

# Asymptotic Analysis of Instant Messaging Service with Relay Nodes

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**Abstract**—In this paper, we provide complete end-to-end delay analyses including the relay nodes for instant messages. Message Session Relay Protocol (MSRP) is used to provide congestion control for large messages in the Instant Messaging (IM) service. Large messages are broken into several chunks. These chunks may traverse through a maximum number of two relay nodes before reaching destination according to the IETF specification of the MSRP relay extensions. We discuss the current solutions of sending large instant messages and introduce a proposal to reduce message flows in the IM service. We consider virtual traffic parameter i.e., the relay nodes are stateless non-blocking for scalability purpose. This type of relay node is also assumed to have input rate at constant bit rate. We provide a new scheduling policy that schedules chunks according to their previous node's delivery time stamp tags. Validation and analysis is shown for such scheduling policy. The performance analysis with the model introduced in this paper is simple and straight forward, which lead to reduced message flows in the IM service.

**Keywords**—Instant messaging, stateless, chunking, MSRP.

## I. INTRODUCTION

INSTANT messaging (IM) is one of today's most popular services. Thus, it is not a surprise that 3G IP Multimedia Subsystem (IMS) already has this service well supported in its architecture. IM is the service that allows an IMS user to send some content to another user in near-real time. The content in an instant message is typically a text message, but can be an HTML page, a picture, a file containing a song, a video clip, or any generic file. In this paper, we provide complete end-to-end delay analyses including the relay nodes for instant messages.

There are two modes of operation of the instant message service, depending on whether they are stand-alone instant message, not having any relation with previous or future instant message. This mode of IM is referred to as "pager mode". The model is also similar to the SMS (Short Message Service) in cellular networks. The other model is referred to as session based instant message that is sent as part of an existing session, typically established with a SIP (Session Initiation Protocol) INVITE request. Both modes have different

requirements and constraints, hence the implementation of both models.

The IETF (Internet Engineering Task Force) has created an extension to SIP that allows a SIP UA (User Agent) to send an instant message to another UA. The extension consists of a new SIP method named MESSAGE. The SIP MESSAGE method (RFC 3428 [1]), is able to transport any kind of payload in the body of the message, formatted with an appropriate MIME (Multipurpose Internet Mail Extensions) type. 3GPP TS 23.228 [21] already contains requirements for Application Servers (ASs) to be able to send textual information to an IMS terminal. 3GPP TS 24.229 [2] introduces support for the MESSAGE method extension. The specification mandates IMS terminals to implement the MESSAGE method [1] and to allow implementation to be an optional feature in ASs.

The work over instant messaging [4, 5, 6] observed so far lacks a thorough analysis of the scalable behavior of the nodes involved in providing the IM service. The messages of IM may be very large. Large instant messages have disadvantages like service behavior is too slow on low bandwidth links and more importantly, messages get fragmented over some transport protocol and then look at SIP extension that resolve this issue. Even if messages are compressed, sometimes SIP messages can be too large. Another problem with SIP is that the fact that any proxy can change the transport protocol from TCP (Transmission Control Protocol) to UDP (User Datagram Protocol) or other transport protocols and vice versa. The protocols other than TCP and SCTP (Stream Control Transmission Protocol) are not famous for congestion control. If an IMS terminal is sending a large instant message over a transport protocol that does not offer congestion control, the network proxies can become congested and stop processing other SIP requests like INVITE, SUBSCRIBE, etc. Even if a terminal sends large SIP MESSAGE over a transport protocol that implements end-to-end congestion control e.g., TCP, SCTP, the next proxy can switch to UDP and congestion may occur.

To solve the issue of large message passing and congestion control in IM, a limit has been placed on the SIP MESSAGE method such that MESSAGE requests cannot exceed the MTU (Maximum Transmit Unit) minus 200 bytes. If the MTU is not known, this limit is 1300 bytes. Another solution to sending SIP MESSAGE requests with large bodies to use the content indirection mechanism [3]. Content indirection allows replacing a MIME body part with an

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external reference, which is typically an HTTP (Hyper Text Transfer Protocol) URI (Universal Resource Identifier).

Another solution to getting around the size limit problem with MESSAGE is to use session-based IM mode rather than pager mode. Session-based instant message mode uses the SIP INVITE method to establish a session. An IMS terminal establishes a session to send and receive instant messages via Message Session Relay Protocol (MSRP) [20]. MSRP is a simple text-based protocol whose main characteristic is that it runs over transport protocols that offer congestion control. In the IMS, MSRP is implemented in the IMS terminals. Analysis is required to determine the service order of such servers. Our work in this paper is to analyze the delay bound of the relay nodes that implements the MSRP to provide instant messaging service. The benefit of the work lies in the simplicity of the model derivation.

The rest of the paper is organized as follows. Section II provides a review of MSRP. The SEND chunking system, our proposal of scalability over MSRP relay nodes and delay analysis for work conserving non-blocking situation are described in Section III. Finally we conclude the paper in Section IV.

## II. BACKGROUND

There are currently three methods defined in MSRP after the INVITE message is sent for an IM session set up: (i) SEND: sends an instant message of any arbitrary length from one endpoint to another, (ii) VISIT: and endpoint connects to another end point, and (iii) REPORT: endpoint or a relay provides message delivery notifications.

