International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:5, No:3, 2011

Elastic-Plastic Analysis for Finite Deformation of a Rotating Disk Having Variable Thickness with Inclusion

Sanjeev Sharma, Manoj Sahni

***Abstract*—**Transition theory has been used to derive the elastic-plastic and transitional stresses. Results obtained have been discussed numerically and depicted graphically. It is observed that the rotating disk made of incompressible material with inclusion require higher angular speed to yield at the internal surface as compared to disk made of compressible material. It is seen that the radial and circumferential stresses are maximum at the internal surface with and without edge load (for flat disk). With the increase in thickness parameter (k = 2, 4), the circumferential stress is maximum at the external surface while the radial stress is maximum at the internal surface. From the figures drawn the disk with exponentially varying thickness (k = 2), high angular speed is required for initial yielding at internal surface as compared to flat disk and exponentially varying thickness for k = 4 onwards. It is concluded that the disk made of isotropic compressible material is on the safer side of the design as compared to disk made of isotropic incompressible material as it requires higher percentage increase in an angular speed to become fully plastic from its initial yielding.

***Keywords*—**Finite deformation, Incompressibility, Transitional stresses, Elastic-plastic.

I. INTRODUCTION

THIS paper is concerned with the analysis of a rotating disk made of isotropic material with exponentially varying thickness. There are many applications of such type of rotating disks, such as in turbines, rotors, flywheels and with the advent of computers, disk drives. The use of rotating disk in machinery and structural applications has generated considerable interest in many problems in domain of solid mechanics. The analysis of stress distribution in circular disk rotating at high speed is important for a better understanding of the behavior and optimum design of structures. The analysis of thin rotating disks made of isotropic material has been discussed extensively by Timoshenko and Goodier [1]. In the classical theory, solutions for such type of disks made of isotropic material can be found in most of standard textbooks [1]-[5]. Chakrabarty [2] and Heyman [6] solved the problem for the plastic state by utilizing the solution in the elastic range and considering the plastic state with the help of Tresca's, von-Mises or any other classical yield condition. Han [7] has investigated elastic and plastic stresses for isotropic materials with variable thickness. Eraslan [8] has calculated elastic and plastic stresses having variable thickness using Tresca's yield criterion, its associated flow rule and linear strain hardening. Wang [9] has investigated deformation of elastic half rings.

Transition is a natural phenomenon and there is hardly any branch of science or technology in which we do not come across transition from one state to another. At transition, the fundamental structure of the medium undergoes a change. The particles constituting a medium rearrange themselves and give rise to spin, rotation, vorticity and other non-linear effects. This suggests that at transition, non-linear terms are very important and neglection of which may not represent the real physical phenomenon. Therefore transition fields are non-linear, non-conservative and irreversible in nature. Elasticity-plasticity, visco-elastic, creep, fatigue, relaxation are some of the examples of transition in which non-linear terms are very important. At present, such problems like elastic-plastic, creep and fatigue are treated by assuming ad-hoc, semi-empirical laws with the result that discontinuities, singular surfaces, non-differentiable regions have to be introduced over which two successive states of a medium are matched together. In a series of papers, Seth [10]-[12] has given an entirely different orientation to this interesting problem of transition. He has developed a new 'transition theory' of elastic-plastic and creep deformation. The transition theory utilizes the concept of *generalized principal strain measure* and asymptotic solution at critical points or turning points of the differential system defining the deformed field and has been successfully applied to a large number of problems [13]-[19]. The generalized principal strain measure [19] is defined as,

$$e_{ii} = \int_0^{e_{ii}^A} [1-2e_{ii}^A]^{\frac{n}{2}-1} de_{ij}^A = \frac{1}{n}[1-(1-2e_{ii}^A)^{\frac{n}{2}}], \qquad (i,j=1,2,3) \quad (1)$$

where n is the measure and e_{ij}^A is the principal Almansi finite strain components. For n = -2, -1, 0, 1, 2 it gives Cauchy, Green, Hencky, Swainger and Almansi measures respectively.

In this paper an attempt has been made to study the behavior of isotropic thin rotating disk with exponentially variable thickness and edge load using transition theory [10].

Manoj Sahni is with the Jaypee Institute of Information Technology, Noida, U.P.-201301 INDIA (phone: +91-9971541700; e-mail: manoj_sahani117@ rediffmail.com).

Sanjeev Sharma is with the Jaypee Institute of Information Technology, Noida, U.P.- 201301 INDIA (e-mail: sanjiit12@rediffmail.com).

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:5, No:3, 2011

The thickness of the disk is assumed to vary along the radius in the form

$$h = h_0 e^{-\left(\frac{r}{b}\right)^k}$$

where k the geometric parameter and b is the radius of the disk.

II. Objective of the Present Study

In order to explain the elastic-plastic deformation, it is first necessary to recognize the transition state as an asymptotic one and in this work; it is our main aim to eliminate the need for assuming semi-empirical laws, yield condition. We also obtain the constitutive equation corresponding to the transition state.
Borah [16] identified the transition state in which the governing differential equation shows some criticality. The general yield condition of transition is identified from the vanishing of Jacobian of transformation, $\frac{\partial(X,Y,Z)}{\partial(x,y,z)} = 0$, where (X,Y,Z), (x,y,z) are the coordinates of a point in the undeformed and deformed state respectively.

