hand, when $\Omega = 0$, $\Psi = 1$, and $V_T > I$, Manager C has a forgone profit. By integrating the decision variables, the loss function of Manager C is given as follows:⁴

$$\pi(\Omega, \Psi, V_T) = \frac{(\Omega - \Psi)(V_T - I)}{I} \quad (4)$$

where π denotes the loss ratio of Manager C for a simulation trial.

It is important to point out that when there are no delays in collecting project payoffs, V_T is equal to V_t . The variables Ω , Ψ , and π are recorded in each trial. The preceding procedure is to be repeated until the pre-specified simulation trials are completed. In Monte Carlo simulation, the results due to the misspecification of stochastic process are summarized with three performance measures. Let m denote the number of total simulation trials and l be the number of total mistaken decisions within m trials. Thus, if the underlying assumption of stochastic process is misspecified, the probability of making mistaken decisions can be calculated by

$$P(\Omega \neq \Psi) = \frac{l}{m} \quad (5)$$

There are two types of loss ratios when the misspecification of stochastic process occurs, the unconditional expected loss ratio and the conditional expected loss ratio. The *unconditional* expected loss ratio is defined as the average loss ratio out of total simulation trials, expressed as follows:

$$\bar{\pi} = \frac{\sum_{i=1}^l \pi_i}{m} \quad (6)$$

The *conditional* expected loss ratio is referred to as the average loss ratio out of the mistaken decisions, describing the expected value loss of each mistaken decision given that the mistaken decision is made. Therefore, the *unconditional* expected loss ratio is *ex ante* and the conditional expected loss ratio is *ex post*. The *conditional* expected loss ratio is mathematically expressed as follows:

$$\bar{\pi} |_{\Omega \neq \Psi} = \frac{\sum_{i=1}^l \pi_i}{l} \quad (7)$$

⁴ Refer to [13] for a similar form of loss function.

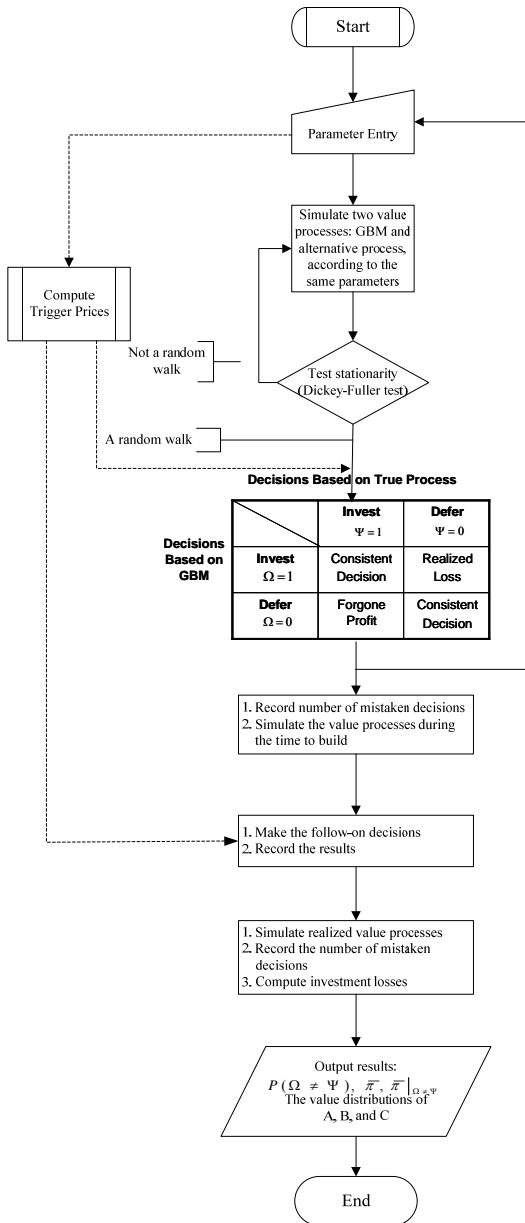


Fig. 1 The Simulation Procedure

It is obvious that the *conditional* expected loss ratio can be alternatively derived from the *unconditional* expected loss ratio in Equation (6) divided by the probability of making the mistaken decisions in Equation (5), expressed as follows:

$$\bar{\pi} |_{\Omega \neq \Psi} = \frac{\bar{\pi}}{P(\Omega \neq \Psi)} \tag{8}$$

The final output of simulation is the NPV distributions generated from realized project payoffs due to three different types of investment behavior. Figure 1 illustrates the thorough simulation procedure.

B. Stochastic Processes

Since stochastic processes are regarded as major sources of uncertainty in the evaluation of capital investments, here we introduce a variety of stochastic processes for later applications. The specifications of each stochastic process are presented both in continuous time and in discrete time.

(A) Geometric Brownian Motion

The most widely applied stochastic process is GBM which accounts for a continuous form of random walk. The main property of this class of stochastic process is that the rate of return is normally distributed, implying a lognormal distribution of the project value. The continuous-time version of a GBM is given below:

$$dV = \alpha V dt + \sigma V dz \tag{9}$$

where α , σ , and dz denote drift rate, instantaneous volatility, and an increment of a standard Wiener process, respectively.

