A Scatter Search and Help Policies Approaches for a New Mixed Model Assembly Lines Sequencing Problem

N. Manavizadeh , M. Rabbani , H. Sotudian , F. Jolai

Abstract—Mixed Model Production is the practice of assembling several distinct and different models of a product on the same assembly line without changeovers and then sequencing those models in a way that smoothes the demand for upstream components. In this paper, we consider an objective function which minimizes total stoppage time and total idle time and it is presented sequence dependent set up time. Many studies have been done on the mixed model assembly lines. But in this paper we specifically focused on reducing the idle times. This is possible through various help policies. For improving the solutions, some cases developed and about 40 tests problem was considered. We use scatter search for optimization and for showing the efficiency of our algorithm, experimental results shows behavior of method. Scatter search and help policies can produce high quality answers, so it has been used in this paper.

Keywords—mixed model assembly lines, Scatter search, help policies, idle time, Stoppage time

I. INTRODUCTION

MIXED model assembly lines are a type of production line with different models on the same line. These models have some similar characteristics that let us to assemble them on a same line. Fig. 1 presents difference between the types of assembly lines. Single model lines, multi model lines and mixed model lines can assemble different types of products without changing setup.
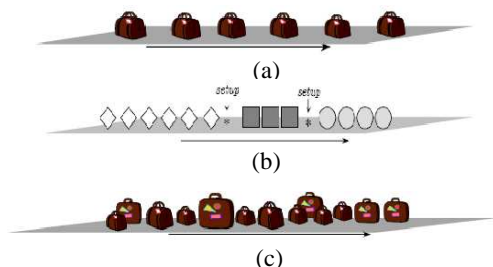

Fig. 1 (a) Single model assembly line (b) multi model assembly line (c) mixed model assembly line

Implementing a mixed model assembly line requires solving two different problems:
a. Balancing the line
b. Determining the productions sequence

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In this paper, we assume our lines are fixed and we cannot change the balance of the lines and we should only solve the sequencing problem. Mixed model sequencing problems focus on some different objective functions
a. Trying to have a smooth line and minimizing work overload.
b. As different models sequence in line and set up time will be a critical factor so minimizing set up time is another objective function.
c. When we have not fixed station length, minimizing line length is an objective function.
d. For minimizing the costs, workers idle time should be minimized.
e. As the importance of machines time and number of products that must be produced minimizing stoppage time is another objective function.

All of above items can be considered as cost manner. At this paper we are going to minimize stoppage time and idle time simultaneously in a special line. Characteristics of this line described in section 2.

Reference [1] has Considered Minimizing stoppage time and proofed some lemma for upper and lower bound of problem also OHNO solved problem by using branch-and-bound method for small size problems [14],[15]. In order to improve objective function Giovanni Celanoa et al implemented some help policies that each of them can decrease total Stoppage time [2]. To decrease stoppage time by using some policies or utility workers jae kyu yoo used aid of relief man and formulates a new mixed model sequencing problem also VincentGiard implemented some utility workers and the number of these workers and the number of sequence dependent setups are to be optimized simultaneously through a cost function [3], [4].

Considering total stoppage time and total idle time simultaneously as objective function has come in several researches. Moreno described a special line with some characteristics and then modeled a dynamic formulation for minimizing total stoppage time and idle time simultaneously this model has great advantages. This paper used close stations but PAN2 considered a line with open and close stations[5], [16]. Actually using close stations in line compared with open stations in line.

As the impact of setup time on stoppage time and idle time Yeo Keun Kim and Siwon Kim assumed a sequence dependent set up time on line for that objective[6], [17]. Kara et al. presented a multi-objective approach for balancing and sequencing mixed-model U-lines to simultaneously minimize the absolute deviations of workloads across workstations, part usage rate, and cost of setups is presented [7]. To increase the performance of the proposed algorithm, a newly developed neighborhood generation method is also employed.

Since the performance measures considered in the study are conflicting with each other, the proposed algorithm suggests much flexibility and more realistic results to decision makers. Karabatı and Sayın studied the problem in an assembly line environment with synchronous transfer of parts between the stations. They formulated the assembly line balancing problem with the objective of minimizing total cycle time by incorporating the cyclic sequencing information. They showed that the solution of a mathematical model that combines multiple models into a single one by adding up operation times constitutes a lower bound for this formulation [8].Although many researchers have focused on stoppage time as an objective function. Tavakkoli-Moghaddam et al and Alireza Rahimi-Vahed et al assumed three objective functions: a) total utility work. B) Total production rate variation. And c) total setup cost [9], [10]. In addition, optimization of production rate and number of setups considered in [11]. Fattahi and Salehi presented a hybrid metaheuristic algorithm based on the simulated annealing (SA) approach to solve the problem. The problem was to minimize the idle and utility time cost with a variable launching interval between products on the assembly line [12]. Battini et al. proposed an innovative balancing–sequencing step-by-step procedure that aims to optimize the assembly line performance and at the same time contain the buffer dimensions in function of different market demand and production mix. The model is validated using simulation software and an industrial application is presented [13]. In section 2 we described our assumptions, in section 3 and 4 a dynamic formulation developed. In section 5 four new help policies are described for improvement of objective function. In section 6 and 7 optimization approach described and 40 runs of problem implemented.