MSRP does not impose any restriction on the size of an instant message. If an IMS user, Alice wants to deliver a very large message, she can split the message into chunks and deliver each chunk in a separate SEND request. The message ID corresponds to the whole message, so the receiver can also use it to reassemble the message and tell which chunks belong with which message.

Long chunks may be interrupted in mid-transmission to ensure fairness across shared transport connections. This chunking mechanism allows a sender to interrupt a chunk part of the way through sending it. The ability to interrupt messages allows multiple sessions to share a TCP connection, and for large messages to be sent efficiently while not blocking other messages that share the same connection, or even the same MSRP session. Any chunk that is larger than 2048 octets MUST be interruptible [20].

Another characteristic of MSRP is that, MSRP messages do not traverse SIP proxies. This is an advantage, since SIP proxies are not bothered with proxying large instant messages. Also, MSRP does not run over UDP or any other transport protocol that does not offer end-to-end congestion control. It supports instant messages to traverse zero, one or two MSRP relays (see Fig. 1). The relay extension of MSRP is defined in [7].

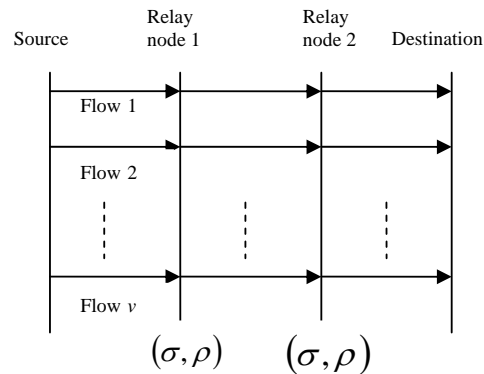


Fig. 1 IM with maximum 2 relay nodes

The default is that SEND messages are acknowledged hop-by-hop. Each relay node that receives a SEND request acknowledges receipt of the request before forwarding the content to the next relay or the final target. When sending large content, the client may split up a message into smaller pieces; each SEND request might contain only a portion of the complete message. For example, when Alice sends Bob a 4GB file called "file.mpeg", she sends several SEND requests each with a portion of the complete message. Relays can repack message fragments en-route. As individual parts of the complete message arrive at the final destination client, the receiving client can optionally send REPORT requests indicating delivery status. MSRP nodes can send individual portions of a complete message in multiple SEND requests. As relays receive chunks they can reassemble or re-fragment them as long as they resend the resulting chunks in order.

A series of papers [8-10] have studied the capacity scaling in relay networks. These works quantify the impact of large wireless relay networks in terms of signal-to-noise ratio. Most of the work focuses on characterizing one relay node only. The work of H. Bolcskei *et al* in [10] demonstrated that significance performance gains can be obtained in wireless relay networks employing terminals with multiple-input multiple-output (MIMO) capability. However, these works do not address the issue of characterizing traffic parameter in relay nodes where the relay nodes do not keep the transaction states. A signification challenge is to schedule the large chunks and characterize the traffic parameters under delay bounds.

In any IMS network the capacity (memory/storage) is large for IM communication. Large messages have to be broken down into chunks to overcome the fixed size limit fact. Real time service of IM is always desirable. However, issues arise if a) the relay nodes in between source and destination IMS terminals possess slow links b) traffic order gets distorted before reaching relay nodes and c) relay nodes maintains transaction states. Therefore efficient service discipline of the chunks of IM is necessary. In an IM system, transmission time typically depends on the number of chunks in messages. The number of broken chunks in a large message is not fixed.

Thus analysis of such system is not trivial. In this work, we explore asymptotic delay characteristics of instant messages when the messages traverse via relay nodes.

The end to end delay bound of IMS instant messages indeed requires much attention. Although, the study of the fundamental frameworks, namely Integrated and Differentiated services have a long history, defining the flow characteristic of an IMS instant message traversing the relay nodes (maximum number of relay nodes is two for an IMS terminal [7]) under MSRP is not trivial due to the arbitrary number of chunks in IM messages. We analyze the end-to-end delay for an IM under work conserving situation. The delay bound is useful to compute the IM end-to-end transmission time.

### III. MODELING

The large sized SEND messages in IM, MSRP delivers in several SEND messages, where each SEND contains one chunk of the overall message. The crucial aspect in this paper is the ordering of SEND chunks at the relay nodes if they do not keep transaction states of a chunk flow for scalability purpose.

Long chunks are interrupted in mid-transmission to ensure fairness across shared transport connections. To support this, MSRP uses a boundary-based framing mechanism. The start line of an MSRP request contains a unique identifier that is also used to indicate the end of the request. Included at the end of the end-line, there is a flag that indicates whether this is the last chunk of data for this message or whether the message will be continued in a subsequent chunk. There is also a Byte-Range header field in the request that indicates that the overall position of this chunk inside the complete message.

This chunking mechanism allows a sender to interrupt a chunk part of the way through sending it. The ability to interrupt messages allows multiple sessions to share a TCP connection, and for large messages to be sent efficiently while not blocking other messages that share the same connection, or even the same MSRP session. As mentioned before that any chunk that is larger than 2048 octets MUST be interruptible. While MSRP would be simpler to implement if each MSRP session used its own TCP connection, there are compelling reasons to conserve connection. For example, the TCP peer may be a relay device that connects to many other peers. Such a device will scale better if each peer does not create a large number of connections. The chunking mechanism only applies to the SEND method, as it is the only method used to transfer message content [20]. We call the chunking mechanism i.e., breaking one large SEND message into several SEND messages a SEND system.