III. Governing Equations

We consider a thin disk of constant density with central bore of radius 'a' and external radius 'b'. The disk is rotating with angular speed 'ω'about an axis perpendicular to its plane and passed through the center of the disk. A case of plane stress is taken in which the axial stress T_{ZZ} is zero. The disk is assumed to be symmetric with respect to the mid plane. The displacement components in cylindrical polar co-ordinates are given by [11].

$$u = r(1-\beta); \quad v = 0; \quad w = dz \tag{2}$$

where β is a function of $r = \sqrt{x^2 + y^2}$ only and d is a constant. The finite strain components are given as

$$e_{rr}^A = \frac{\partial u}{\partial r} - \frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2 = \frac{1}{2}\left[1-(\beta + r\beta')^2\right]$$

$$e_{\theta\theta}^A = \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2}\left(1-\beta^2\right)$$

$$e_{zz}{}^A = \frac{\partial w}{\partial z} - \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{2}\left[1-(1-d)^2\right]$$

$$e_{r\theta}{}^A = e_{\theta z}{}^A = e_{zr}{}^A = 0 \tag{3}$$

where $\beta' = \frac{d\beta}{dr}$.

On substitution of equation (3) in (1), the generalized components of strain are given as

$$e_{rr} = \frac{1}{n}\left[1-(\beta + r\beta')^n\right], \quad e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right]$$

$$e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right], \quad e_{r\theta} = e_{\theta z} = e_{zr} = 0 \tag{4}$$

The stress-strain relations for isotropic material are given as

$$T_{ij} = \lambda\delta_{ij} I_k + 2\mu e_{ij}, \quad (i, j, k = 1,2,3) \tag{5}$$

where T_{ij} and e_{ij} are the stress and strain components respectively, λ and μ are the Lame's constants, $I_k = e_{kk}$ is the first strain invariant and δ_{ij} is the Kronecker's delta.
Equation (5) for this problem becomes

$$T_{rr} = \frac{2\lambda\mu}{\lambda + 2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2\mu e_{rr}$$

$$T_{\theta\theta} = \frac{2\lambda\mu}{\lambda + 2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2\mu e_{\theta\theta}$$

$$T_{zz} = T_{r\theta} = T_{\theta z} = T_{zr} = 0 \tag{6}$$

Substituting equation (3) in (5), the strain components in terms of stresses are obtained as

$$e_{rr} = \frac{1}{2}\left[1-(r\beta' + \beta)^2\right] = \frac{1}{E}\left[T_{rr} - \left(\frac{1-C}{2-C}\right)T_{\theta\theta}\right]$$

$$e_{\theta\theta} = \frac{1}{2}\left[1-\beta^2\right] = \frac{1}{E}\left[T_{\theta\theta} - \left(\frac{1-C}{2-C}\right)T_{rr}\right]$$

$$e_{zz} = \frac{1}{2}\left[1-(1-d)^2\right] = -\left(\frac{1-C}{2-C}\right)\frac{1}{E}\left(T_{rr} - T_{\theta\theta}\right)$$

$$e_{r\theta} = e_{\theta z} = e_{zr} = 0 \tag{7}$$

where E is the Young's modulus and C is the compressibility factor of the material. In terms of Lame's constant they are given as

$$C = \frac{2\mu}{(\lambda + 2\mu)} \quad and \quad E = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)}$$

Substituting equation (4) in (6), we get the stresses as

$$T_{rr} = \frac{2\mu}{n}\left[3 - 2C - \beta^n\left\{1 - C + (2-C)\left(\frac{r\beta'}{\beta} + 1\right)^n\right\}\right]$$

$$T_{\theta\theta} = \frac{2\mu}{n}\left[3 - 2C - \beta^n\left\{2 - C + (1-C)\left(\frac{r\beta'}{\beta} + 1\right)^n\right\}\right]$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0 \tag{8}$$

Equations of equilibrium are all satisfied except

$$\frac{d}{dr}\left(hrT_{rr}\right) - hT_{\theta\theta} + \rho r^2\omega^2 h = 0 \tag{9}$$

where ρ density of material and h is the exponentially variable thickness of the disk.
Using equation (8) in (9), we get a non-linear differential equation in β as

$$(2-C)\, nP\beta^{n+1}(P+1)^{n-1}\frac{dP}{d\beta} = \frac{n\rho\omega^2 r^2}{2\mu} + \beta^n\left\{k\left(\frac{r}{b}\right)^k - nP\right\}$$
$$\left\{1-C+(2-C)(P+1)^n\right\} + \beta^n\left\{1-(P+1)^n\right\} - k(3-2C)\left(\frac{r}{b}\right)^k \tag{10}$$

where $r\beta' = \beta P$ (P is a function of β and β is a function of r). Transition or turning points of P in equation (10) are $P \to -1$ *and* $P \to \pm\infty$. The boundary conditions are:

$$u = 0 \;\; at \;\; r = a$$
$$T_{rr} = \sigma_0 \; at \; r = b \tag{11}$$

IV. Solution Though The Principal Stress

It has been shown [13]-[19] that the asymptotic solution through the principal stress leads from elastic to plastic state at the transition point $P \to \pm\infty$, we define the transition function R as

$$R = \frac{n}{2\mu}T_{\theta\theta} = \left[(3-2C) - \beta^n\left\{2-C+(1-C)(P+1)^n\right\}\right] \tag{12}$$

Taking the logarithmic differentiation of equation (12) with respect to r and using equation (10), we get

$$\frac{d}{dr}(\log R) = \frac{\beta^n\left(\dfrac{1-C}{2-C}\right)\left[\begin{array}{l}\left(\dfrac{n\rho\omega^2 r^2}{2\mu\beta^n} + k\left(\dfrac{r}{b}\right)^k\right)\left\{1-C+(2-C)(P+1)^n\right\} \\ -nP(1-C) + \left\{1-(P+1)^n\right\} - \dfrac{k(3-2C)}{\beta^n}\left(\dfrac{r}{b}\right)^k\end{array}\right] + nP\beta^n(2-C)}{r\left[3-2C-\beta^n\left\{2-C+(1-C)(P+1)^n\right\}\right]} \tag{13}$$