II. ASSUMPTIONS

We consider an assembly line by following conditions:

A. Characteristics of the stations:

- This line has close stations. it means that each station has two fixed boundaries and conveyor moves from left to right in up-lines and moves from left to right in down-lines.

- When a worker cannot finish the task on a part in the station, the line is stopped. So in this paper, we considered some help policies and then compared them together. These help policies means that additional workers help the critical worker so concurrent work on a part is allowed

- Processing time is deterministic in the stations and we do not have parallel stations.

- Set up time & set up cost is considered. Set up time is sequence dependent and it needs worker so setting up will be done in station boundaries.

B. Characteristics of the line:

- Number of station is given and all stations are the same.

- Because of help policies velocity of workers is considered.

C. Objective function:

Our objective function is to minimize stoppage time and idle time simultaneously.

This line has K stations and one worker assigned to each station. N products (parts) should be produced in line; these parts are from M models.

III. NOTATION AND PARAMETERS

t_m^k	Operation time in station k for model M
$\pi(n)$	The nth unit in sequence
L^k	Length of workstation k
p_n^k	Starting position of nth unit in station kth from left boundary of station k
f_n^k	completing position of nth unit in station kth from left boundary of station k
st_n^k	Stoppage time caused by worker k while he has been working on nth
it_n^k	Idle time of worker k after completing nth
$t\_p_n^k$	Time that worker k begins working on nth
$t\_f_n^k$	Time that worker k finishes working on nth
$t\_fw_n^k$	Time the operator meets the (n+1)th part when he/she has finished the nth unit or the time when he arrives at the left boundary of the station k even though the (n+1)th has not yet arrived
S	Conveyor stoppage time
t_{fs}	Time when the conveyor stops
L_{labs}^k	Right side position of station k
L_{sabs}^n	Position of the nth unit from the left boundary of the first station when the conveyor stoppage happens
S_{mr}^k	Set up time in station k when model r sequence after m
X_{mr}^k	$\begin{cases} 1 & \text{if model r sequence after m} \\ 0 & \text{O.W} \end{cases}$
v_c	Speed of the conveyer
v_k	Speed of worker k
v_u	Speed of utility worker
t_c	Cycle time
pos^k	Position of worker k when line stopped
pos_t^u	Position of utility worker when line stopped
s_n^k	position of starting set up for nth unit in station kth from left boundary of station k
$t\_set_n^k$	Time that worker k begins set up on nth

IV. EDITORIAL POLICY THE MODEL FORMULATION

So with above explanation, we have this objective function:

$$\sum_{k=1}^{K} \sum_{n=1}^{M} st_n^k + it_n^k$$

Conveyor moves with speed of v_c in the line and before first stoppage occurs, we have following equations:

$$f_n^k = p_n^k + v_c \cdot t_{\pi(n)}^k \qquad (1)$$

$$st_n^k = 0 \qquad (2)$$

Distance between two parts is $v_c t_c$ so according to following equations we can calculate distance which worker should walk:

$$\frac{d_k}{v_k} = \frac{t_c v_c - d_k}{v_c} \text{ or } d_k = \frac{v_c v_k t_c}{v_c + v_k} \qquad (3)$$

$$it_n^k = (\frac{1}{v_c} + \frac{1}{v_k}) \max (d_k - f_n^k, 0) \qquad (4)$$

When two different models are assembled in line so we will have sequence dependent set up time and distance which worker walks for setting up and we have:

$$stu_{mr}^k = \sum_{m=1}^{n} \sum_{r=1}^{n} S_{mr}^k \cdot X_{mr}^k \cdot v_c \qquad (5)$$

$$s_{n+1}^k = \max(f_n^k - d_k, 0) \qquad (6)$$

$$p_{n+1}^k = s_{n+1}^k + stu_{mr}^k \qquad (7)$$

$$t\_f_n^k = t\_p_n^k + t_{\pi(n)}^k \qquad (8)$$

If $f_n^k - d_k < 0$ so idle time exists and worker is idle at boundaries of station so we will have following equations:

$$t\_fw_n^k = t\_f_n^k + \frac{f_n^k}{v_k} \qquad (9)$$

$$t\_set_{n+1}^k = t\_fw_n^k + it_n^k \qquad (10)$$

$$t\_p_{n+1}^k = t\_set_{n+1}^k \frac{stu_{mr}^k}{v_c} \qquad (11)$$

And if $f_n^k - d_k > 0$ we do not have idle time then equations are as below:

$$t\_fw_n^k = t\_f_n^k + \frac{d_k}{v_k} \qquad (12)$$

$$t\_set_{n+1}^k = t\_fw_n^k \qquad (13)$$

$$t\_p_{n+1}^k = t\_set_{n+1}^k + \frac{stu_k^n}{v_c} \qquad (14)$$

If kth worker is not able to finish the task in kth station, it means that $p_n^k + v_c t_{\pi(n)}^k > L^k$ and we have following equations for this station:

$$f_n^k = L^k \qquad (15)$$

$$st_n^k = \frac{1}{v_c}(p_n^k + v_c t_{\pi(n)}^k - L^k) = S \qquad (16)$$

$$it_n^k = 0 \qquad (17)$$

$$s_{n+1}^k = L^k - d_k \qquad (18)$$

$$p_{n+1}^k = t\_set_{n+1}^k + stu_k^n \qquad (19)$$

$$t\_f_n^k = t\_p_n^k + t_{\pi(n)}^k \qquad (20)$$

$$t_{fs} = t\_f_n^k - S \qquad (21)$$

$$t\_fw_n^k = t\_f_n^k + \frac{d_k}{v_k} \qquad (22)$$

$$t\_set_{n+1}^k = t\_fw_n^k \qquad (23)$$

$$t\_p_{n+1}^k = t\_set_{n+1}^k + \frac{stu_k^n}{v_c} \qquad (24)$$

When k th worker stops the line, in other stations we will have one of following situations

Case1:

In case 1 Worker has finished his work and now walking to left boundary of station so we have following situations in case 1

$$pos^k = f_n^k - v_k(t_{fs} - t_{f_n^k}) \qquad (25)$$

Case 1-1:

Next part is in station. In case 1-1 we assume that $\widetilde{t_{\pi(n)}^k}$ is remaining walking time and operator have enough time to work on next part so equations are:

$$\widetilde{t_{\pi(n)}^k} = \frac{pos^k - L_{sabs}^{n+1}}{v_k} \qquad (26)$$

$$it_n^k = 0 \qquad (27)$$

$$s_{n+1}^k = f_n^k - d_k - \frac{v_c v_k}{v_c + v_k} \widetilde{t_{\pi(n)}^k} \qquad (28)$$

In case 1-1 we have two different conditions. Case 1-1-a occurs at some conditions that the line starts when worker is in setting up the line because:

$$p_{n+1}^k = s_{n+1}^k + (\frac{stu_{mr}^k}{v_c} - S - t_{fs} + t_{set_{n+1}}^k)v_c \qquad (29)$$

$$t\_set_{n+1}^k \leftarrow t_{fs} + S \qquad (30)$$

$$stu_{mr}^k \leftarrow stu_{mr}^k - v_c(t_{fs} + S - t_{set_n}^k) \qquad (31)$$

And in case 1-1-b line starts after set up finished and now operator works on part so:

$$t_{\pi(n+1)}^k \leftarrow t_{\pi(n+1)}^k - (S - \widetilde{t_{\pi(n)}^k} - stu_{mr}^k) \qquad (32)$$

$$t\_fw_n^k = t\_set_{n+1}^k = t\_f_n^k + \frac{f_n^k - p_{n+1}^k - stu_{mr}^k}{v_k} \qquad (33)$$

$$p_{n+1}^k = f_n^k - d_k - \frac{v_k v_c \widetilde{t_{\pi(n)}^k}}{v_c + v_k} \qquad (34)$$

$$t\_p_{n+1}^k = t_{fs} + S \qquad (35)$$

Case 1-2:

In this case worker after walking do not have enough time to reach next part so:

$$it_n^k = 0 \qquad (36)$$

$$p_{n+1}^k = \frac{1}{v_c + v_k}\{v_c v_k(t\_f_n^k - t_{fs} - S) + v_c(L_{labs}^{k-1} + f_n^k + v_k L_{sabs}^{n+1}) - L_{labs}^{k-1} + stu_{mr}^k \qquad (37)$$

$$t\_fw_n^k = t\_set_{n+1}^k = \frac{1}{v_c + v_k}\{v_k t\_f_n^k + v_c(t_{fs} + S + L_{labs}^{k-1} - L_{sabs}^{n+1} + f_n^k\} \qquad (38)$$

$$s_{n+1}^k = \frac{1}{v_c + v_k}\{v_c v_k(t\_f_n^k - t_{fs} - S) + v_c(L_{labs}^{k-1} + f_n^k + v_k L_{sabs}^{n+1}) - L_{labs}^{k-1} \qquad (39)$$

$$t\_p_{n+1}^k = t\_fw_n^k + \frac{stu_{mr}^k}{v_c} \qquad (40)$$

Case1-3:

In this case, operator moves to assemble the next part after finishing previous part but next part has not came in the station, So he/she arrives at left boundary of station and was became idle.

$$it_n^k = (\frac{1}{v_c} + \frac{1}{v_k})(d_k - f_n^k) + S \qquad (41)$$

$$p_{n+1}^k = stu_{mr}^k \qquad (42)$$

$$s_n^k = 0 \qquad (43)$$

$$t\_fw_n^k = \frac{f_n^k}{v_k} + t\_f_n^k \qquad (44)$$

$$t\_set_{n+1}^k = t\_fw_n^k + it_n^k \qquad (45)$$

$$t\_p_{n+1}^k = t\_set_{n+1}^k + \frac{stu_{mr}^k}{v_c} \qquad (46)$$

Case 1-4:

Worker is not idle at left boundary of station because the next part will be entered into the station, so the line is run and worker does not reach to boundaries so:

Case 2:

In this case Operator is working on a part and remaining time is $\widetilde{t_{\pi(n)}^k}$.

$$pos^k = p_n^k + v_c(t_{fs} - t\_p_n^k) \tag{52}$$
$$\widetilde{t_{\pi(n)}^k} = t_{\pi(n)}^k - (t_{fs} - t\_p_n^k) \tag{53}$$

Case 2-1:

Operator is working on a part and remaining time ($\widetilde{t_{\pi(n)}^k}$) is more than length of stoppage time. The equations are:

$$p_n^k = p_n^k + v_c(t_{\pi(n)}^k - \widetilde{t_{\pi(n)}^k}) \tag{54}$$
$$t_{\pi(n)}^k \leftarrow \widetilde{t_{\pi(n)}^k} - S \tag{55}$$
$$t\_p_n^k = t_{fs} + S \tag{56}$$

Case 2-2:

This case is like 2-1 but worker has enough time to finish this part and walk downstream to next part (this part has entered station before) and he will work on it. So equations are:

$$f_n^k = p_n^k + v_c(t_{\pi(n)}^k - \widetilde{t_{\pi(n)}^k}) \tag{57}$$
$$st_n^k = 0 \tag{58}$$
$$it_n^k = 0 \tag{59}$$
$$t\_f_n^k = t_{fs} + \widetilde{t_{\pi(n)}^k} \tag{60}$$
$$t\_fw_n^k = t\_set_{n+1}^k = t\_f_n^k + \frac{v_c t_c}{v_k} \tag{61}$$
$$s_{n+1}^k = f_n^k - v_c t_c \tag{62}$$

But in this situation by considering set up we have two cases. Case 2-2-a occurs when line starts at set up time so

$$t\_set_{n+1}^k = t_{fs} + S \tag{63}$$
$$stu_{mr}^k \leftarrow stu_{mr}^k - v_c(S + \widetilde{t_{\pi(n)}^k} + (t_{fw_n}^k - t_{f_n}^k)) \tag{64}$$
$$p_{n+1}^k = s_{n+1}^k + v_c(\frac{stu_{mr}^k}{v_c} - S + \widetilde{t_{\pi(n)}^k} + (t_{fw_n}^k - t_{f_n}^k)) \tag{65}$$

case 2-2-b means that after set up operator has time for working on next part.

$$t_{\pi(n+1)}^k \leftarrow t_{\pi(n+1)}^k - (S - \widetilde{t_{\pi(n)}^k} - t_{fw_n}^k + t_{f_n}^k - \frac{stu_{mr}^k}{v_c}) \tag{66}$$
$$p_{n+1}^k = s_{n+1}^k \tag{67}$$
$$t\_p_{n+1}^k = t_{fs} + S \tag{68}$$

Case2-3:

This case is like case 2-2 but in this case worker does not have enough time to work on next part so we have equations 57 to 62 in addition as bellows:

$$s_{n+1}^k = f_n^k - t_c v_c + v_k v_c \frac{\frac{v_c t_c}{v_k} - S - \widetilde{t_{\pi(n)}^k}}{v_c + v_k} \tag{69}$$
$$p_{n+1}^k = s_{n+1}^k + stu_{mr}^k \tag{70}$$
$$t\_fw_n^k = t\_set_{n+1}^k = t\_f_n^k + \frac{f_n^k - s_{n+1}^k}{v_k} \tag{71}$$
$$t\_p_{n+1}^k = t\_fw_n^k + \frac{stu_{mr}^k}{v_c} \tag{72}$$

Case 2-4:

In this case after finishing the part, worker comes back but next part does not enter to station yet and worker reach to station boundary and become idle so we have following equations:

$$f_n^k = p_n^k + v_c(t_{\pi(n)}^k - \widetilde{t_{\pi(n)}^k}) \tag{73}$$
$$st_n^k = 0 \tag{74}$$
$$it_n^k = (\frac{1}{v_c} + \frac{1}{v_k})\max\{d_k - f_n^k + \frac{v_c v_k}{v_c + v_k}(S - \widetilde{t_{\pi(n)}^k}), 0\} \tag{75}$$
$$t\_f_n^k = t_{fs} + t_{\pi(n)}^k \tag{76}$$
$$p_{n+1}^k = \max\{f_n^k - \frac{v_c v_k}{v_c + v_k}(S - \widetilde{t_{\pi(n)}^k}) - d_k, 0\} + stu_{mr}^k \tag{77}$$

$$it_n^k = 0 \tag{47}$$
$$s_n^k = \left(\frac{1}{v_c + v_k}\right)\{v_c v_k(t_{f_n}^k - t_{fs} - S) + v_c(L_{labs}^{k-1} + f_n^k) + v_k L_{sabs}^{n+1}\} - L_{labs}^{k-1} \tag{48}$$
$$p_{n+1}^k = s_{n+1}^k + stu_{mr}^k \tag{49}$$
$$t\_fw_n^k = t\_set_{n+1}^k = \left(\frac{1}{v_c + v_k}\right)\{v_k t\_f_n^k + v_c(t_{fs} + S) + L_{labs}^{k-1} - L_{sabs}^{n+1} + f_n^k\} \tag{50}$$
$$t\_p_n^k = t\_fw_n^k + \frac{stu_{mr}^k}{v_c} \tag{51}$$

$$t\_fw_n^k = t\_f_n^k + \frac{f_n^k}{v_k} \tag{78}$$
$$t\_set_{n+1}^k = t\_fw_n^k + it_n^k \tag{79}$$
$$t\_p_{n+1}^k = t\_set_{n+1}^k + stu_{mr}^k \tag{80}$$
$$s_{n+1}^k = 0 \tag{81}$$

Case 2-5:

This case is like case 2-4 but worker does not reach to station's boundary on time, because the line has been started. So we have equations 73 to 77 and bellows:

$$t\_fw_n^k = t\_f_n^k + \frac{f_n^k - p_{n+1}^k}{v_k} \tag{82}$$
$$t\_set_{n+1}^k = t\_fw_n^k + it_n^k \tag{83}$$
$$t\_p_{n+1}^k = t\_fw_n^k + it_n^k + \frac{stu_{mr}^k}{v_c} \tag{84}$$
$$s_n^k = f_n^k - \frac{v_c v_k}{v_c + v_k}(S - \widetilde{t_{\pi(n)}^k}) - d_k \tag{85}$$

Case 3:

In this case, worker has finished n th part and now is idle at left boundary of station so:

$$it_n^k = it_n^k + S \tag{86}$$
$$t\_p_{n+1}^k = t\_fw_n^k + it_n^k + \frac{stu_k^n}{v_c} \tag{87}$$
$$s_n^k = 0 \tag{88}$$

Case 4:

In this case at the time of stoppage, worker is setting up the line. $\widetilde{stu_{mr}^k}$ is the remaining time of set up.

$$\widetilde{stu_{mr}^k} = stu_{mr}^k - v_c(t_{fs} - t\_set_{n+1}^k) \tag{89}$$
$$pos^k = s_{n+1}^k + v_c(t_{fs} - t\_set_{n+1}^k \tag{90}$$

Case 4-1:

And we know that $\widetilde{stu_{mr}^k} > S$.

$$stu_k^n = \widetilde{stu_{mr}^k} - Sv_c \tag{91}$$
$$t\_set_{n+1}^k = t_{fs} + S \tag{92}$$
$$s_{n+1}^k = pos^k \tag{93}$$

Case 4-2:

Worker has enough time to work on part after setting up but working on part will not be finished. So equations are:

$$p_n^k = s_n^k \qquad (94)$$

$$t\_p_n^k = t_{fs} + \widetilde{stu_{mr}^k} \qquad (95)$$

$$t_{\pi(n)}^k = t_{\pi(n)}^k - (S - \widetilde{stu_{mr}^k}) \qquad (96)$$

Case 4-3:

Worker after doing set up and working on part moves downstream and next part has entered to station and worker has time to work on next part.

$$f_n^k = pos^k \qquad (97)$$

$$p_{n+1}^k = f_n^k - v_c t_c \qquad (98)$$

$$t\_fw_n^k = t_{fs} + S - \widetilde{stu_{mr}^k} - t_{\pi(n)}^k - \frac{v_c t_c}{v_k} \qquad (99)$$

$$t\_p_{n+1}^k = t\_fw_n^k + \frac{\widetilde{stu_{mr}^k}}{v_c} \qquad (100)$$

$$s_n^k = p_{n+1}^k \qquad (101)$$

$$t\_set_{n+1}^k = t\_fw_n^k \qquad (102)$$

In this situation, if operator is in setting up of next part, the line will be started and we will have case 4-3-a

$$stu_{mr}^k \leftarrow stu_{mr} - v_c(t_{fs} + S - t\_fw_n^k) \qquad (103)$$

$$p_{n+1}^k = stu_{mr} + s_{n+1}^k \qquad (104)$$

$$t\_set_{n+1}^k = t_{fs} + S \qquad (105)$$

and for 4-3-b, equations are

$$t_{\pi(n+1)}^k = t_{fs} + S - t\_p_{n+1}^k \qquad (106)$$

$$t\_p_{n+1}^k = t_{fs} + S \qquad (107)$$

Case 4-4:

This case is like case 4-3 but worker does not have enough time to reach next part so:

$$f_n^k = pos^k \qquad (108)$$

$$p_{n+1}^k = f_n^k - \frac{v_k\{v_c t_c - v_k(S - \widetilde{stu_n^k} + t_{\pi(n)}^k)\}}{v_k v_c} + stu_{mr}^k \qquad (109)$$

$$t\_fw_n^k = t\_set_{n+1}^k = \frac{f_n^k - s_{n+1}^k}{v_k} \qquad (110)$$

$$t\_p_{n+1}^k = t\_fw_n^k + \frac{stu_{mr}^k}{v_c} \qquad (111)$$

Case 4-5:

When operator finish working on this part walk downstream. Because next part hasn't entered to station Operator become idle at left boundary of station.

$$f_n^k = pos^k \qquad (112)$$

$$t\_fw_n^k = t\_f_n^k + \frac{f_n^k}{v_k} \qquad (113)$$

$$p_{n+1}^k = stu_{mr}^k \qquad (114)$$

$$it_n^k = t_{fs} + S - t\_fw_n^k + (v_c t_c - f_n^k)/v_c \qquad (115)$$

$$t\_p_{n+1}^k = t\_fw_n^k + it_n^k + \frac{stu_{mr}^k}{v_c} \qquad (116)$$

Case 4-6:

This case is like case 4-5 but before reaching the left boundary, the line is started. In addition of above equations we have:

$$it_n^k = 0 \qquad (117)$$

$$s_n^k = f_n^k - \frac{v_k\{v_c t_c - v_k(t_{fs} + S - t\_f_n^k)\}}{v_c + v_k} \qquad (118)$$

$$p_{n+1}^k = s_n^k + stu_{mr}^k \qquad (119)$$

$$t\_fw_n^k = t\_set_{n+1}^k = \frac{f_n^k - s_{n+1}^k}{v_k} \qquad (120)$$

$$t\_p_{n+1}^k = t\_fw_n^k + \frac{stu_{mr}^k}{v_c} \qquad (121)$$

V. HELP POLICY

When line stoppage occurs we can apply some help policies to decrease stoppage time. We should consider two important points for choosing a help policy, first we should focus on length of helping time so if a help policy is chosen, and stoppage time will decrease because the worker helps the critical worker to finish the tasks, but another important factor is calculation time. If we want to test all stations for finding the best helping time then calculation time will increase obviously so if a help policy can balance between these two factors then it will be the best help policy. Since In our objective function we have two parts, help policies do not have affect on set up time, then for choosing the best help policies we should consider stoppage time and idle time together. In fact if a help policy is used to decrease the idle time, then stoppage time and idle time will decrease simultaneously. So we can have the following policies that each of them focuses on a special aspect of problem [2].

a) Central PC (CP): in this way when the line is stopped, a central PC for each station calculates the helping time. This calculation depends on distance between stations and position of worker who should be helped. If a worker is working on a part or in setting up now, first he/she should finish his work and then go for help but if he/she is idle or in moving between stations, he should go for help immediately. per above explanations we will have two cases below:

- Operator in walking or idle.