For the simulation purpose, the discrete-time version of GBM is expressed as follows:

$$\Delta \ln V = v \Delta t + \sigma \sqrt{\Delta t} \varepsilon \tag{10}$$

where Δt and ε represent a small interval of time and a random drawing from a standard normal distribution, respectively, and $v = \alpha - \frac{1}{2} \sigma^2$.

(B) Mean Reversion

Another class of stochastic process is a mean-reverting process which is often used to describe the price behavior of commodity and natural resources. The most prominent property of a mean-reverting process is that its growth rate is not a constant but instead a function of a difference between current value and long-run mean, suggesting that growth rate in effect responds to disequilibrium. Dixit and Pindyck [3] examine the value of an investment opportunity whose value follows a mean-reverting process. As there are many ways to specify a mean-reverting process, Dixit and Pindyck's specification is somewhat arbitrary but convenient to find a "quasi-analytical" solution for the value of the project. The formation of this specific mean-reverting process is given below:

$$dV = \eta (\bar{V} - V) V dt + \sigma V dz \tag{11}$$

where η denotes a speed of mean reversion and \bar{V} is a long-run mean.

Equation (11) can be discretized into the following equation:

$$\Delta \ln V = \left[\eta (\bar{V} - V) - \frac{1}{2} \sigma^2 \right] \Delta t + \sigma \sqrt{\Delta t} \varepsilon \tag{12}$$

⁵ Since GBM is log-normally distributed, a more explicit form of Equation (10) is given below:

$$V_{t+\Delta t} = V_t \left[e^{(v\Delta t + \sigma\sqrt{\Delta t}\varepsilon_t)} \right]$$

(C) Mixed Diffusion-Jump

In general, a mixed diffusion-jump process consists of a GBM and a Poisson jump component. There are a variety of forms of a mixed diffusion-jump process, one of which is proposed by [12] in the financial option pricing problem and then applied by [18] in the context of evaluating an investment opportunity with competitive arrivals. A mixed diffusion-jump process in continuous time could be expressed as follows:

$$dV = (\alpha - \lambda k)Vdt + \sigma Vdz + Vdq_3 \quad (13)$$

where dq_3 is an increment of a Poisson jump process with a mean arrival rate λ such that

$$dq_3 = \begin{cases} \varphi & \text{with a probability of } \lambda dt \\ 0 & \text{with a probability of } 1 - \lambda dt \end{cases} \quad (14)$$

where $\varphi \sim N(k, \sigma_\varphi)$ denotes a proportional jump relative to V if a jump occurs.

Note that the Poisson jump term dq_3 is assumed to be independent of dz such that $E(dq_3 dz) = 0$. Equation (14) also reveals that the actual growth rate of such a mixed diffusion-jump process is not α but instead $\alpha - \lambda k$ in order to adjust the influence of a Poisson event. For the simulation purpose, the discrete-time version of the mixed diffusion-jump process is given as follows:

$$\Delta \ln V = (v - \lambda k)\Delta t + \sigma\sqrt{\Delta t}\varepsilon + D_3 \quad (15)$$

where D_3 denotes an increment of a Poisson jump in discrete time with a mean arrival rate λ such that

$$D_3 = \begin{cases} \varphi & \text{with a probability of } \lambda \Delta t \\ 0 & \text{with a probability of } 1 - \lambda \Delta t \end{cases} \quad (16)$$

It is worth noting that [11] and [3] also propose a mixed diffusion-jump process with the sign of the jump term changed into negative to describe the situation in that the project becomes suddenly worthless when a major competitor of the same product enters the market.

(D) Jump Amplitude Process

Another type of jump processes are jump amplitude processes which are suggested by [16] to describe the characteristics of R&D investments. The jump amplitude process differs from other jump processes in that the impacts of information arrivals can not be foreseen such that jump direction and jump size are stochastic by nature. A jump amplitude process can be mathematically expressed as follows:

$$dV = \alpha Vdt + Vdq_4 \quad (17)$$

where dq_4 an increment of a stochastic jump process. The jump term, dq_4 , is characterized by a parameter of jump intensity λ such that

$$dq_4 = \begin{cases} \psi & \text{with a probability of } \lambda dt \\ 0 & \text{with a probability of } 1 - \lambda dt \end{cases} \quad (18)$$

where ψ denotes a proportional jump relative to V .

By definition, $\psi = X\Gamma$ where $X = 1$ or -1 , $P(X = 1) = p$, and $\Gamma | X \sim Wei(\gamma_X, 2)$. The discrete-time version of a jump amplitude process is modeled as follows:

$$\Delta \ln V = \alpha \Delta t + D_4 \quad (19)$$

where D_4 denotes an increment of a stochastic jump component in discrete time with a mean arrival rate λ , and D_4 is expressed by

$$D_4 = \begin{cases} \psi & \text{with a probability of } \lambda \Delta t \\ 0 & \text{with a probability of } 1 - \lambda \Delta t \end{cases} \quad (20)$$

Since a jump amplitude process allows both positive and negative jumps, the estimation of the probability of up-jumps and down-jumps is important in specifying the process. By assuming $P(X=1)=0.5$, this means there is a 50-50 chance of up-jump and down-jump.