*Proposal:* Traditional MSRP [20] may be used without traditional session set up in IMS to provide the congestion control. Also, MSRP relay nodes should not keep transaction states for the SEND chunks.

The benefit of this proposed technique contains reduced

message flows in the network as well as gaining scalability at the relay nodes. In order to comply with this, we propose the following scheduling. The detail analysis is provided below that captures the delay bound of the relay nodes.

We provide analysis of one relay node first in terms of aggregate flows of two SEND message chunk flows and delay bound, which will later be used to compute delay bound for SEND systems with two relay nodes, source and destination. We assume the following for the analysis. One IMS source terminal sends multiple large instant messages (SEND) to the same destination via two relay nodes. Each of the SEND messages is broken into small SEND chunks. Again, it is to be noted that we are assuming relay nodes do not keep transaction states of the chunks. Though, the IETF draft [7] specifies that the relays may keep transaction states for a very short time, it will be expensive to keep such states for the relay nodes if there is huge number of clients being served and traffic flows are massive. We are only interested in busy traffic situation for the derivation of properties in this research and thus server analysis with stateless assumption is more practical. We assume message chunks are served according to the order of delivery time tag of the previous node and all chunks are treated as if they belong to a single flow due to the elastic and massive flows of SEND chunk messages in a flow. Thus performance analysis of an individual flow of SEND system at a relay node can be achieved by analyzing the aggregated flows at this node.

Here we adopt a scheduler which services a job according to delivery time stamps of the pervious node. We aim to provide work conserving but stateless scheduling of the chunks. Every message chunks has message id that identifies which SEND message it belongs. The source node generates the chunks and delivers them to a relay node. The ordering is considered to be the order as the source node generates chunks for the first relay node and then first relay node for the second relay node and so on. During the delivery time these chunks may receive time stamp tags. These chunks may reach / propagate to the relay node out of order, and hence the arrival times of the chunks to the relay node may not always be in order of the order id of the message chunks. Only when the source initiates chunks, it is guaranteed that the order ids are the same as the delivery order of the chunks.

Let the propagation delay and link capacity of any link are 0 and  $l_c$ , respectively. The sequence of chunks of a SEND system transmitted by a source to a destination is referred to as a flow of SEND system (Fig. 1). The paths via relay nodes are predetermined as defined in the MSRP relay extensions [7] (The relay nodes are authorized by explicitly by the end terminals). Let, at a relay node chunk  $k$  of a flow of SEND system  $i$  is attached with a time stamp tag according to

the delivery time from the previous node of  $X_i^k + \left( \frac{S_i}{\alpha_i} \right)$

where  $\alpha_j$ ,  $S_i$ , and  $X_i^k$  are the input rate, chunk size and

the arrival time of chunk  $k$  of flow of SEND system  $i$  respectively; the delivery order time stamp of chunk  $k$  of flow of SEND system  $i$  is updated at the next relay node with an increment of  $\left(\frac{S_{\max}}{l_c}\right) + \left(\frac{S_i}{\alpha_i}\right)$ , and chunks are served at the increment order of their previous node's delivery order time tag, where,  $S_{\max}$  is the maximum size of chunk in all flows. Under these conditions, it is easy to perceive that the worst case delay of a flow of SEND system  $i$  at any relay node is no longer than  $\left(\frac{S_{\max}}{l_c}\right) + \left(\frac{S_i}{\alpha_i}\right)$  [11].

We adopt the characteristic of traffic model in [12, 13] which has been widely adopted for characterizing network traffic. If the total traffic of a flow  $F(t_1, t_2)$  arriving in the time interval  $(t_1, t_2]$  is bounded by:

$$F(t_1, t_2) \leq \sigma + \rho(t_2 - t_1) \quad (1)$$

Then the flow of SEND system is referred to as conforming to the traffic parameter  $(\sigma, \rho)$  [12]. Here the assumptions are under non-overflow condition with a flow injection to a leaky bucket with parameters of buffer size,  $\sigma$  and output rate  $\rho$ . In other words,  $\rho$  is the average traffic rate in the long run and  $\sigma$  is the burst bound of the flow of SEND system  $(\sigma, \rho)$ . It is practical to assume that the links of relay nodes will be subject to delay bound in terms of propagation delay. We consider a chunk to be arrived only after its last bit has arrived to a relay node and the delivery time of a chunk at a node is the time when the last bit of the chunk leaves the relay node. Note that we are considering the input traffic as the constant bit rate for the relay nodes in this section.