Taking the asymptotic value of equation (13) as $P \to \pm\infty$ and integrating, we get

$$R = A_1 r^{-\frac{1}{2-C}} e^{\left(\frac{r}{b}\right)^k} \tag{14}$$

where A_1 is a constant of integration, which can be determined by the boundary condition.
From equation (12) and (14), we have

$$T_{\theta\theta} = \frac{2\mu}{n} A_1 r^{-\frac{1}{2-C}} e^{\left(\frac{r}{b}\right)^k} \tag{15}$$

Substituting equation (15) in (9) and integrating, we get

$$T_{rr} = \frac{2\mu A_1(2-C)}{n(1-C)} r^{-\frac{1}{2-C}} e^{\left(\frac{r}{b}\right)^k} - \frac{\rho\omega^2 f(r)}{r} e^{\left(\frac{r}{b}\right)^k} + \frac{B_1}{rh_0} e^{\left(\frac{r}{b}\right)^k} \tag{16}$$

where B_1 is constant of integration and $f(r) = \int r^2 e^{-\left(\frac{r}{b}\right)^k} dr$.

Substituting equations (15) and (16) in second equation of (7), we get

$$\beta = \sqrt{1 - \frac{2}{E}\left(\frac{1-C}{2-C}\right) e^{\left(\frac{r}{b}\right)^k}\left\{\frac{\rho\omega^2}{r} f(r) - \frac{B_1}{rh_0}\right\}} \tag{17}$$

Substituting equation (17) in (2), we get

$$u = r - r\sqrt{1 - \frac{2}{E}\left(\frac{1-C}{2-C}\right) e^{\left(\frac{r}{b}\right)^k}\left\{\frac{\rho\omega^2}{r} f(r) - \frac{B_1}{rh_0}\right\}} \tag{18}$$

where $E = \dfrac{2\mu(3-2C)}{(2-C)}$ is the Young's modulus in terms of compressibility factor. Using boundary condition (11) in equation (16) and (18), we get

$$B_1 = h_0\rho\omega^2 f(a),$$
$$A_1 = \frac{n(1-C)}{2\mu(2-C)}\left[\frac{\sigma_0}{e} + \frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right] b^{\frac{1}{2-C}} \tag{19}$$

Substituting the values of constant of integration A_1 and B_1 from equation (19) in equations (15), (16) and (18) respectively, we get the transitional stresses and displacement as

$$T_{\theta\theta} = \left[\frac{\sigma_0}{e} + \frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right]\frac{(1-C)}{(2-C)}\left(\frac{b}{r}\right)^{\frac{1}{2-C}} e^{\left(\frac{r}{b}\right)^k} \tag{20}$$

$$T_{rr} = \left[\left\{\frac{\sigma_0}{e} + \frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right\}\left(\frac{b}{r}\right)^{\frac{1}{2-C}} - \frac{\rho\omega^2\{f(r)-f(a)\}}{r}\right] e^{\left(\frac{r}{b}\right)^k} \tag{21}$$

$$u = r - r\sqrt{1 - \frac{2}{E}\left(\frac{1-C}{2-C}\right)\frac{\rho\omega^2}{r}\{f(r)-f(a)\} e^{\left(\frac{r}{b}\right)^k}} \tag{22}$$

From equation (20) and (21), we get

$$T_{rr} - T_{\theta\theta} = \left[\frac{\sigma_0}{e} + \frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right]\left(\frac{b}{r}\right)^{\frac{1}{2-C}} e^{\left(\frac{r}{b}\right)^k}\frac{1}{2-C} - \frac{\rho\omega^2\{f(r)-f(a)\}}{r} e^{\left(\frac{r}{b}\right)^k} \tag{23}$$

From equation (23), it is seen that $\left|T_{rr} - T_{\theta\theta}\right|$ is maximum at the internal surface (i.e. at $r = a$), therefore yielding will take place at the internal surface of the disk and equation (23) become

$$\left|T_{rr}-T_{\theta\theta}\right|_{r=a}=\left|\left[\frac{\sigma_0}{e}+\frac{\rho\omega^2\{f(b)-f(a)\}}{b}\right]\left(\frac{b}{a}\right)^{\frac{1}{2-C}}e^{\left(\frac{a}{b}\right)^k}\cdot\frac{1}{2-C}\right|\equiv Y\,(say)$$

and the angular speed necessary for initial yielding is given by

$$\Omega_i^2=\frac{\rho\omega_i^2b^2}{Y}=\left[\frac{(2-C)}{e^{\left(\frac{a}{b}\right)^k}}\left(\frac{a}{b}\right)^{\frac{1}{2-C}}-\frac{T_o}{e}\right]\frac{b^3}{\{f(b)-f(a)\}} \tag{24}$$

where $T_o=\sigma_o/Y$

The disk becomes fully plastic $(C\to 0)$ at the external surface (i.e. at r = b) and equation (23) become

$$\left|T_{rr}-T_{\theta\theta}\right|_{r=b}=\left|\left[\frac{\sigma_0}{2}-\left(\frac{\rho\omega^2}{2b}\right)e\{f(b)-f(a)\}\right]\right|=Y^*$$

Angular speed required for the disk to become fully plastic is given by

$$\Omega_f^{\ 2}=\frac{\rho\omega_f^2b^2}{Y^*}=\frac{\left[-2+T_o^*\right]}{e}\frac{b^3}{\left[f(b)-f(a)\right]} \tag{25}$$

where $w_f=\frac{\Omega_f}{b}\sqrt{\frac{Y^*}{\rho}}$ and $T_0^*=\sigma_o/Y^*$.