IV. SIMULATION RESULTS

To analyze the trigger price effect and the misspecification effect, a few parameter values are applied as the base case: the investment cost (I) are \$100; the risk free rate (r) is 8%; the annual cost of waiting (δ) is 4%; and the licensing period (τ) lasts 5 years. In addition, the initial project value (V_0) is assumed to be \$100 based on the idea that real options matter in investment decisions only when the NPV of the project is near zero. The simulation is based on 10,000 trials. The analysis then starts by assuming that there are no collection lags, i.e., $T=0$.

A. Consequence of Adopting Optimal Investment Trigger When Project Payoffs are Collected Immediately

The simulation result which summarizes the descriptive statistics of the distributions of Manager A and B is given in Table 2. Also, Figure 2-5 display the histograms of the distributions of Manager A and B, which demonstrate the trigger price effect under a GBM, MR, MX, or JA process, respectively. As mentioned, the distribution of Manager A basically reflects the realized project payoffs of an actual stochastic process, while the distributions of Manager B represent the realized project payoffs when the trigger price is adopted as the optimal investment rule. Based on the parameter values of the base case, the trigger prices of interest, V_{GBM}^* , V_{MR}^* , V_{MX}^* , and V_{JA}^* , are given by 278.08, 120.91, 143.96, and 217.50, respectively. By comparing the value distributions of Manager A and Manager B, it is apparent to observe how the optimal investment rules influence the distributions of corporate value.

According to Figure 2 – 5, the most obvious result is that the left side of the distributions of Manager B under all stochastic processes is truncated at the value of zero, indicating that the optimal investment policies effectively help management to prevent the downside risk of investment. This finding can be alternatively observed from a comparison between the skewness coefficient of Manager B and that of Manager A. As indicated in Table 2, the distributions of Manager B under any of the stochastic processes in general have higher coefficients of skewness and kurtosis than those of Manager A, hence suggesting that the adoption of optimal investment triggers leads to the fat-tailed, right-skewed distributions of corporate value. This evidence suggests that the adoption of optimal investment triggers has a positive impact on project payoffs of investment.

TABLE II DESCRIPTIVE STATISTICS OF THE DISTRIBUTIONS OF MANAGER A AND B

True Process	Manage	Min	Max	Mean	STD
GBM	A	-78.44	488.87	22.35	57.56
	B	0.00	488.87	4.62	33.10
Mean Reversion	A	-34.94	36.55	-2.16	9.96
	B	0.00	36.55	0.32	2.89
Mixed Jump	A	-100	471.94	-54.03	70.24
	B	0.00	471.94	10.29	34.97
Jump Amplitude	A	-60.53	255.00	26.00	33.96
	B	0.00	255.00	3.08	21.73

Note: As a Percentage of Investment Cost

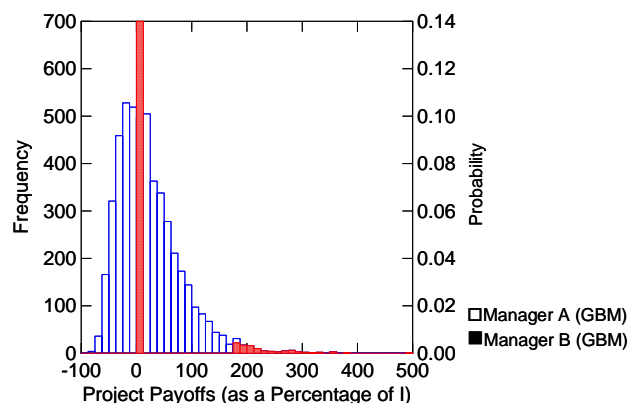


Fig. 2 The Effect of Adopting Optimal Investment Triggers, Given a GBM

Table 2 also reveals that both the mean and the standard deviation of realized net payoffs of Manager B are in general lower than those of Manager A, thus implying that when the optimal investment trigger is applied as a decision-making tool, investment risk can be reduced at the expense of lowering the expected rate of return. This is because the trigger price (V^*) is in general much higher than the investment cost (I), and adopting the trigger price as the optimal investment policy may result in the consequence that most investment opportunities are rejected with zero-payoff, thus lowering the mean and the variation of realized payoffs.