If we consider steady state of the network i.e., traffic load less than one then a chunk will only be delayed at a node if there is a chunk being served or there are chunks waiting in the buffer with earlier delivery time stamps, we assume that the start time of each busy period is initialized at 0. Here, a busy period is an interval of time during which the transmission queue of the output link is continuously backlogged which is consistent with [12]. For a lossless SEND system backlog at time  $t$  is  $R(t) - R^*(t)$  where  $R(t)$  is an increasing function that has a derivative  $\frac{dR}{dt} = r(t)$  such that  $R(t) = \int_0^t r(s)ds$  ( $r$  is called the rate function). It is always possible to map a continuous time model  $R(t)$  to a discrete time model  $R(n)$ ,  $n \in \mathbb{N}$  by choosing a time slot  $\delta$  and sampling by  $R(n) = Z(n\delta)$ . We define the virtual delay at time  $t$  as the inf norm of  $d(t) = \inf \{\tau \geq 0 : R(t) \leq R^*(t + \tau)\}$ . The virtual delay at

time  $t$  is the delay that would be experienced by a bit arriving at time  $t$  if all bits received before it are served before it. For  $d(t)$  being very small we have  $R^*(t + d(t)) = R(t)$ . Thus for a SEND system flow with input rate  $\alpha$  and service function  $\rho$ , the backlog for all  $t$  satisfies the following sup norm:

$$\text{Proposition 1: } R(t) - R^*(t) \leq \sup_{s \geq 0} \{\alpha(s) - \rho(s)\} \quad (2)$$

*Proof:* According to the definition of  $R^*(t)$ , we have

$$R^*(t) \leq \inf_{0 \leq s \leq t} [R(t - s) + \rho(s)] \quad (3)$$

Thus,

$$R(t) - R^*(t) \leq R(t) - \inf_{0 \leq s \leq t} [R(t - s) + \rho(s)] \quad (4)$$

i.e.,

$$\begin{aligned} R(t) - R^*(t) &\leq \sup_{0 \leq s \leq t} [R(t) - R(t - s) + \rho(s)] \\ &\leq \sup_{0 \leq s \leq t} [\alpha(s) + \rho(t - s)] \end{aligned} \quad (5)$$

And proposition 1 is proved.

The previous node's delivery time stamp tag of each chunk lags behind its arrival time at any relay node. Note that chunks are served by the order of their previous node's delivery time stamps which is our assumption. Thus, the delay for each chunk to traverse the network remains the same only if the time stamps of all chunks are increased by a constant  $D$  at the previous node. We can assume that, if the burst of each flow of SEND system is bounded and the capacity of any link is no less than the average rate of the flows traversing the link, there exists a worst case delay bound in the network, i.e., the worst case delay of a flow of SEND system to traverse any pair of relay nodes is bounded.

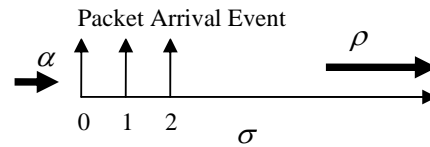


Fig. 2 Intuition of the traffic model

Since we consider that the relay node need not keep transaction states of the SEND flows, chunks in the buffer are served by the order of their delivery time stamp tags, not their arrival times. There is also no distinct relation between the delivery time stamp of a chunk and its arrival time. Thus, a chunk with an earlier delivery time stamp than another chunk, though it arrives later, may be served first. This may happen due to the well-known traffic distortion problem [12]. Therefore in this regard, it is more reasonable to evaluate a chunk's delay with reference to its previous delivery node's time stamp, rather than its arrival time at the current node. We need to characterize traffic of this kind of scheduling for the relay nodes. We modify and extend the characteristics of

$(\sigma, \rho)$  with additional assumptions to simplify the model for SEND systems for deployment purpose.

*Definition:* We define the parameter  $(\sigma, \rho)$  such that the total traffic of the flow of chunks, whose time stamps are in the range of  $(t_1, t_2]$ , is no larger than  $\sigma + \rho(t_2 - t_1)$  (See Fig. 2).

Assuming that chunks are ordered by their previous node's delivery time stamps as  $C_1, C_2, \dots, C_k, \dots$  ( $Y_i \geq Y_j$ , if  $i > j$ , where  $Y_i$  is the previous node's delivery time tag of chunk  $C_i$ ). For any two

chunks  $C_m$  and  $C_k$ , ( $k \geq m$ ),  $\sigma + \rho Y_k - Y_m \geq \sum_{i=m}^k S_i$ ,

where  $S_i$  is the size of  $C_i$ .

A chunk may receive service as long as there is no chunk in the buffer when it arrives. Thus, it is necessary to take into account the arrival time of a chunk to characterize traffic in a relay node. Therefore, we define the traffic parameter for any two chunks of a flow as follows: for any two chunks  $C_k$  and  $C_m$  of a flow

( $k \geq m \geq 1$ ),

$$\sigma + \rho(Y_k - \max\{Y_{m-1}, \min\{X_m, X_{m+1}, \dots, X_k\}\}) \geq \sum_{i=m}^k S_i$$

where  $X_i$  is the arrival time of chunk  $C_i$ ,  $i=1, 2, \dots$ ; we refer to  $F(t_1, t_2) = \sigma + \rho(t_2 - t_1)$  in the time interval  $(t_1, t_2]$  as the traffic function of this flow with the traffic parameter  $(\sigma, \rho)$ . We apply the additive property of  $(\sigma, \rho)$  traffic model [12] to obtain the following:

*Proposition 2:* Given two flows with traffic parameters  $(\sigma_1, \rho_1)$  and  $(\sigma_2, \rho_2)$  the traffic parameter of the aggregated traffic of the two flows is  $(\sigma_1 + \sigma_2, \rho_1 + \rho_2)$ .