We introduce the following non-dimensional components as

$$R=\frac{r}{b},\ R_0=\frac{a}{b},\ \sigma_r=\frac{T_{rr}}{Y},\ \sigma_\theta=\frac{T_{\theta\theta}}{Y},\ and\ \bar{u}=\frac{u}{b}$$

Transitional stresses, angular speed and displacement can be obtained from equations (20)-(22) and (24) in non-dimensional form as

$$\sigma_\theta=\left[\frac{T_o}{e}+\Omega_i^{\ 2}\int_{R_0}^{1}R^2e^{-R^k}dR\right]\frac{(1-C)}{(2-C)}R^{-\frac{1}{2-C}}e^{R^k} \tag{26}$$

$$\sigma_r=\left[\frac{T_o}{e}+\Omega_i^{\ 2}\int_{R_0}^{1}R^2e^{-R^k}dR\right]R^{-\frac{1}{2-C}}e^{R^k}-\frac{\Omega_i^{\ 2}}{R}e^{R^k}\left[\int_{R_0}^{R}R^2e^{-R^k}dR\right] \tag{27}$$

$$\bar{u}=R-R\sqrt{1-\frac{2(1-C)Y}{E(2-C)}\frac{\Omega_i^{\ 2}.e^{R^k}}{R}\int_{R_0}^{R}R^2e^{-R^k}dR} \tag{28}$$

$$\Omega_i^{\ 2}=\left[\frac{(2-C)R_0^{\frac{1}{2-C}}}{e^{R_o^k}}-\frac{T_0}{e}\right]\frac{1}{\int_{R_0}^{1}R^2e^{-R^k}dR} \tag{29}$$

Stresses, displacement and angular speed for fully-plastic state $(C\to 0)$ are obtained from equations (26)-(28) and (25) as

$$\sigma_\theta=\frac{1}{2}\left[\frac{T_o}{e}+\Omega_f^2\int_{R_0}^{1}R^2e^{-R^k}dR\right]R^{-\frac{1}{2}}e^{R^k} \tag{30}$$

$$\sigma_r=\left[\frac{T_o}{e}+\Omega_f^2\int_{R_0}^{1}R^2e^{-R^k}dR\right]R^{-\frac{1}{2}}e^{R^k}-\frac{\Omega_f^2}{R}e^{R^k}\left[\int_{R_0}^{R}R^2e^{-R^k}dR\right] \tag{31}$$

$$\bar{u}=R-R\sqrt{1-\frac{Y}{E}\frac{\Omega_f^2}{R}\int_{R_0}^{R}R^2e^{-R^k}dR\,e^{R^k}} \tag{32}$$

$$\Omega_f^{\ 2}=\frac{\left[-2+T_o^*\right]}{e}\frac{1}{\int_{R_0}^{1}R^2e^{-R^k}dR} \tag{33}$$

TABLE I
ANGULAR SPEED REQUIRED FOR INITIAL YIELDING AND FULLY PLASTIC STATE WITH DIFFERENT EDGE LOADING (FLAT DISK)

	C	T_0=0		% increase	T_0=0.2		% increase
		Ω_i^2	Ω_f^2		Ω_i^2	Ω_f^2	
R_0=0.5	0	4.84	6.86	41.421	4.16	6.17	48.244
	0.25	4.04	6.86	69.828	3.35	6.17	84.112
	0.5	3.24	6.86	111.65	2.55	6.17	141.63
	0.75	2.46	6.86	178.58	1.77	6.17	247.53

V. NUMERICAL ILLUSTRATION AND DISCUSSION

In fig. 1, curves have been drawn between angular speed $\left(\Omega_i^2\right)$ and various radii ratios $R_o=(a/b)$ for different compressibility factors $(C=0,0.25,0.5,0.75)$ and variable thickness (k=0, 2, 4). It is observed that the rotating disk made of incompressible material with inclusion require higher angular speed to yield at the internal surface as compared to disc made of compressible material and this behavior remains the same with increase in edge load $(T_0=0.1,\ 0.2)$. With the increase in edge load, the angular speed required for initial yielding decreases. From table 1, it is seen that for isotropic compressible material, high percentage increase in angular speed is required to become fully plastic as compared to rotating disk made of incompressible material. In figs. 2-4, curves have been drawn between the transitional stresses, displacement against the radii ratio. It is seen that with the increase in compressibility factor, radial as well as circumferential stresses decreases. From fig. 5, it is observed that the radial and circumferential stresses are maximum at the internal surface (for flat disk i.e. k = 0). With the increase in thickness parameter (k = 2, 4), the circumferential stress is maximum at the external surface. With edge load the behavior remains the same. Similar graph is also obtained by *Güven* [20] for rotating disk with rigid inclusion.