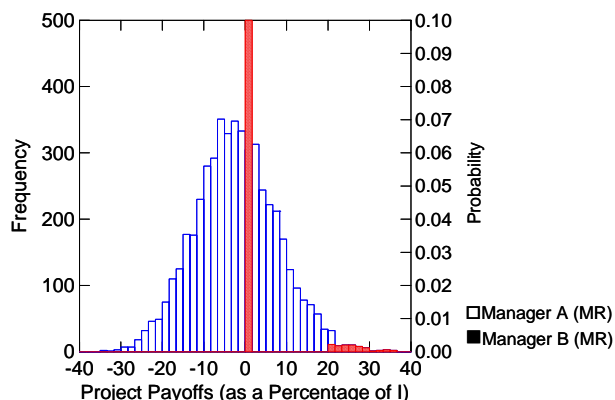


Fig. 3 The Effect of Adopting Optimal Investment Triggers, Given an MR Process

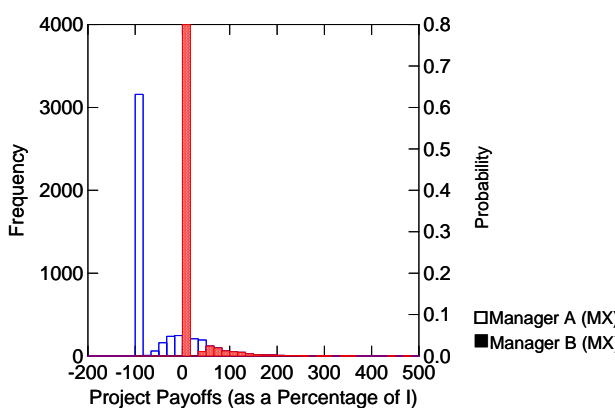


Fig. 4 The Effect of Adopting Optimal Investment Triggers, Given an MX Process

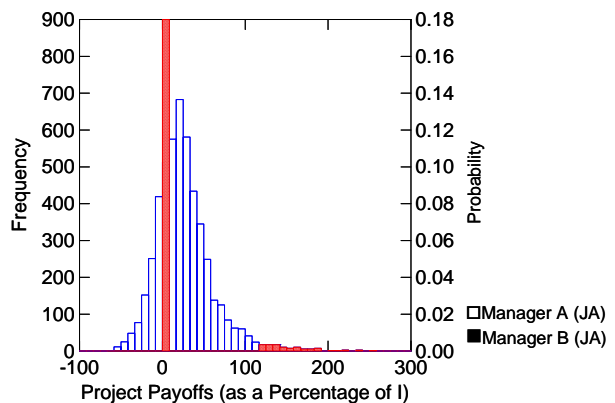


Fig. 5 The Effect of Adopting Optimal Investment Triggers, Given a JA Process

To sum up, the adoption of optimal investment triggers has a major impact on investment by truncating the downside risk and skewing the value distributions rightward. Furthermore, investment risk, in terms of the variation in project values, can be greatly reduced by the optimal investment policy, along with

a lower rate of return in consequence of rejecting most projects with zero-payoff. This result is consistent across all the stochastic processes of interest.

B. Misspecification Effect When Project Payoffs are Collected Immediately

By applying the same parameter values in the base case, the misspecification effect of stochastic process, i.e., mistaking an alternative process for a GBM, on investment is further explored. In addition to the value distributions of Manager B and C, three performance measures, based on investment losses, are also reported to gauge the probability of making mistaken decisions and loss ratios. The simulation results are given in Table 3, Table 4, and Figure 6–8: Table 3 summarizes the descriptive statistics of the distributions of Manager B and C; Table 4 provides three performance measures due to the misspecification effect; and Figure 6 – Figure 8 illustrate the histograms of the value distributions caused by the misspecification, given that the actual stochastic process is an MR, MX, and JA process, respectively.

There are several findings which can be generalized from the results. Firstly, the misspecification of mistaking an alternative process for a GBM in general shifts the value distributions to the left. This finding can be easily observed from the lower means and the negative skewness coefficients of Manager C in Table 3. Similarly, the histograms in Figure 6–8 also illustrate that the value distributions of Manager C are left-skewed.

Secondly, the probabilities of making the mistaken decisions, $P(\Omega \neq \Psi)$, given the actual processes are an MR, MX, and JA, are 3.26%, 12.22% and 3.60%, respectively. Since the GBM trigger, V_{GBM}^* , is much higher than the other triggers, V_{MR}^* , V_{MX}^* , and V_{JA}^* , $P(\Omega \neq \Psi)$ mostly results from the mistaken decisions of $\Omega = 0$ and $\Psi = 1$, i.e., mistakenly deferring the project while it should be taken on. $P(\Omega \neq \Psi)$ under an MX process is higher than that under the other two processes. The main reason is that $P(\Omega \neq \Psi)$ under an MX process also includes a higher probability of $\Omega = 1$ and $\Psi = 0$, i.e., mistakenly taking on the project while it should be deferred. It is known that an MX process is identical to a GBM as Poisson jumps are not considered by management. Therefore, there is a positive probability that management takes on an investment opportunity that has already been appropriated by potential competitors.

TABLE III DESCRIPTIVE STATISTICS OF THE DISTRIBUTIONS OF MANAGER B AND C

True Process	Manager	Min	Max	Mean	STD
Mean Reversion	B	0.00	36.55	0.32	2.89
	C	-36.55	20.54	-0.33	3.19
Mixed Jump	B	0.00	47.94	10.29	34.97
	C	-100.00	111.43	-1.04	11.33
Jump Amplitude	B	0.00	255.00	3.08	21.73
	C	-45.10	119.47	0.34	4.63

Note: As a Percentage of Investment Cost

Thirdly, Table 4 indicates that the conditional loss ratios, $\bar{\pi}|_{\Omega \neq \Psi}$, for the MX and JA processes tend to be higher. This is mainly because both MX and JA processes contain a jump

component with an average jump size of 10%. Our sensitivity analysis reveals that $\bar{\pi}|_{\Omega \neq \Psi}$ increases with the jump size.

TABLE IV THREE PERFORMANCE MEASURES OF THE MISSPECIFICATION EFFECT

True Process\ Measures	$P(\Omega \neq \Psi)$	$\bar{\pi}$	$\bar{\pi} _{\Omega \neq \Psi}$
Mean Reversion	3.26%	0.47	14.39
Mixed Jump	12.22%	11.53	94.37
Jump Amplitude	3.60%	3.49	96.95

Note: Loss ratios are expressed as a percentage of investment cost.