*Proof:* Assume that chunks are ordered by their delivery order. Given any two chunks  $C_k$  and  $C_m$  ( $k \geq m$ ) of the aggregated flow, assuming chunks  $C_{i_1}, C_{i_2}, \dots$  and  $C_{i_n}$ , ( $i_1 < i_2 < \dots < i_n$  and  $n \leq (k - m + 1)$ ) belong to flow 1, and the rest of the chunks  $C_{j_1}, C_{j_2}, \dots$  and  $C_{j_p}$ , ( $j_1 < j_2 < \dots < j_p$  and  $p \leq (k - m + 1)$ ) belong to flow 2. Thus for the virtual traffic parameter, we have

$$\begin{aligned} & \sigma_1 + \rho_1(Y_{i_n} - \max\{Y_{i_{n-1}}, \min\{X_{i_1}, X_{i_2}, \dots, X_{i_n}\}\}) \\ & \geq \sum_{z=i_1}^{i_n} S_z \end{aligned} \quad (6)$$

$$\begin{aligned} & \sigma_2 + \rho_2(Y_{j_p} - \max\{Y_{j_{p-1}}, \min\{X_{j_1}, X_{j_2}, \dots, X_{j_p}\}\}) \\ & \geq \sum_{z=j_1}^{j_p} S_z \end{aligned} \quad (7)$$

Since,

$$\begin{aligned} & \max\{Y_{i_n}, Y_{j_p}\} = Y_k, \\ & \min\left\{\begin{aligned} & \min\{X_{i_1}, X_{i_2}, \dots, X_{i_n}\}, \\ & \min\{X_{j_1}, X_{j_2}, \dots, X_{j_p}\} \end{aligned}\right\} \end{aligned} \quad (8)$$

=)  $\min\{X_m, X_{m+1}, \dots, X_k\}$  and

$$\min\{Y_{i_{n-1}}, Y_{j_{p-1}}\} = Y_{m-1},$$

We have,

$$\begin{aligned} & (\sigma_1 + \sigma_2) + (\rho_1 + \rho_2) \\ & \times (Y_k - \max\{Y_{m-1}, \min\{X_m, X_{m+1}, \dots, X_k\}\}) \\ & \geq [\sigma_1 + \rho_1 \times (Y_{i_n} - \max\{Y_{i_{n-1}}, \min\{X_{i_1}, X_{i_2}, \dots, X_{i_n}\}\})] \\ & + [\sigma_2 + \rho_2 \times (Y_{j_p} - \max\{Y_{j_{p-1}}, \min\{X_{j_1}, X_{j_2}, \dots, X_{j_p}\}\})] \geq \sum_{z=m}^k S_z \end{aligned} \quad (9)$$

*Application of Proposition 2:* If the function of all traffic flows are known, the virtual traffic aggregated function can be derived by Proposition 2.

However, the chunk pattern may be distorted at a relay node. In such case, we can provide the following relation for a flow in terms of worst case delay of the outgoing traffic.

*Proposition 3:* Assume that the traffic parameter of the input traffic of a SEND chunk flow at a relay node is  $(\sigma, \rho)$  and the worst case delay to traverse a relay node is  $D$  (let the mean service time of a chunk at this current node is  $d$ ). We can characterize the output traffic of this flow as  $(\sigma', \rho)$  where the buffer requirement is  $\sigma' = \max\{0, \rho(D - d) + L_{\max}\} + \sigma$ .

*Proof:* Assume that chunks are ordered by their delivery times at this current node, i.e., for chunks  $C_k$  and  $C_m$  ( $k \geq m$ ,  $T_k \geq T_m$ ) where the delivery order time tag of chunk  $C_i$ ,  $i = 1, 2, \dots$ , is  $T_i$  and is also the arrival time of  $C_i$  of the output traffic. As the worst case delay of a chunk is  $D$ , we have the following relation:

$$T_i \leq Y_i + D \quad (10)$$

Again, since the delivery order of each chunk is delayed by  $d$  and  $\sigma' = \max\{0, \rho(D - d) + S_{\max}\} + \sigma$ , for any two chunks  $k$  and  $m$  ( $k \geq m \geq 1$ ), we get

$$\begin{aligned} & \sigma' + \rho[Y_k + d - \max\{Y_{m-1} + d, T_m\}] \\ & \geq \sigma' + \rho[Y_k + d - \max\{Y_{m-1} + d, Y_m + D\}] \end{aligned} \quad (11)$$

$$\geq \min\{\sigma + \rho(Y_k - Y_{m-1}), \sigma + S_{\max} + \rho(Y_k - Y_m)\}$$

And

$$\begin{aligned} & \sigma + \rho[Y_k - \max\{Y_{m-1}, \min[X_m, X_{m+1}, \dots, X_k]\}] \geq \sum_{i=m}^k S_i \\ & \text{i.e., } \rho(Y_k - Y_{m-1}) \geq \sum_{i=m}^k S_i \end{aligned} \quad (12)$$

Now let the previous node's delivery order of a chunk  $C_i$ ,  $i = 1, 2, \dots$ , at the outgoing link of the relay node is:

$$Y'_i = Y_i + D + \left(\frac{S_{\max}}{\rho}\right). \text{ Thus from Eq. (11) and (12) we have:}$$

$$\begin{aligned} & \sigma + \rho[Y'_k - \max\{Y'_{m-1}, \min[T_m, T_{m+1}, \dots, T_k]\}] \\ & \geq \min\{\sigma + \rho(Y_k - Y_{m-1}), \sigma + L_{\max} + \rho(Y_k - Y_m)\} \\ & \geq \min\left\{\sum_{i=m}^k S_i, \sum_{i=m+1}^k S_i + S_{\max}\right\} \geq \sum_{i=m}^k S_i \end{aligned} \quad (13)$$

Thus the characteristic of traffic parameter for worst case Delay  $D$  is  $(\sigma', \rho)$  and proposition 3 is proved.