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:5, No:3, 2011

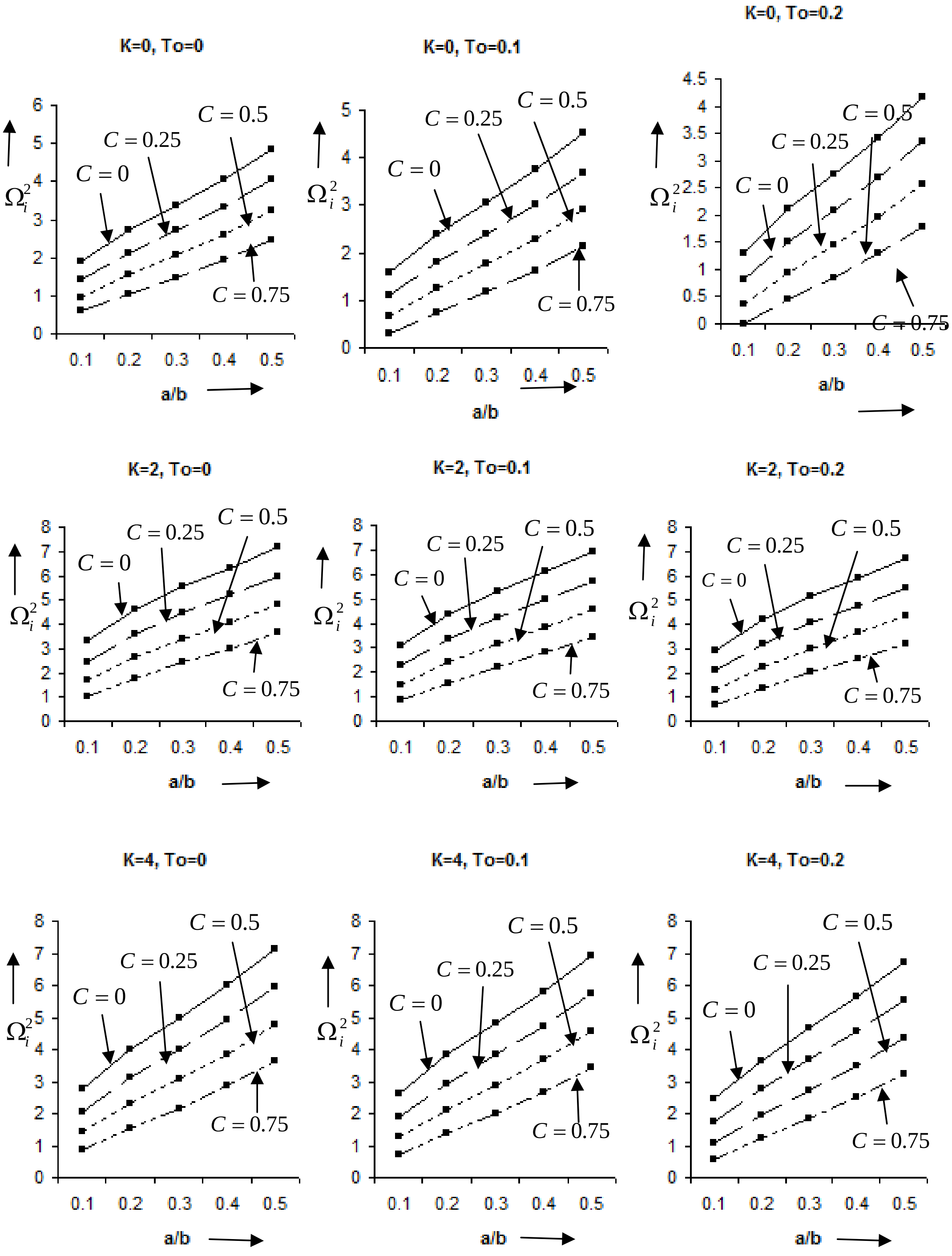

Fig. 1 Angular speed required for initial yielding at the internal surface of the rotating disk with variable thickness (k = 0, 2, 4) and edge loading (T_o = 0, 0.1, 0.2).

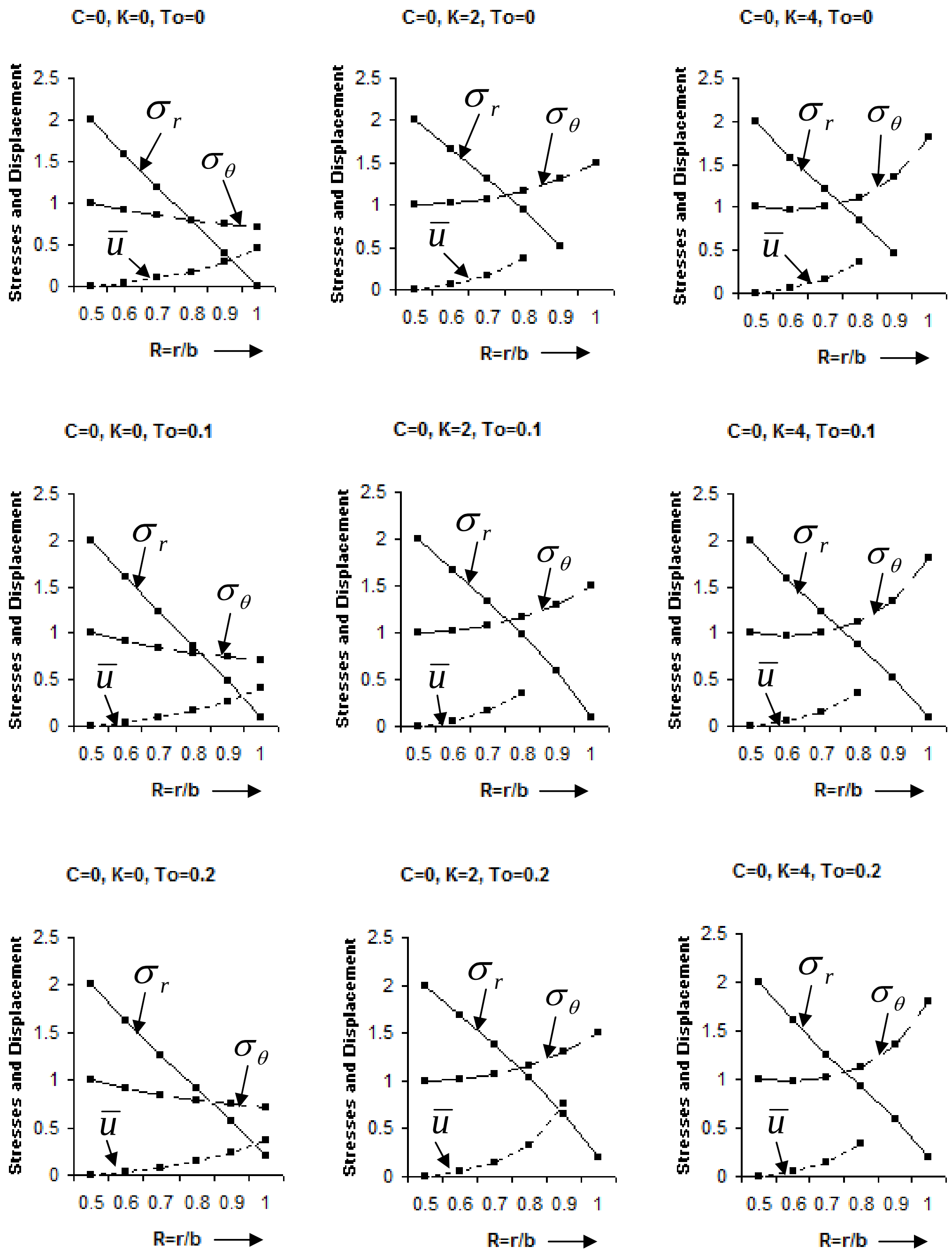

Fig. 2 Transitional stresses and displacement in a thin rotating disk along the various radii ratio (R = r/b) with compressibility (C = 0) for variable thickness (K = 0, 2, 4) and edge load (To = 0, 0.1, 0.2).