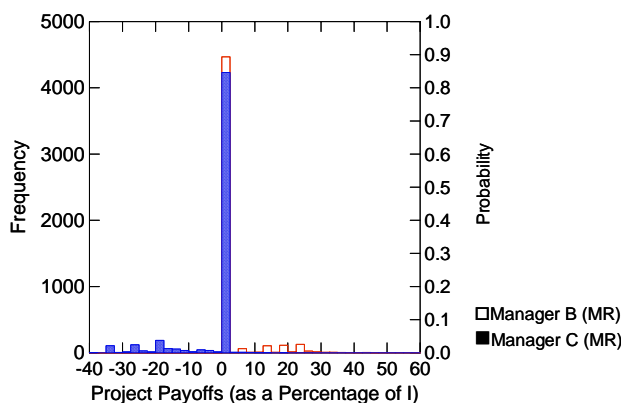


Fig. 6 The Effect of Misspecifying an MR for a GBM

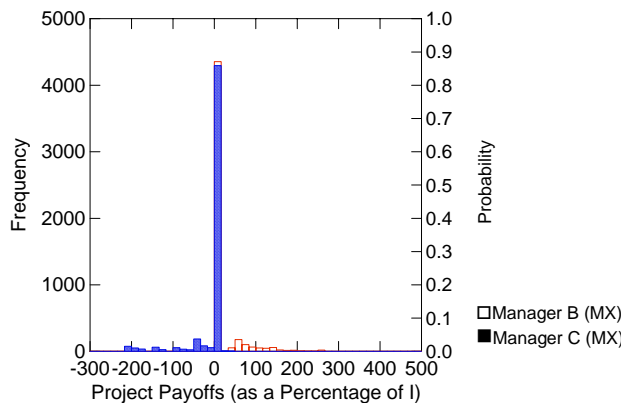


Fig. 7 The Effect of Misspecifying an MX for a GBM

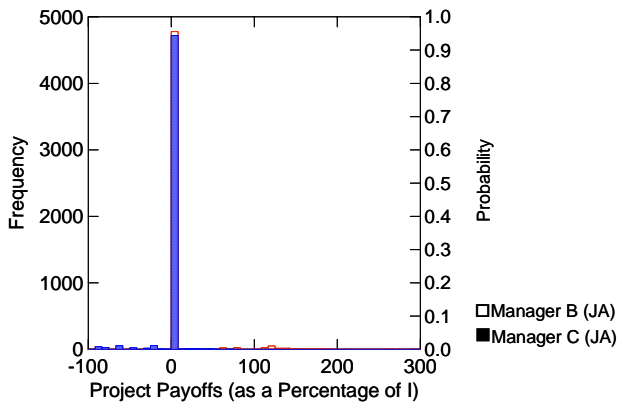


Fig. 8 The Effect of Misspecifying a JA for a GBM

To sum up, the misspecification effect has a major impact on realized project payoffs of investment in the way of moving the value distributions leftward with a lower expected rate of return. The misspecification effect also results in a positive probability of making mistaken decisions, depending on the actual stochastic process. Finally, it is demonstrated that the GBM assumption may cause a greater loss in value in the misspecification of stochastic process where the actual stochastic process contains a jump component.

C. Consequence of Adopting Optimal Investment Trigger When There are Collection Lags

Thus far, the analysis is based on the assumption that the project is delivered without time lags and thus project payoffs are immediately obtained after the decision to invest is made. However, capital investments, in reality, take time to build and to deliver, and are thus termed construction lags and delivery lags, respectively.[15] In addition to these two types of investment lags, there is the third type of investment lags, collection lags, which describe the delays in collecting project payoffs. Even where there are no construction lags and delivery lags, capital investments are still exposed to uncertainty due to collection lags. In particular, the presence of collection lags is one of the major characteristics that distinguish real options from financial options. Earlier studies on irreversible investments mostly center on the effect of construction lags and delivery lags on the value of options. For example, Majd and Pindyck [10] find that the time-to-build effect tends to decrease the delay on investment due to waiting and increase the option value under uncertainty. Bar-Ilan and Strange [1] demonstrate that construction lags may offset uncertainty and more uncertainty may speed up investment in the presence of long lags.

While Majd and Pindyck [10], Bar-Ilan and Strange [1], and others focus on investigating the effects of construction lags and delivery lags on investment in terms of the value of options, few studies have addressed the issues of collection lags. The analysis in the preceding subsections can be easily extended to investigate the trigger price effect and the misspecification effect on investment in the presence of collection lags. For this purpose, the collection lags are assumed to be 5 years, i.e., $T=5$, after the investment decision is made.