The virtual delay  $D$  to traverse a relay node by a flow of SEND system can be bounded by input, output rate of that flow, if not considered constant as before. Let,  $\delta(s) = \inf\{\tau \geq 0 : \alpha(s) \leq \rho(s + \tau)\}$  and  $h(\alpha, \rho)$  is the

supremum of all values of  $\delta(s)$  assuming  $\alpha, \rho$  are the input and output rate respectively and  $\alpha \leq \rho$ . Then the following relation provides a delay bound for a flow of SEND system to traverse a relay node.

**Proposition 4:** Assume a flow constrained by arrival rate  $\alpha$  traverses a relay node that offers a service curve  $\rho$ . The virtual delay  $d(t)$  for all  $t$  satisfies  $d(t) \leq h(\alpha, \rho)$ .

**Proof:** Consider some fixed  $t \geq 0$ ; for all  $\tau < d(t)$ , we have from virtual delay relation,  $R(t) > R^*(t + \tau)$ . The service rate property at time  $t + \tau$  implies that there is some  $s_0$  such that  $R(t) > R(t + \tau - s_0) + \rho(s_0)$ . It follows from this later equation that  $t + \tau - s_0 < t$ . Thus

$$\begin{aligned} & \alpha(\tau - s_0) \geq [R(t) - R(t + \tau - s_0)] > \rho(s_0). \quad \text{Thus} \\ & \tau \leq \delta(\tau - s_0) \leq h(\alpha, \rho). \text{ This is true for all } \tau < d(t) \text{ and} \\ & \text{thus } d(t) \leq h(\alpha, \rho) \text{ and proposition 4 is proved.} \end{aligned}$$

Next we analyze the worst case delay bound of all SEND

chunk flows to traverse a relay node.

**Proposition 5:** Let,  $C_k^i$  be the  $k$ th chunk of flow  $i$  and assume that the chunks are ordered according to their current node's delivery order time tag. Define  $\varepsilon_i = \max\{\min_{k \geq m+1} \{Y_{m-1}^i - \min\{X_m^i, X_{m+1}^i, \dots, X_k^i\}\}, 0\}$  where  $Y_m^i$  and  $X_m^i$  are the delivery time tag from previous node and the arrival time at current node of  $C_m^i$ ;  $S_{\max}$  be the maximum size of a chunk. Assume that the input traffic of a relay node consists of flows  $1, 2, \dots, v$ , whose traffic parameters are  $(\sigma_i, \rho_i)$  respectively and the capacity of the output link of

this node is  $l_c, l_c \geq \sum_{i=1}^v \rho_i$ . Under these assumptions, the worst case delay bound at a current relay node is:

$$\frac{1}{l_c} \left[ \sum_{i=1}^v (\sigma_i - \rho_i \theta_i) + S_{\max} \right] \quad (14)$$

**Proof:** For any chunk  $C_k$  if we assume  $m$  to be the (biggest integer  $k \gg m > 0$  such that  $Y_k < Y_m$  and  $T_k > T_m$  where  $Y_i$  and  $T_i$  are the previous node's delivery time tag and the delivery time of  $C_i$  at current node. Thus

$$Y_m > Y_k \geq Y_i, \quad \text{for all } m < i < k \quad (15)$$

$$T_k > T_i \geq T_m, \quad \text{for all } m < i < k \quad (16)$$

In other words,  $C_m$  is transmitted before chunks  $C_{m+1}, \dots, C_k$ ; however, its previous node's delivery time tag is greater than that of chunks  $C_{m+1}, \dots, C_k$ . Thus

$$\min\{X_{m+1}, \dots, X_k\} > T_m - \frac{S_m}{l_c} \quad (17)$$

Since,  $C_{m+1}, \dots, C_k$  arrive after  $T_m - \left(\frac{S_m}{l_c}\right)$  and depart

before  $C_k$  at the current relay node, we have

$$T_k = T_m + \frac{\sum_{i=m+1}^k S_i}{l_c} \quad (18)$$

Note that  $Y_i \geq X_i$  for all  $i = 1, 2, \dots$ , and thus

$$Y_k \geq Y_i \geq X_i \geq T_m - \left(\frac{S_m}{l_c}\right) \text{ for } i = m+1, \dots, k-1.$$

Furthermore we have the traffic

$$\begin{aligned}\varepsilon_i &= \max\left\{\min_{k \geq m+1} \{Y_{m-1}^i - \min\{X_m^i, X_{m+1}^i, \dots, X_k^i\}\}, 0\right\} \\ &= \min\{X_m^i, X_{m+1}^i, \dots, X_k^i\} + \varepsilon\end{aligned}$$

$$\text{function, } \leq \max\{Y_{m-1}^i, \min\{X_m^i, X_{m+1}^i, \dots, X_k^i\}\}$$

i.e.,

$$\sigma_i + \rho_i [Y_k^i - (\min\{X_m^i, X_{m+1}^i, \dots, X_k^i\} + \varepsilon_i)] \geq \sum_{j=1}^k S_j^i \quad (19)$$