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:5, No:3, 2011

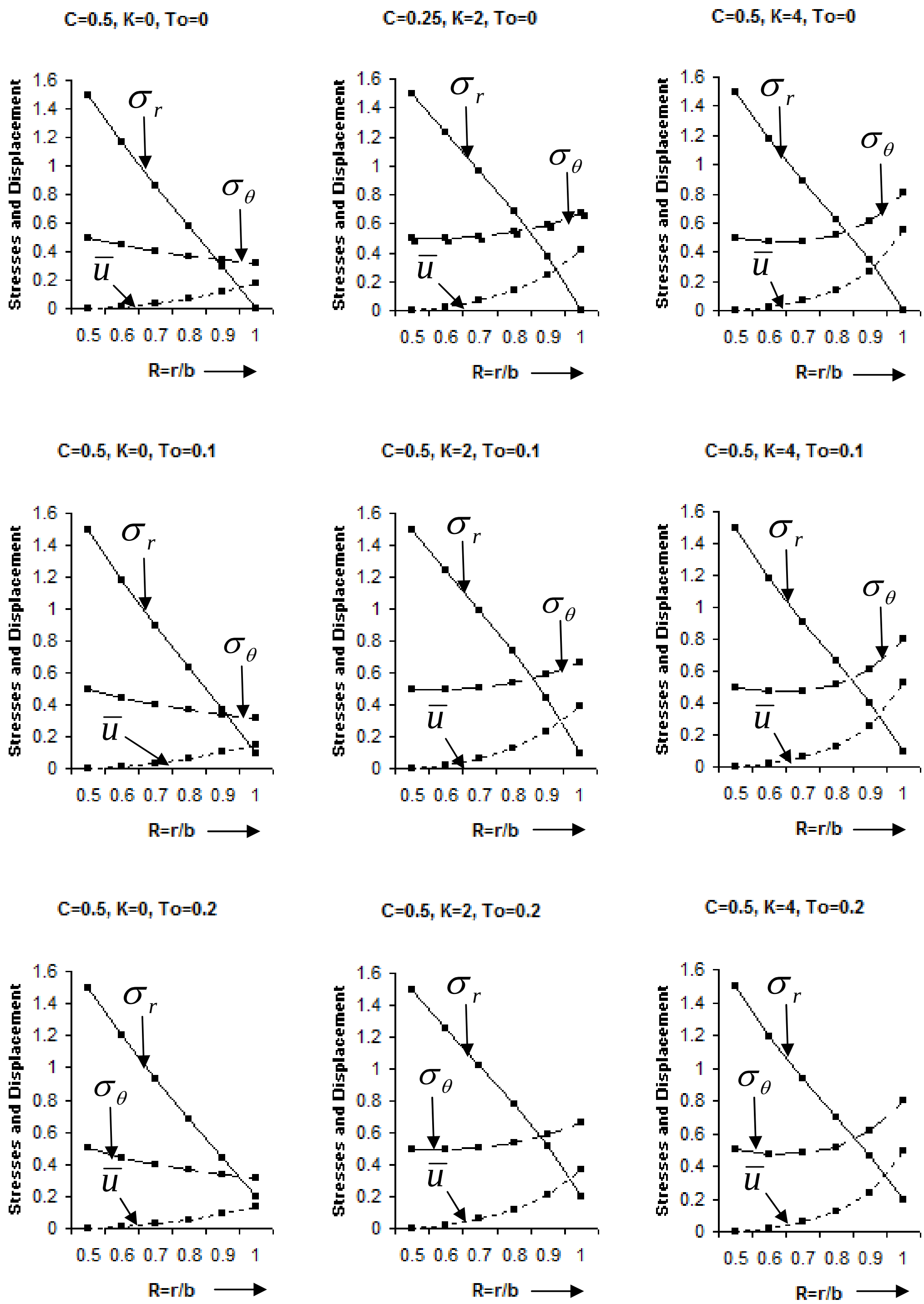

Fig. 3 Transitional stresses and displacement in a thin rotating disk along the various radii ratio (R = r/b) with compressibility (C = 0.5) for variable thickness (K = 0, 2, 4) and edge load (To = 0, 0.1, 0.2).