The results of adopting optimal investment triggers in the presence of collection lags are summarized in Table 5 and Figures 9 – 12: Table 5 provides the descriptive statistics; and Figures 9 – 12 illustrate the histograms of the distributions of Manager A and B, given that the underlying process follows a GBM, MR, MX, or JA process, respectively. It can be easily seen that the value distributions of Manager A in the existence of collection lags are justified by the optimal investment rules in a very similar way to that assumed no collection lags. Firstly, as shown in Table 5, the coefficients of skewness and kurtosis of Manager B are in general much higher than those of Manager A, hence suggesting that the distributions of corporate value are relatively right-skewed, fat-tailed when the optimal investment rules are applied. This finding is consistent with the value distributions in Figure 9 – 12. Secondly, optimal investment triggers also help management prevent the downside risk in the way of truncating the value distributions truncated with a lower rate of return and a lower variation in project returns, according to the means and standard deviations reported in Table 5. However, when collection lags exist, the value distributions of Manager B are no longer truncated at zero, but instead there is a slight chance that the rate of project return may become negative. The intuition underlying the result is that there is a positive probability that the project value may drop to a lower level after the project is triggered. Thus, there is always uncertainty due to collection lags, i.e., the period between τ and T , even though the investment decisions are made optimally at time τ . By comparing the standard deviations in Table 2 to those in Table 5, it becomes apparent that the variation in project returns increases in the presence of collection lags. Our sensitivity analysis indicates that the longer the collections lags are, the higher the variation in project returns is.

To sum up, the adoption of optimal investment triggers, in the presence of collection lags, still has a major impact on investment by moving the value distributions to the right. However, the existence of collection lags may expose investment to higher uncertainty in terms of realized project payoffs, compared to the case of no collection lags.

TABLE V DESCRIPTIVE STATISTICS OF THE DISTRIBUTIONS OF MANAGER A AND B

True Process	Manager	Min	Max	Mean	STD
GBM	A	-91.62	977.20	48.93	104.77
	B	-11.25	977.20	9.38	62.77
Mean Reversion	A	-40.06	33.65	-1.94	9.85
	B	-40.06	31.54	-0.88	6.53
Mixed Jump	A	-100	924.79	-44.70	96.24
	B	-51.73	924.79	13.74	59.89
Jump Amplitude	A	-85.46	868.31	55.69	88.21
	B	-11.26	868.31	5.70	43.74

Note: As a Percentage of Investment Cost

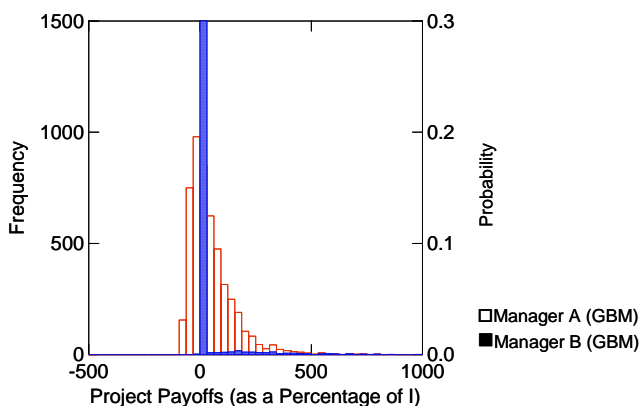


Fig. 9 The Effect of Adopting Optimal Investment Triggers When Collection Lags Exist, Given a GBM Process

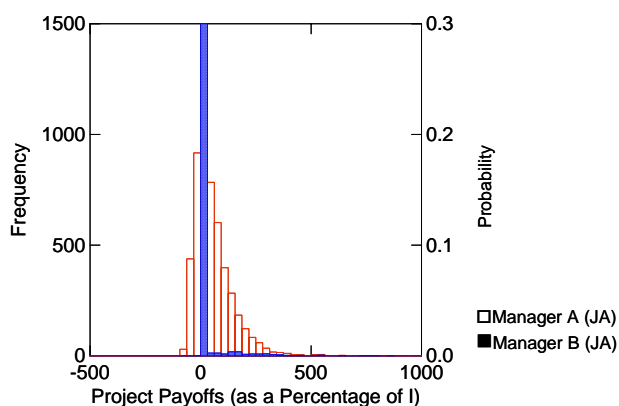


Fig. 12 The Effect of Adopting Optimal Investment Triggers When Collection Lags Exist, Given a JA Process

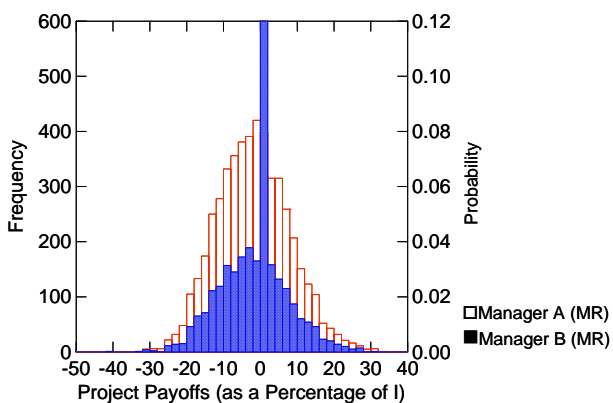


Fig. 10 The Effect of Adopting Optimal Investment Triggers When Collection Lags Exist, Given an MR Process

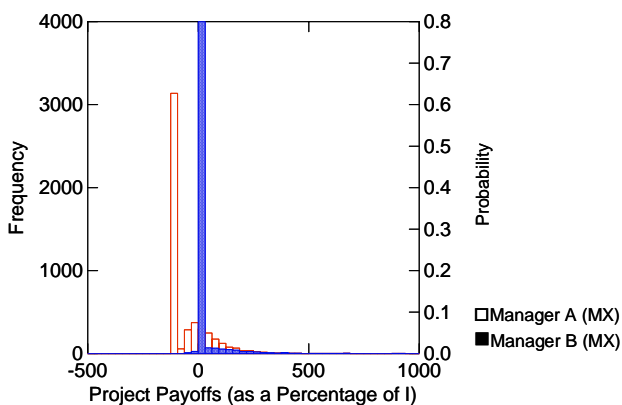


Fig. 11 The Effect of Adopting Optimal Investment Triggers When Collection Lags Exist, Given a MX Process