Since, chunks  $C_{m+1}, \dots, C_k$  comprise the chunks of flows  $1, 2, \dots, v$ , we have

$$\sum_{i=1}^v \left\{ \sigma_i + \rho_i [Y_k^i - (\min\{X_m^i, X_{m+1}^i, \dots, X_k^i\} + \varepsilon_i)] \geq \sum_{i=m+1}^k S_i \right\}$$

i.e.,

$$\sum_{i=m+1}^k S_i \leq \sum_{i=1}^v (\sigma_i - \rho_i \varepsilon_i) + \left( \sum_{i=1}^v \rho_i \right) \left[ R_k - \left( T_m - \frac{S_m}{l_c} \right) \right] \quad (20)$$

From Eq. (18) and Eq. (20) we have

$$\begin{aligned}T_k &= T_m + \frac{\sum_{i=m+1}^k S_m}{l_c} \\ &\leq T_m + \frac{\left( \sum_{i=1}^v \rho_i \right) \left[ Y_k - \left( T_m - \frac{S_m}{l_c} \right) \right] + \sum_{i=1}^v (\sigma_i - \rho_i \varepsilon_i)}{l_c} \\ &\leq Y_k + \frac{S_{\max}}{l_c} + \frac{\sum_{i=1}^v (\sigma_i - \rho_i \varepsilon_i)}{l_c} \quad (21)\end{aligned}$$

If there does not exist such  $m$ , then  $C_1, \dots, C_{k-1}$  all leave the node before  $C_k$  and thus have

$$T_k = \frac{\sum_{i=1}^k S_i}{l_c} \leq \frac{\left( \sum_{i=1}^v \rho_i \right) Y_k + \sum_{i=1}^v (\sigma_i - \rho_i \varepsilon_i)}{l_c} \quad (22)$$

i.e.,

$$T_k - Y_k \leq \frac{\sum_{i=1}^v (\sigma_i - \rho_i \varepsilon_i)}{l_c}$$

$$\text{Thus the delay is bounded by } \frac{\sum_{i=1}^v (\sigma_i - \rho_i \varepsilon_i)}{l_c} + \frac{S_{\max}}{l_c}$$

and proposition 5 is proved.

*Application of proposition 3 and proposition 5:* The proposed propositions are straight forward for performance analysis. From the above relation, we can also characterize the outgoing traffic parameter of a relay node for a given propagation delay,  $E_p$ . Let  $E_p$  be the propagation delay of a

chunk of a flow of SEND system  $(\sigma, \rho)$  i.e., the propagation delay of a chunk from a relay node to the next relay node. Then the worst case delay of a flow of SEND system is  $D + E_p$  if this is the first relay node i.e., there is no update at

the previous node of this flow of SEND system. Here we assume that this is the first relay node and the flows arrive from the source directly to this node. In this case if all of the chunks of the flow are updated / serviced by an increment  $d$  at the relay node, then input traffic parameter for the next relay node is  $(\sigma', \rho)$  where buffer requirement is  $\sigma' = \max\{0, \rho(D + E_p - d) + S_{\max}\} + \sigma$ .

The delay bound of proposition 5 can further be tightened.

For instance, if  $\left( \sum_{i=1}^v \frac{\alpha_i}{l_c} \right) \rightarrow 0$ , then the worst case delay

bound would be  $\frac{\left( \sum_{i=1}^v \sigma_i + S_{\max} \right)}{l_c}$ . On the other hand,

if  $\varepsilon = \min_i \{\varepsilon_i\}$ , and the delivery time tag at the pervious node of all chunks are decreased by  $\varepsilon$ , then the traffic functions of all flows remain the same and the actual worst case delay bound from proposition 5 is

$$\frac{\left( \sum_{i=1}^v \sigma_i + S_{\max} \right)}{l_c} - \varepsilon. \text{ Therefore, it is possible to tighten the}$$

worst case delay as well in this instance. If all chunks' delivery time stamps at the previous node are increased or decreased by a constant at the entrance to a relay node, their delivery time remains unchanged. If all chunks' previous node's delivery time tag decreased by  $\varepsilon$ , applying proposition 5, for any chunk  $C_k$  we have the following:

$$T_k - (Y_k - \varepsilon) \leq \frac{\sum_{i=1}^v [\sigma_i - \rho_i (\varepsilon_i - \varepsilon)] + S_{\max}}{l_c} \quad (23)$$

i.e.,

$$T_k - Y_k \leq \frac{\sum_{i=1}^v [\sigma_i - \rho_i (\varepsilon_i - \varepsilon)] + S_{\max}}{l_c} - \varepsilon$$

i.e., the worst case delay is bounded by

$$\frac{\sum_{i=1}^v [\sigma_i - \rho_i(\varepsilon_i - \varepsilon)] + S_{\max}}{l_c} - \varepsilon$$

Now if we take the propagation delay into account, the increment for flow of SEND system  $n$ ,  $1 \leq n \leq v$ , should be

$$\frac{\sum_{i=1}^v [\sigma_i - \rho_i(\varepsilon_i - \varepsilon)] + S_{\max}}{l_c} - \varepsilon + E_{n,i} \text{ where } E_{n,i} \text{ is the}$$

propagation delay of flow of SEND system  $n$  to traverse the link between relay node  $i$  and its next adjacent relay node.