- Worker is working on a part and after finishing the current task, he/she can help to critical worker.

For each station we can calculate the stoppage time, also we calculate maximum helping time which worker should help critical worker in each station. By this policy we can be sure that the best worker has selected.

b) Vicinity help (VH): Although CP policy can be so good in decreasing stoppage time but it is obvious that in this policy calculation time will be increased. If we want to notice on calculation time then we can only calculate for neighbor stations. In other hand, by considering distance between stations neighbor worker is the best for help.

c) Idle worker help (IWH): because in our objective function in addition of stoppage time we have idle time so if we use an idle worker in our help policy, so stoppage time and idle time will decrease simultaneously. In IWH policy we only use calculations for idle worker and if we do not have idle workers so nobody will help critical worker.

d) Utility worker (UW): in this help policy we want to apply a new worker for help. We assumed that in addition of station workers, we have another worker that we named him/her utility worker. A utility worker is an operator that in ordinary times is idle but when stoppage line occurs then he goes and helps the critical worker. Although idle time will increase but stoppage time and calculation time will decrease more than usual policies [2].

VI. Optimization Approach

Since our problem is NP-Hard so for big size problems we can use scatter search as a Meta heuristic algorithm. For above problem we used a scatter search for minimizing our objective function. Scatter search is a revolutionary algorithm and it can fit for mixed model assembly line problem so good. Scatter search has 5 important element that each of them reflect on optimization as well.

- Diversification method that make P initial solutions. For this part of algorithm we made random initial solutions and then improved them by local search.

- Improvement method in our scatter search is local search and by using the method, the initial solutions and other solutions will be improved. Local search deals with improvement of objective function but separation among solution will be another important factor. So as the importance of separation reference set divided into two parts which the first part has the best objective values and the second part has the most diverse solutions.

- Reference set update method is a way for selecting a set of best solutions of initial solutions at first and then in each steps we can select better solutions for reference set. We used a static reference update method. Reference set is made of two kinds of answers some of them have good objective function and some of them have the maximum division.

- Subset generation method that is a way for selecting some subsets to combine them and make a new solution.

- Solution combination method is a way to make a new solution of subsets that we already selected from reference set. After combination we used improvement method again on each of new solutions to compare them with reference set to join this solution to the first set or not.

VII. Experimental Result

In this section, the effectiveness of help policies are tested in the problem and a wide set of tests has been implemented. Those experimental results have been tested by implementing two classes of problems:

a. Small sized problem: experiments are carried out on small sized problem. we have four stations and four models but number of parts and their MPS are variable from ten to thirteen in small size problems.

b. Large sized problems: the second class of experiments are implemented on large sized problems that each of them has ten stations and five models but as above, number of parts will be variable between twenty to twenty three.

4 different problems have been solved for each help policies and without help policies, so for each class we solved 20 problems. The problems were executed for two classes are totally 40 runs. The cycle time and speed of conveyor is 600 and 1 respectively. The lengths of stations have been selected out from a uniform distribution [720; 960] and processing time of each model selected from uniform distribution $[0.4l_k; 0.95l_k]$. Also set up times and speed of workers has been selected out from a uniform distribution [240; 360] and [200,300] respectively. As we mentioned in help policies, Utility worker is in some situations so speed of utility worker has been equaled to 300.

TABLE I
PROBLEM SETS

Problem	I	MPS	No. Feasible solutions
1	10	(3,2,2,3)	25200
2	11	(4,3,2,2)	69300
3	12	(5,2,3,2)	166320
4	13	(1,3,5,4)	360360
5	20	(6,2,2,5,5)	58663725120
6	21	(5,3,3,4,6)	684410126400
7	22	(4,4,5,6,3)	3764255695200
8	23	(7,6,2,4,4)	6184134356400

Set of problems have shown in TableI with their MPS. These experimental results are solved by using scatter search and results are shown in table II. According to table II, the objective values have been presented in different help policies in small and large size problems.