D. Misspecification Effect When There are Collection Lags

In this subsection, the misspecification effect on investment is examined with collection lags allowed. Table 6 reports the descriptive statistics and Table 7 exhibits the performance measures. Since the histograms of the project payoffs of Manager C cannot illustrate the misspecification effect in an apparent way, the normalized density curves are demonstrated in Figure 13–15, given the actual process is MR, MX or JA, respectively.

There are several findings revealed from the result. Firstly, from the means reported in Table 6, the misspecification of stochastic process appears to have a relatively minor effect on lowering the expected rate of return. When the actual process is an MR, the investment behavior of Manager B does not dominate that of Manager C in terms of the expected rate of return. The density curves in Figures 13–15 also indicate a consistent result that the misspecification effect appears to have a relatively minor impact on the project values. Secondly, in terms of the standard deviations of Manager C in Table 3 and 6, there is in general a greater variation in the realized project payoffs when the collection lags exist. Thirdly, the coefficients of skewness indicate the misspecification effect, in the existence of collection lags, still has an influence on investment in a similar way of no collection lags by shifting the distributions of corporate value leftward, yet the moving of the distributions is less obvious than the case of no collection lags. Fourthly, there is still a positive $P(\Omega \neq \Psi)$ in the three types of misspecification, while both $\bar{\pi}$ and $\bar{\pi}|_{\Omega \neq \Psi}$, in the presence of collection lags, are amplified in consequence of the uncertainty due to the collection lags.

TABLE IX DESCRIPTIVE STATISTICS OF THE DISTRIBUTIONS OF MANAGER B AND C

True Process	Manage	Min	Max	Mean	STD
Mean Reversion	B	-36.41	37.91	0.04	10.12
	C	-37.91	37.62	1.79	9.96
Mixed Jump	B	-406.31	928.16	53.59	92.81
	C	-928.16	594.72	43.05	98.14
Jump Amplitude	B	-712.28	984.71	-43.94	95.05
	C	-984.71	397.06	-52.72	91.00

Note: As a Percentage of Investment Cost

To sum up, the misspecification of stochastic process, in the presence of collections lags, has an effect on shifting the distributions of corporate value leftward. However, the shifting of the left-skewed distributions, in the existence of collection lags, is relatively minor, compared to the situations where the project payoffs are collected immediately after the decision is made. In addition, collection lags appear to increase the loss ratios in consequence of uncertainty during the lag time.

TABLE VII THREE PERFORMANCE MEASURES OF THE MISSPECIFICATION EFFECT

True Proc/Measure	$P(\Omega \neq \Psi)$	$\bar{\pi}$	$\bar{\pi} _{\Omega \neq \Psi}$
Mean Reversion	3.84%	3.71	96.61
Mixed Jump	12.34%	32.27	261.51
Jump Amplitude	4.06%	8.58	172.98

Note: Loss ratios are expressed as a percentage of investment cost.

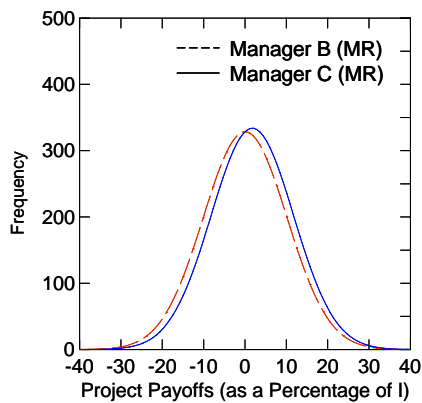


Fig. 13 The Effect of Misspecifying an MR for a GBM (Collection Lags)

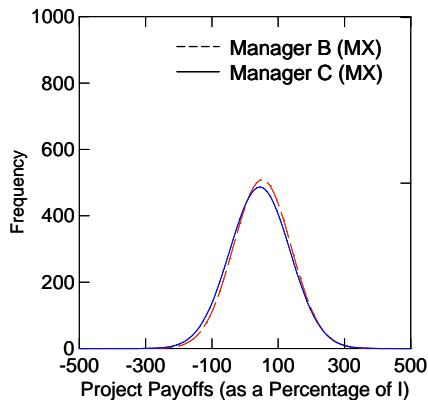


Fig. 14 The Effect of Misspecifying an MX for a GBM (Collection Lags)

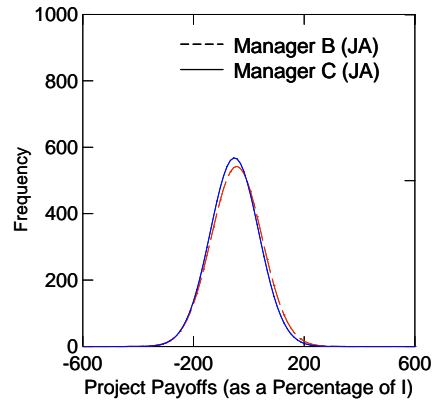


Fig. 15 The Effect of Misspecifying a JA for a GBM (Collection Lags)