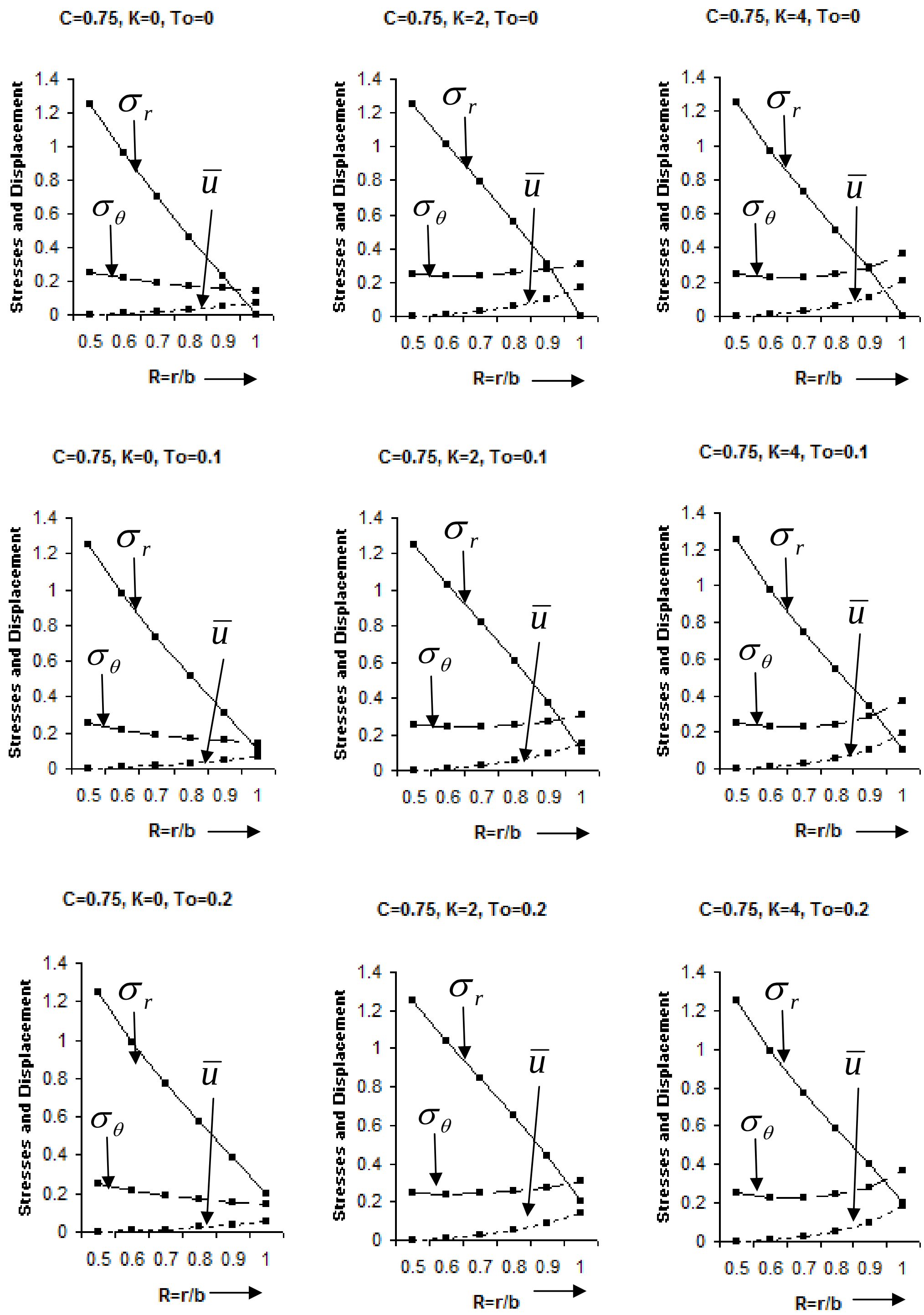

Fig. 4 Transitional stresses and displacement in a thin rotating disk along the various radii ratio (R = r/b) with compressibility (C = 0.75) for variable thickness (K = 0, 2, 4) and edge load (To = 0, 0.1, 0.2).