*Example:* In order to further analyze the proposed propositions, consider two cases.

Let two flows; flow 1 and flow 2 are contending for the bandwidth of a link with a capacity of  $\frac{2S}{l_c}$ . The reserved

bandwidths of the two flows are both  $\frac{S}{l_c}$ , and all chunks are of size  $S$ . However, the inter-arrival times of two consecutive chunks of flows 1 and 2 are  $l_c$  and  $\frac{l_c}{2}$ , respectively. Assume that the first chunks of both flows arrive at time 0, and the arrival time of the  $k$ th chunk of flow  $i$ ,  $i=1,2$ , is  $X_i^k$ , where

$X_i^k = (k-1)l_c$  if  $i=1$ , and  $X_i^k = \frac{l_c(k-1)}{2}$  if  $i=2$ . The

previous node's delivery time tag attached to the  $k$ th chunk of flow  $i$  is, however,  $l_c k$ , which is independent of  $i$  and will make each flow of SEND system attain its reserved bandwidth. Therefore, it can be observed that the worst case delay of flow 1 is  $l_c$ , and it is infinity for flow 2. However, if the previous node's delivery time tag of the  $k$ th chunk of flow  $i$ ,  $i=1,2$ , is set to  $l_c + X_i^k$ , then the worst case delays of both flows become infinity. We can observe such characteristic from the propositions we derived. The delivery order at the previous node attached to the  $k$ th chunk of flow 2 are i)  $l_c k$  and ii)  $l_c + X_i^k$  respectively. In the first case the traffic

parameters of the two flows are  $\left(0, \frac{S}{l_c}\right)$  and  $\left(0, \frac{S}{l_c}\right)$  i.e.,

same. By the aggregate property from proposition 2, we have

the traffic parameter of the aggregate flow as  $\left(0, \frac{2S}{l_c}\right)$  and

by proposition 5 the delay bound of any chunk is  $\frac{l_c}{2}$  since

$\varepsilon_1 = \varepsilon_2 = 0$ . Therefore, since the delivery order at the previous node of a chunk lags behind its arrival time, bounded

by  $l_c$  and infinity in flows 1 and 2 respectively, and then the worst case delay of the flows are  $\frac{3l_c}{2}$  and infinity

respectively. In the later case, the traffic parameter of flow 2 is  $\left(\infty, \frac{S}{l_c}\right)$ . Thus the aggregate traffic flow is

$\left(\infty, \frac{2S}{l_c}\right)$  and the worst case delay is infinity. Thus, we see

that the worst case delays of both flows become infinity according to our analysis.

Proposition 5 provides bound for all the flows in a relay node. If we know the guaranteed minimum throughput of a flow of SEND system  $\rho$ , then theorem 1 of [14] can be applied to define the end-to-end guarantee of a SEND system flow in the interval  $[t_1, t_2]$  as follows:

*Proposition 6:* If a flow  $v$  of SEND system provides lower-bound on throughput of the form:  $W_{v,H}(t_1, t_2) \geq \rho_v(t_2 - t_1) - \phi$  where,  $\phi \geq 0$ ;  $\rho_v, H$  are rate reserved for flow  $v$  and number of relay nodes along the path of  $v$ ; then its also provides to flow  $v$  an end-to-end delay guarantee of the form:  $R_{v,2}^k - EAT(C_v^k) \leq \frac{\phi + S_{\max}^v}{\rho_v}$ ,

where  $R_{v,2}^k, EAT(C_v^k), S_{\max}^v$  are delivery time stamp tag of chunk  $k$  of flow  $v$  from the second relay node, expected arrival time of chunk  $k$  of flow  $v$  and maximum size of the chunk of flow  $v$  respectively.

*Proof:* see Theorem 1 of [14].

From the above analysis using proposition 5, we can find the end-to-end delay bound for an IMS source to an IMS destination terminal using two relay nodes as:

$$\sum_{r_n=1,2} \left[ \frac{\sum_{i=1}^v (\sigma_{r_n i} - \rho_{r_n i} \varepsilon_i) + S_{\max}}{l_c} \right] + \sum_{i=1}^v \rho_{2i} E_{2,e} \quad (24)$$

Where,  $r_n$  represents the index of relay nodes,  $\rho_{2i}$  is the traffic rate of flow  $i$  reaching the end destination terminal  $e$  from the second relay node, and  $E_{2,e}$  is the propagation delay for a chunk of a flow to reach from the second relay node to the destination end terminal. Eq. (24) achieves the goal of our work in stateless work conserving situation. Note that the design and analysis of the above work are consistent with [14-16] with the traffic parameter behaving as virtual clock arrivals as shown in [17]. Li and Knightly [18, 19] provided analysis for multihop stateless scheduling, but the simplicity of our analysis is perhaps preferred to be deployed regarding virtual traffic flows.



## IV. CONCLUSIONS

Providing instant messaging in real time is indeed open challenge today. Previous works on relay nodes are centered on one node only. We have shown a complete end-to-end delay evaluation that includes two relay nodes (maximum number that a source MRSP terminal can select) for buffer non-blocking situation. In the analysis constant bit rate was considered to be consistent with Cruz's [12, 13] famous work conserving virtual traffic parameter model  $(\sigma, \rho)$ . With our model, the performance evaluation of end-to-end delay for large instant messages becomes straight forward.

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