V. CONCLUDING REMARKS

This paper explores the consequence of adopting optimal investment triggers and the effect of misspecification of stochastic process on investment, particularly on the distributions of corporate values. The general result is that adopting optimal investment triggers may increase corporate values by shifting the value distributions rightward, and the misspecification effect may decrease corporate values by shifting the value distributions leftward. Specifically, the adoption of optimal investment triggers has a significant impact on investment to such an extent that the downside risk of investment is truncated at the project value of zero, thereby moving the value distributions rightward. It has also been found that investment risk, in terms of the variation in project values, can be reduced by applying the optimal trigger, along with a lower rate of return in consequence of rejecting most projects with zero-payoff. On the other hand, the effect of misspecification of stochastic process also has a major impact on investment in the way that it moves the value distributions leftward. Furthermore, the misspecification effect causes a positive probability of making mistaken decisions and losses in value, depending on the actual stochastic process. The losses in value become especially substantial when the actual stochastic process contains a jump component.

Finally, by extending the analytical framework to the situations where collection lags are in place, we found that both the consequence of adopting optimal triggers and the effect of misspecification, in the presence of collection lags, still have an impact on investment. However, since collection lags appears to bring about higher uncertainty during the collection period, the adoption of optimal triggers does not truncate the distributions at the value of zero, thus leading to a higher variation in project values. In addition, the misspecification of stochastic process, in the presence of collections lags, has a relatively minor effect on shifting the distributions of corporate value leftward, compared to the case of no collection lags. However, collection lags increase the conditional and unconditional loss ratios in consequence of increased uncertainty. Therefore, collection lags reduce the influence of adopting optimal investment triggers and the misspecification

effect on investment in a different way.

REFERENCES

- [1]. Bar-Ilan, Avner and Strange, William C. "Investment Lags." *American Economic Review* 86 (1996), 610-622.
- [2]. Corman, Linda. "To Wait or Not to Wait." *CFO Magazine* 13, 5 (1997), 91-94.
- [3]. Dixit, Avinash K. and Pindyck, Robert S. *Investment under Uncertainty* (1994). Princeton University Press, New Jersey, USA.
- [4]. Faulkner, Terrence W. "Applying 'Option Thinking' to R&D Valuation." *Research Technology Management* 39, 3 (1996), 50-56.
- [5]. Geske, Robert. "The Valuation of Corporate Liabilities as Compound Options," *Journal of Financial and Quantitative Analysis* 12 (1977), 541-552.
- [6]. Geske, Robert. "The Valuation of Compound Options." *Journal of Financial Economics* 7 (1979), 63-81.
- [7]. Hamilton, James D. *Time Series Analysis*. Princeton University Press (1994). New Jersey, USA.
- [8]. Jensen, Kjeld and Warren, Paul. "The Use of Options Theory to Value Research in the Service Sector." *R&D Management* 31, 2 (2001), 173-180.
- [9]. Lint, Onno and Pennings, Enrico. "An Option Approach to the New Product Development Process: a Case Study at Phillips Electronics." *R&D Management* 31, 2 (2001), 163-172.
- [10]. Majd, Saman and Pindyck, Robert S. "Time to Build, Option Value, and Investment Decisions." *Journal of Financial Economics* 18, 1 (1987), 7-27.
- [11]. McDonald, Robert and Siegel, Daniel. "The Value of Waiting to Invest." *Quarterly Journal of Economics* 101 (1986), 707-728.
- [12]. Merton, Robert C., "Option Pricing When Underlying Stock Returns are Discontinuous." *Journal of Financial Economics* 3 (1976), 125-144.
- [13]. Morgan, M. Granger and Henrion, Max. *Uncertainty*. Cambridge University Press (1990). Cambridge, UK.
- [14]. Nichols, Nancy A. "Scientific Management at Merck: an Interview with CFO Judy Lewent." *Harvard Business Review* 72 (1994), 88-99.
- [15]. Peeters, Marga. "Investment Gestation Lags: the Difference between Time-to-Build and Delivery Lags." *Applied Economics* 28 (1996), 203-208.
- [16]. Pennings, Enrico and Lint, Ono. "The Option Value of Advanced R&D." *European Journal of Operational Research* 103, 1 (1997), 83-94.
- [17]. Sender, Gary L. "Option Analysis at Merck." *Harvard Business Review* 72 (1994), 92.
- [18]. Trigeorgis, Lenos. "Valuing the Impact of the Uncertain Competitive Arrivals on Deferrable Real Investment Opportunities." Working Paper, Boston University (1990).
- [19]. Turvey, Calum G. "Mycogen as a Case Study in Real Options," *Review of Agricultural Economics* 23, 1 (2001), 243-264.
- [20]. Wang, Yungchih G. "Topics in Investment Appraisal and Real Options" PhD Thesis, Imperial College, University of London (2004).

Dr. George Yungchih Wang received his PhD in Finance and Economics from Imperial College, University of London, UK, and his MBA from University of Connecticut, USA. He is currently an assistant professor at National Kaohsiung University of Applied Sciences, Taiwan, and is also a visiting professor at University of Wisconsin, La Crosse, USA. His major research area is in corporate finance, investment appraisal, and corporate governance.