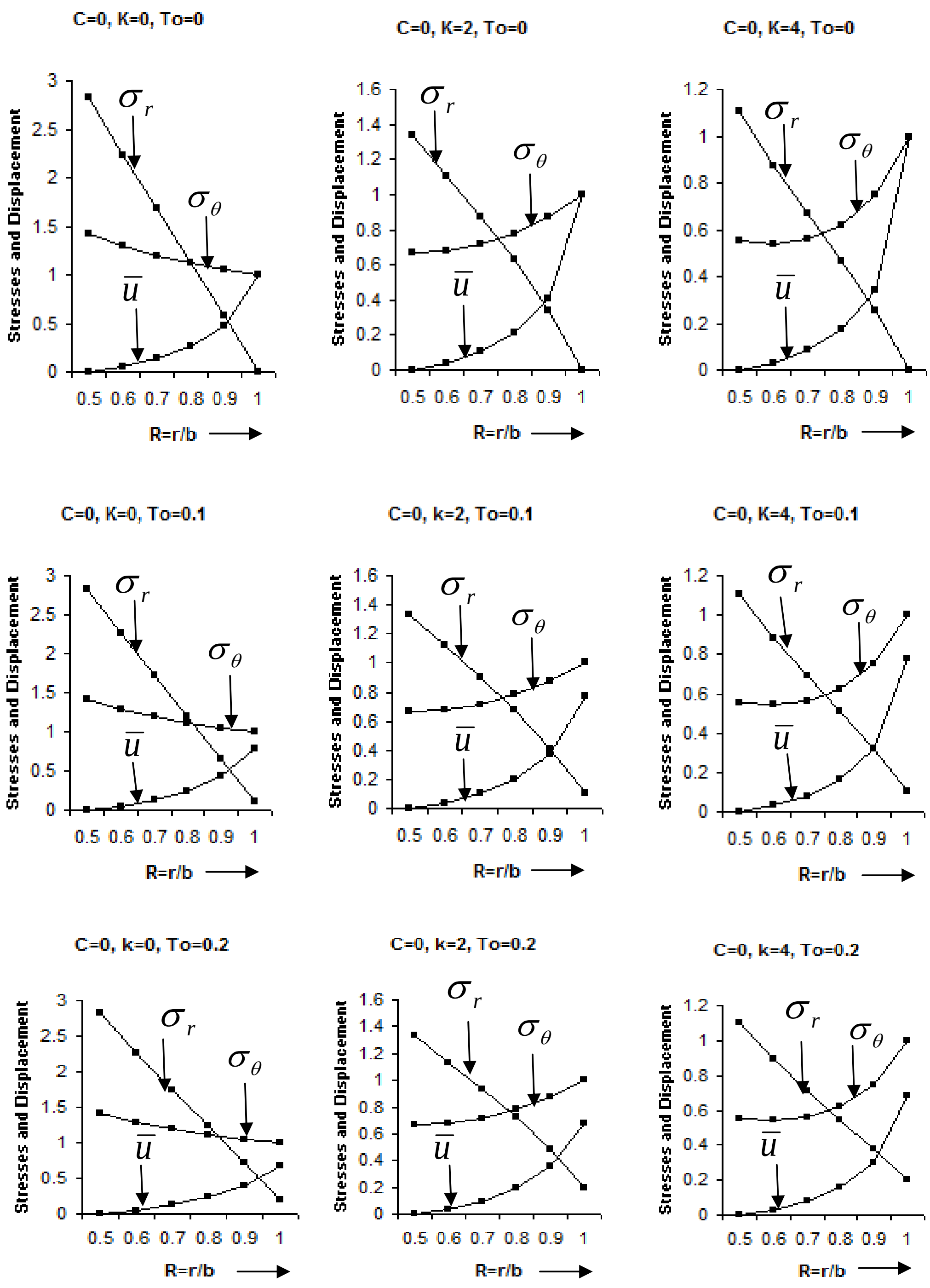

Fig. 5 Plastic stresses and displacement in a thin rotating disk along the various radii ratio (R = r/b) for variable thickness (K = 0, 2, 4) and edge load (To = 0, 0.1, 0.2).

References

[1] S.P. Timoshenko, J.N. Goodier, *Theory of Elasticity*, 3rd Edi., New York: McGraw-Hill Book Coy., London, 1951.
[2] J. Chakrabarty, *Applied Plasticity*, Springer Verlag, Berlin, 2000.
[3] W. Han, B.D. Reddy, *Plasticity, Mathematical Theory and Numerical Analysis*, Springer Verlag, Berlin, 1999.
[4] R.B. Hetnarski, J. Ignaczak, *Mathematical Theory of Elasticity*, Taylor and Francis, 2003.
[5] I.S. Sokolinikoff, *Mathematical Theory of Elasticity*, 2nd Edi., New York: McGraw-Hill Book Co., 1950.
[6] J. Heyman, "Plastic design of rotating discs", *Proc. Inst. Mech. Engs.*, 1958, pp. 531-546.
[7] R.P.S. Han, Yeh Kai-Yuan, "Analysis of High-Speed Rotating Disks with Variable Thickness and Inhomogenity", *Transactions of the ASME*, 61, pp. 186-191, 1994.
[8] A.N. Eraslan, Y.Orcan, "Elastic-plastic Deformation of a rotating solid disk of exponentially varying thickness", *Mechanics of Materials*, vol. 34, pp. 423-432, 2002.
[9] Xiu'e Wang, Xianjun Yin, "On Large Deformations of Elastic Half Rings", *WSEAS Transactions on Applied and Theoretical Mechanics*, vol. 2(1), pp. 24, 2007.
[10] B.R. Seth, "Transition Theory of Elastic-Plastic Deformation, Creep and Relaxation", *NATURE*, vol. 195, No. 4844, pp. 896-897, 1962.
[11] B. R. Seth, "Transition Analysis of Collapse of Thick-walled Cylinder", *ZAMM*, 50, pp. 617-621, 1970.
[12] B. R. Seth, "Creep Transition", *Jr. Math. Phys. Sci.*, vol. 8, pp. 1-2, 1972.
[13] S. Hulsarkar, "Transition Theory of Creep Shells under Uniform Pressure", *ZAMM*, Vol. 46, pp. 431-437, 1966.
[14] S.K. Gupta, V.D. Rana, "Thermo Elastic-Plastic and Creep Transition in Rotating Cylinder", *J. Math. Phy. Sci.*, 23(1), pp. 71-90, 1989.
[15] S.K. Gupta, Pankaj, "Thermo Elastic-plastic Transition in a thin rotating Disc with inclusion", *Thermal Science Scientific Journal*, 11(1), pp.103-118, 2007.
[16] B.N. Borah," Thermo Elastic-plastic transition", *Contemporary Mathematics*, vol. 379, pp. 93-111, 2005.
[17] S. Sharma, "Elastic-plastic Transition of Non-homogeneous Thick-walled Circular Cylinder under Internal Pressure", *Def. Sc. Journal*, vol. 54, No. 2, 2004.
[18] S. Sharma, M. Sahni, "Creep Transition of Transversely Isotropic Thick-walled Rotating Cylinder", *Adv. Theor. Appl. Mech.*, vol. 1(7), pp. 315-325, 2008.
[19] Pankaj, Sonia R. Bansal, "Elastic-Plastic Transition in a Thin Rotating Disc with Inclusion", *Proceedings of World Academy of Science, Engineering and Technology*, Vol. 28, April 2008.
[20] U. *Güven*, "Elastic-plastic rotating disk with rigid inclusion", *Mech. Struct. and Mach.*, vol. 27, pp. 117-128, 1999.

Dr Sanjeev Sharma obtained his Ph. D. from Himachal Pradesh University, Shimla in 2000. Presently, he is working as Assistant Professor in the Department of Mathematics, Jaypee Institute of Information Technology University, Noida, India. His area of research include: Applied mathematics, solid mechanics, elastic-plastic and creep theory. He has published fourteen research papers in national/ international journals and presented four research papers in national/ international conferences.

Dr. Manoj Sahni obtained his Ph. D. from JIIT Noida, India. Presently, he is working as Lecturer in the Department of Mathematics, Jaypee Institute of Information Technology University (JIIT), Noida. His area of research includes Applied Mathematics, Elastic-plastic and creep theory. He has published six research papers in international/ national journals and three research papers in national/ international conferences.