TABLE II
OBTAINED RESULTS FROM SCATTER SEARCH

	Small sized		Large sized	
No help	4033	3832	33551	56827
	3374	3894	54792	37327
CP	14	89	4406	2493
	186	249	2521	3418
VH	18	89	4406	2493
	186	249	2593	3485
IWH	25	89	4406	3485
	186	249	3447	4222
UW	27	97	8653	9189
	83	299	9809	9456

TABLE III
MEAN VALUES FOR EACH CLASS OF PROBLEM

	Mean value for small sized	Mean value for large sized
No help	3783.25	45624.25
CP	134.5	3209.5
VH	135.5	3244.25
IWH	137.25	3890
UW	126.5	9276.75

In Table III, the mean values for each small and large size problems have been presented. So in each policy and in each size, we introduced a value to compare the results. As is shown in table III, values in the case of No help has a great difference with case that help policies are implemented on them. Different help policies have a different effect on the objective functions so we used a two way ANOVA for both small sized and large sized problems. In following tables we considered policies as treatment and size of MPS as block so in each table effect of policies are tested. Table IV presents ANOVA table on small sized problem by considering F-contribution and comparing it with critical F factor. It is obvious that for small sized problems help policies present the same answer. But as showed in Fig. 2 different policy has different behaviors. CP, IWH and VH methods present the same answer but generally UW method presents different answer. By considering figure 2 and table 4 it will be advised that using one of CP, VH or IWH can be so useful implementing of them will be so easier than other policies in small sized problems and VH method can be the best choice.

TABLE IV
A TWO WAY ANOVA FOR SMALL SIZED PROBLEMS

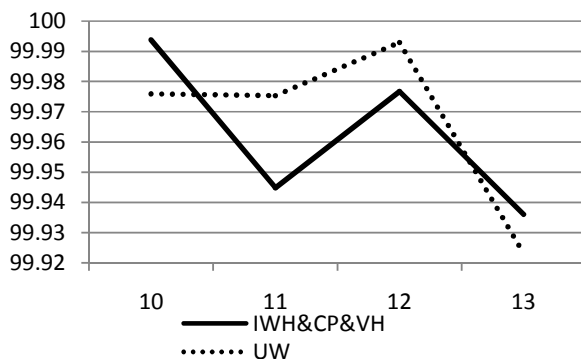| Source variation	Sum of squares	Degrees of freedom	Mean square	F
Help policies	272.19	3	90.72	0.084
Size of MPS	126248.18	3	42082.72	
Errors	9717.56	9	1079.72	
Total	136237	15		

Fig. 2 Percentage Variation diagram for small sized

TABLE V
A TWO WAY ANOVA FOR LARGE SIZED PROBLEMS

Source variation	Sum of squares	Degrees of freedom	Mean square	F
Help policies	103100935.25	3	34366978.42	89.42
Size of MPS	2848655	3	949551.75	
Errors	3458899.25	9	384322.13	
Total	109408489	15		

Per table V and F-contribution it is obvious that in large sized problems policies do not behave the same. So in this situation for finding out that which policy is different from other policies we used Least Significant Difference (LSD)

approach. Results are in table VI and we can conclude that UW had difference with other policies. In large sized problems due to Fig. 3 VH cannot be so good because it does not have a smooth manner so in large sized problem CP or IWH method can be proper ones.

TABLE VI
TEST OF LSD

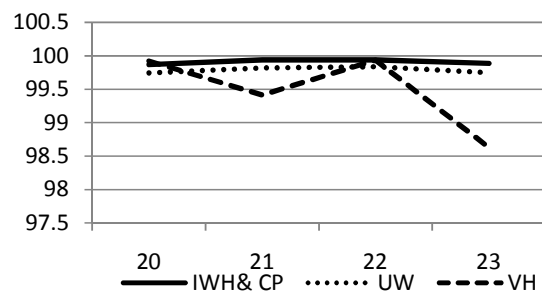|	LSD=991.57	
	Different means	equal or not
CP-VH	34.75	
CP-IWH	680.5	
CP-UW	6067.25	*
VH-IWH	645.25	
VH-UW	6032.5	*
IWH-UW	5386.75	*

Fig. 3 Percentage variation diagram for large sized problem
Per figures 2 and 3, the variations in large size problems are smaller than small size problems

VIII. CONCLUSION

Mixed Model Production is the practice of assembling several distinct and different models of a product on the same assembly line without changeovers and then sequencing those models in a way that smoothes the demand for upstream components. In this paper, we considered an objective function which minimizes total stoppage time and total idle time and it was presented sequence dependent set up time. For improving the solutions, some help policies developed and about 40 tests problem was considered. Experimental results show the big effect of help policies on optimizing objectives. Although in small sized problems all help policies has a same effect but in large sized problems their behavior is different. As above we can say that for small sized problems VH is the best policy and for large sized problems IWH or CP can be the best choice.

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